Operations Management

Dr. Indedeep Singh
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 44
Sequencing Problems – II

[FL] Friends welcome to session 44 in our course on operations management and currently we are in the ninth week of discussion and we are currently discussing important topics related to production control. Now, as you are aware that there are different functions that fall under the category of production control and we are trying to understand each one of them and to name a few of them we are currently focusing on sequencing.

Then scheduling is another important function and expediting as the name suggests is speeding up our operations that is we need to speed up the way we are doing the work; when we are lacking behind our schedule and how to schedule that we will cover in the last session that will be focused on master production schedule during this week and in today's session our target is sequencing.

So, basically sequencing scheduling and expediting all are in relation to that time we have to see that how many machines are available with us, which machine is going to be assigned for a particular job and how to decide that which job will go to which particular machine and that will be based on the time required for processing on a particular machine.

That is our important we can say target during the sequencing of different jobs on machines which we have already covered in our previous two session. The first session was on a production controlled where we focused on the various functions that fall under production control. Then we focused our attention on the most important part that is sequencing that there are number of jobs number of machines how to decide that which job has to go to which machine.

Then we have seen different criteria that is usually followed in industry try to solve some problems related to the different criteria even one of the problem we compared the performance of the three different criteria which are used for sequencing and then we learnt the problem of assigning jobs 2 the machines based on a simple algorithm and there we solved one problem where 2 machines were available with us and we had 5 jobs which had to be sequenced on these 2 machines prior to that we solved a problem for one workstation also.

So, today our focus will be that we will be covering as you have seen the topic sequencing problems 2; our problem today will be that there will be 3 machines and an end jobs that will be available with us. So, we have n jobs that are already lying in our shop floor we have to a sequence these n jobs to these 3 machines, how would the sequence should be taken into account? How the sequence must be planned?

What will be the objective criteria here, now in any case there are 3 machines available with us there are maybe n jobs available with us we have to sequence these n jobs on these 3 machines, definitely this can be done even a person who has no knowledge of operations management can very easily solve this problem he can start distributing the jobs among various machines depending upon the order in which they have to be processed on these 3 machines and finally, at after some time maybe after a given time he will be able to solve this problem.

Now, what is the you can say importance in studying this topic that we are trying to scientifically, mathematically, logically plan that how these n jobs must be allocated or sequenced to these 3 machines. So, what can be our criteria our criteria is that we have to minimize the total elapsed time which is required for this 7 or maybe 8 or n number of jobs to be completed on given 3 machines in a specific order.

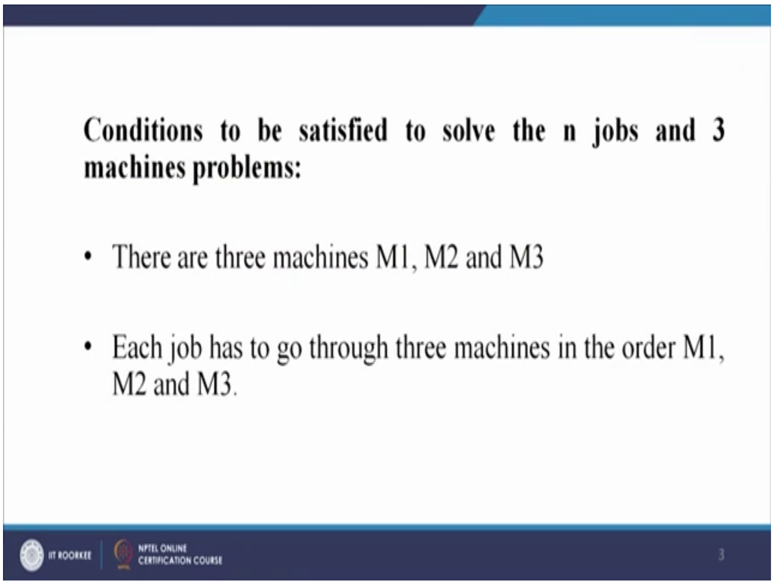
Also we want to minimize the idle time that the machines are having during the complete cycle when these jobs and jobs are sequenced on these 3 machines. So, two important performance criteria for this type of situation are the total elapsed time as well as the idle time for the various machines and both have to be minimized.

So, today our target will be that in the next maybe 20, 25 minutes we are able to understand the sequence or the steps that we have to follow in order to solve this problem of n jobs on 3 machines. So, we have already understood the importance of sequencing in our previous session. So, directly we will focus today on the problem that is the problem of n jobs and m machines.

So, here you can see it is not m machines m is equal to 3 in this case because we can also solve problems in which we have n jobs and m machines, but our focus today is m is equal to 3 that is n jobs and 3 machines and that is the problem which is our target today.

Now for this problem to be solved mathematically we need to have a criteria.

(Refer Slide Time: 05:40)

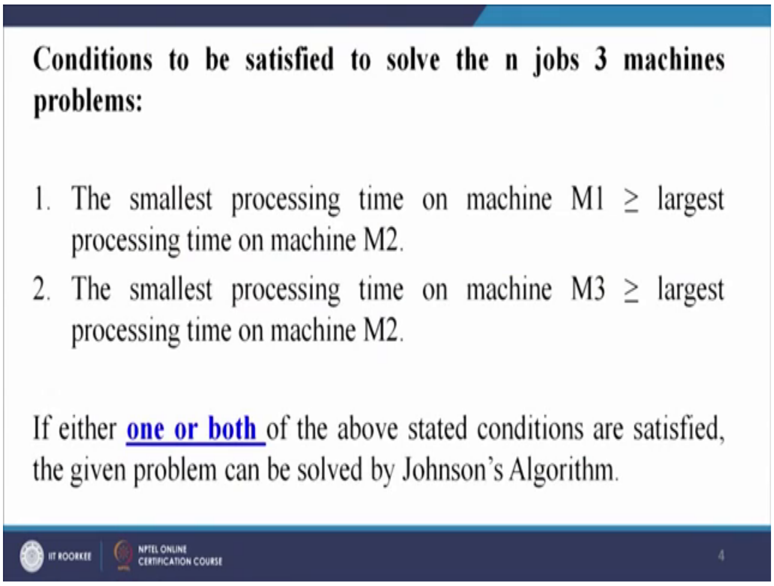


So, there has to be conditions to be satisfied to solve the n jobs and 3 machines problems, now these are the conditions or we can say assumptions that are there for solving this type of problem and the assumptions or conditions are that there are 3 machines available M 1, M 2 and M 3. So, the 3 machines problem has to be solved. So, 3 machines are available with us.

The next condition is that each job has to go through 3 machines in the order M 1, M 2 and M 3. Now if you remember our previous session we have seen that 2 machines and n number of jobs. So, n jobs the example that we have taken was that there are 5 jobs and 2 machines available and there also this condition was to be met that the sequence of operations have to be taken into account.

That is first the jobs will be processed on M 1, then the jobs will be processed on M 2, similar is the case here, here we can see that each job has to go through 3 machines and the order is also fixed that is M 1, M 2 and M 3. Now the conditions to be satisfied to solve n jobs 3 Machines problem these are another additional two conditions which have to be satisfied.

(Refer Slide Time: 07:05)



So, the first condition is the smallest processing time on machine 1, so as you remember we have seen that usually we have the data in the form of a table and we have the jobs in the first column maybe we can name the jobs as a b c d and e and f goes to maybe n and then in the next column we have machine 1 and what is the processing time for each job on machine 1 is given in first column and then in the second column we have machine 2 and the time or the processing time required for each job on machine 2 is also given.

Now, we can see in column 2 we can find out what is the minimum processing time for on machine1 for each and every job. So, each and every job means that which job has got the minimum processing time on machine 1 that we can find out in column one. Similarly, which job has the minimum processing time on machine 2, we can very easily locate that by looking at the values in column 2.

Now, this is a situation where we have 3 machines and n number of jobs. So, here we will have 3 columns in column 1 the processing times for all the jobs for machine 1, in column 2 the processing times for all the jobs for machine 2 and in column 3 the processing times for all the jobs for machine 3. So, we have 3 machines n number of jobs and the corresponding times are available in the form of a table.

Now, we have to focus on all the 3 columns and on each column we will see what is the minimum and the maximum processing time and that we have to compare here and 2 conditions have to be satisfied. So, condition 1 you can see the smallest processing time on machine 1. So, in column 1 whatever is the minimum value it will be specified for a specific job must be greater than or equal to the largest processing time on machine 2.

So, we are not currently considering the third column that is for machine 3, we are focusing only on the first 2 columns that is the processing times for various jobs on in column 1 that is for machine 1 and processing times for all the jobs for machine 2 that is column 2. So, we are focusing on column 1 and column 2 and column 1 minimum value may be suppose it comes for job c.

So, the shortest processing time in column 1 is for job c, it must be greater than equal to largest processing time on a machine 2. So, suppose that is for job d so, for job d the largest processing time is among the various jobs on a machine 2 the job d requires the largest processing time. So, here as per our first condition the shortest time required on machine 1 for any job must be greater than equal to the largest processing time required for any job on machine 2 we will try to see this with the help of an example also.

Similarly, there is second condition the smallest processing time on a machine 3, now we are focusing on column 3 and trying to find out the minimum value in column 3 that will represent to a specific job. So, the minimum or the smallest processing time on a machine 3 must be greater than or equal to the largest processing time on a machine 2. So, on in column 2 we have to focus on the largest processing time only and to a respective job we have to see that for which job we have the largest processing time in column 2 that is for machine 2 and we have to see the smallest processing time in machine 1 that is column 1 and similarly the smallest processing time for machine 3 that is in column 3.

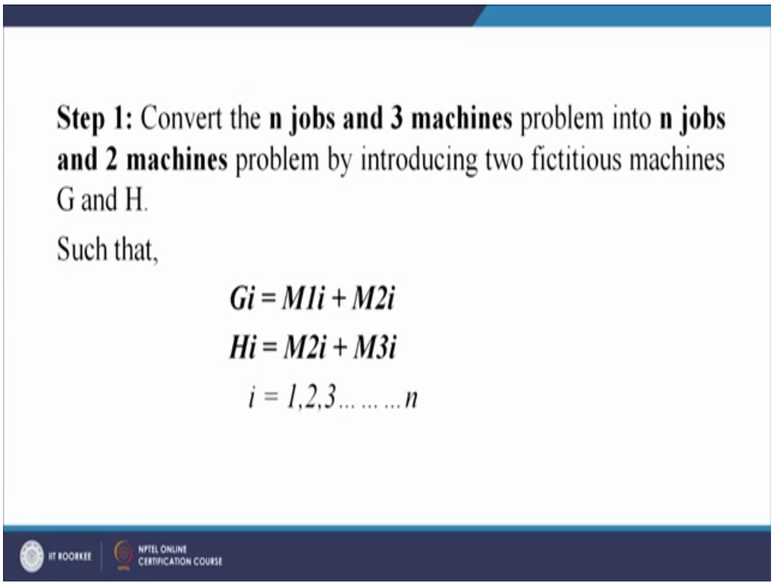
So, to simplify all the things we have to we can summarize that we will have the processing times for all the n jobs in call 3 columns column 1, column 2 and column 3, column 1 will be for machine 1, column 2 for machine 2 and column 3 for machine 3 and in each column we have to see the in first and third column we have to see the minimum processing time value and in column 2 we have to see the maximum processing time value.

So, if this much is clear now the next stage is if either one or both. So, it is this is important either if either one or both of the above stated conditions are satisfied, now out of these 2 conditions that is one and 2 either one or both if both are satisfied very good if even one is satisfied we will go ahead and solve our problem if either one or both of the above stated conditions are satisfied the given problem can be solved by Johnson's algorithm.

So, we will first see as per the data available with us whatever columns we have framed we will see minimum maximum values and compare them as per the 2 conditions mentioned here and try to figure out that whether both conditions are satisfied if even if one is satisfied we will solve the problem using the Johnson's algorithm, now what is Johnson's algorithm that we will see.

Now, the various steps that will be followed suppose now we are assuming that the conditions are satisfied at least one condition is satisfied.

(Refer Slide Time: 13:07)



Now we go to the steps convert the n jobs and 3 machines problem into n jobs and 2 machines problem by introducing 2 fictitious machines G and H. Now you can see that we have a problem at hand if in the problem we n jobs, now n jobs can n can be any number it can be 8, 10, 15, 20 so, n jobs and we have 3 different machines. Now, we are converting this problem into 2 machines problem which we have already solved in our previous session. Now 3 machines problem is now being converted into a 2 machines problem and what are what is the name of the machines any number any alphabet you can use, but in our problem we are using is G and H.

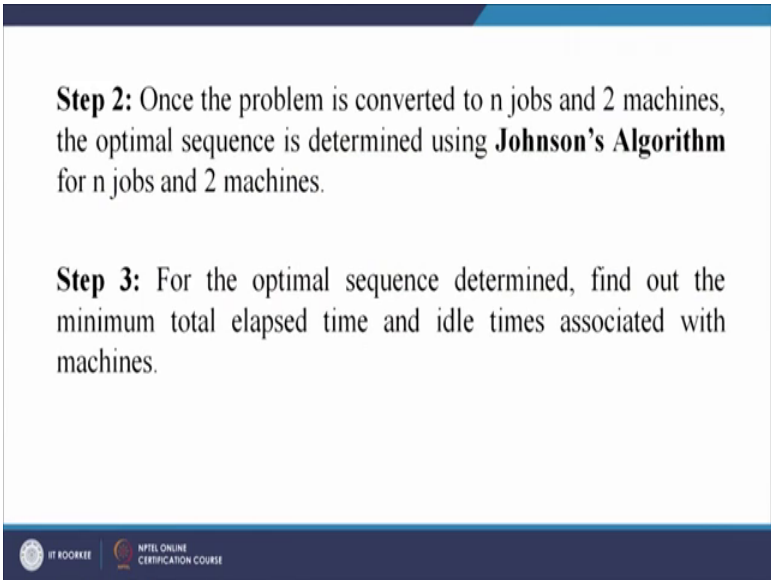
Now, what are these G and H 2 fictitious machines such that Gi that is for machine Gi the processing time will be the processing time for the ith job on M 1 plus the processing time for ith job on machine M 2. So, we will add the first 2 column the processing time in the first 2 column for the ith job now suppose i is equal to 1. For the first job in our problem we have 3 columns, processing times for on machine 1 column 1, processing times on machine 2 column 2 and processing times on a machine 3 column 3.

Now, our processing times are available in all the 3 machines, now suppose for job1 we want to convert it into a fictitious problem with G and H as the two fictitious machines we will say G is one machine and which requires how much time, we will add the column 1 value to the column 2 value that is the processing time required for job 1 on machine 1 and the processing time required for job 1 on machine 2. So, we will add those 2 values and we will get the time required on machine G.

Similarly, for H we will calculate summation of the time required for the same job on machine 2 and the time required for the same job on machine 3. So, we will then get these 3 machines data we will convert it into 2 machines data by adding for machine G first and second column value and for machine H the second and third column value and i can be I will then we again n because here we are solving a problem for n number of jobs to be processed on 3 different types of machine.

So, I think it is easier to calculate because only summation has to be done among the rows in the column for job 1 the first value added to the first the we can say the value in the first column first row plus the value in the second column first row that will give us the time required for machine G and similarly the second plus third value in the very first row the summation of these two will give us the time required for processing job 1 on machine H.

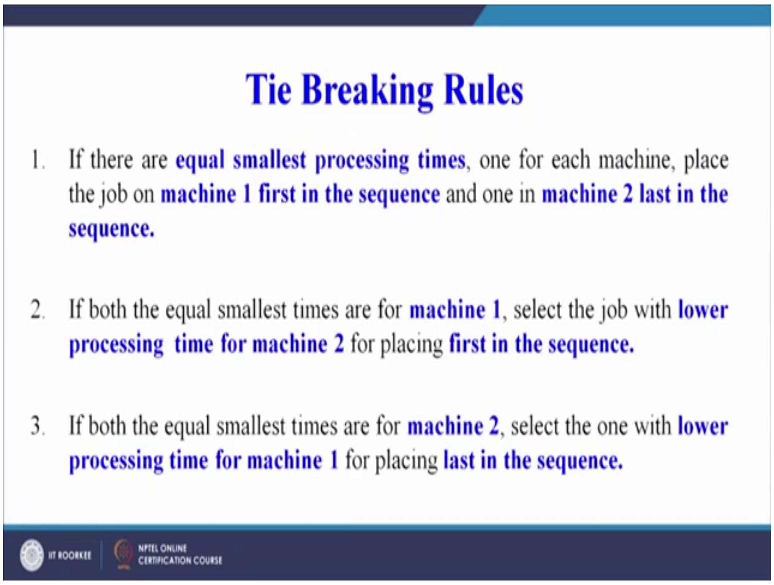
(Refer Slide Time: 16:30)



So, once we have done that, once the problem is converted to n jobs and 2 machines, now what are the 2 machines, 2 machines are G and H the optimal sequence is determined by the Johnson's Algorithm for n jobs and 2 machines which we have already seen in our previous session.

So, the problem becomes fairly simpler now for the optimal sequence determined find out the minimum total elapsed time and idle times associated with the machines which I have already told in the beginning of today's session that our target is to minimize the total elapsed time as well as the idle time for the various machines that are being used for processing.

(Refer Slide Time: 17:12)



Now, let us take important topic that is tie breaking rules many times it may. So, happen that in the same column we have two minimum values, now how to solve or how to break this tie. If there are equal smallest processing times, one for each machine; place the job on machine 1 first in the sequence and one in a machine 2 last in the sequence. Now you can see if there are equal smallest processing times one for each machine.

Now, we have 2 columns or 3 columns, now here we have converted the problem from 3 machines to 2 machines now suppose we focus on a 2 machine problem only. Now we have 2 columns and if suppose there are minimum values is same, for example, 3 hours 3 hours is available or the data is there in column 1 also and 3 hour data is in column 2 also. Now how to schedule so, what we will do?

We will see column 1 and wherever whatever job is corresponding to 3 hours we will schedule it from the left hand side we will schedule it first and for machine 2 whichever job is showing a processing time of 3 hours because 3 is equal to 3 we will schedule it at the end. So, that is you can say first tiebreaking rule when we have the same minimum processing time data or value in 2 columns representing 2 machines.

Now, the second tiebreaking rule is that if both are equal smallest times are for machine 1, now suppose in first column only which is representing machine 1 we have same value that is 3 hours and 3 hours, but 2 jobs are there each requiring 3 hours, but both requiring 3 hours on machine 1 how to break this tie we will look for the corresponding values or the time required for these 2 jobs on the right hand side that is on the machine 2 table or in column 2 select the job with lower processing time for machine 2 and place it first in the sequence.

Now, we have 3 hours and 3 hours in column 1 only, which means that on a machine 1 there are 2 jobs which have 3 hours processing time. So, what we will do, we will see the corresponding values in the second column that is for machine 2, now suppose it is 8 and 10. So, what we will do, which is the minimum value 8 is minimum is less as compared to 10. So, we will schedule this job which has processing time on machine 1 as 3 hours and processing time on machine 2 as 8 hours we will schedule it first or first we will sequence it first.

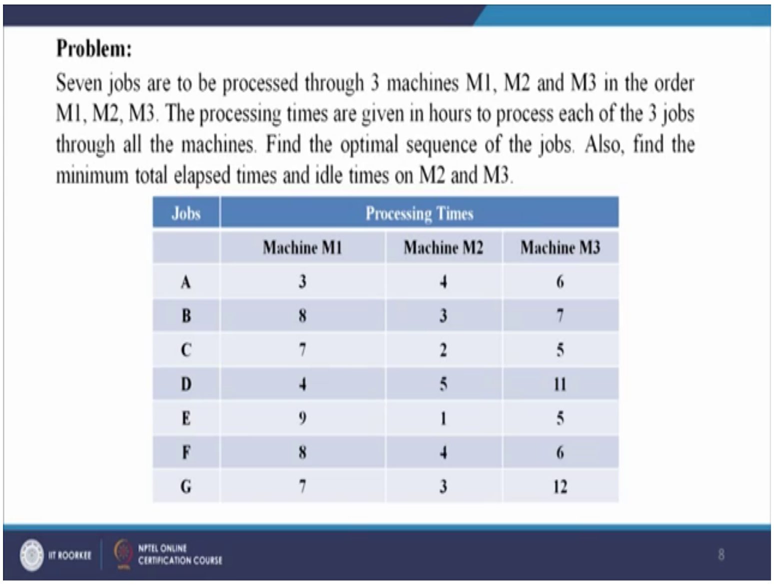
So, again I am reading point 2, that is tiebreaking if both the equal smallest times or equal smallest times are for machine 1 that is in column 1 select the job with lower shortest processing time that was 8 in my example for machine 2 for placing first in the sequence. So, in machine 2 8 was minimum. So, we will the job corresponding to 8 we will sequence first.

Now the next tiebreaking rule is that if both the equal smallest times are for machine 2 now again instead of column 1 now both the minimum or shorting shortest processing times that we find out are in column 2 only. Now what we will do we will see the corresponding values or the corresponding processing times for machine 1 and select one with a lower processing time for machine 1.

So, we will see that again this is reverse of point 2 we will again see the shortest processing times are lying in column 2, now that is for machine 2 we will see the corresponding values for machine1 and in the machine1 we will see which one is minimum and whichever is giving us the minimum value we will sequence it at the end so, that we will place towards the end that is in the last in the sequence.

So, these are the tiebreaking rules and we will now try to solve one problem in today’s session and here we will try to see that how we can solve a problem.

(Refer Slide Time: 21:39)



Problem:

Seven jobs are to be processed through 3 machines M1, M2 and M3 in the order M1, M2, M3. The processing times are given in hours to process each of the 3 jobs through all the machines. Find the optimal sequence of the jobs. Also, find the minimum total elapsed times and idle times on M2 and M3.

Jobs	Processing Times		
	Machine M1	Machine M2	Machine M3
A	3	4	6
B	8	3	7
C	7	2	5
D	4	5	11
E	9	1	5
F	8	4	6
G	7	3	12

Or convert a problem from n number of jobs with 3 machines to n number of jobs to 2 machines and then solve the problem using Johnson's algorithm. As per the problem you can see as I have been using this term columns again and again in this screen you can see that there are 3 4 columns. So, first column is representing the jobs and in this case we have 7 jobs which have to be sequenced and the processing times are given in the next 3 columns that is for machine 1, machine 2 and machine 3.

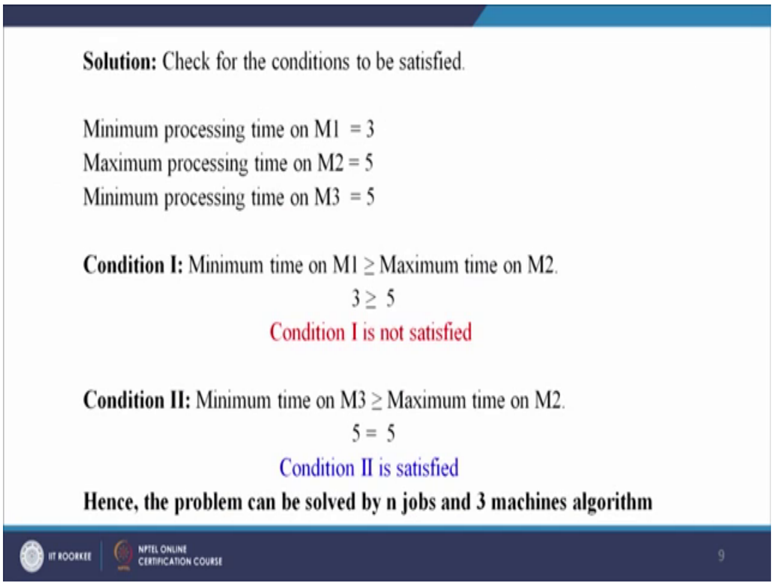
So, 7 jobs are to be processed through 3 machines M 1, M 2 and M 3 in the order is also specified which was one of the conditions that we have seen in the beginning of today's session that is M 1, M 2 and M 3. The processing times given in hours to process each of the 3 each of the 7 jobs through all the machines I think it is a typing error in the process each of the 7 jobs through all the machines. Now find the optimal sequence of the jobs also find the minimum total elapsed times and the idle time zone M 2 and M 3.

So, the 7 jobs we have to sequence on these 3 machines and what we have to minimize we have to find out the minimum time required to complete this project and we have to see that how much time the machines are idle. Now let us see, how we will solve this problem, now as you can see there are 3 machines first we have to check that whether our conditions are satisfied or not. The 2 conditions were the minimum value in this column we can say that is for machine 1 must be greater than or equal to the largest value for machine 2.

So, we can see first let us try to see what is the minimum processing time on machine 1. So, it is 3 so, for job A 3 hours is the minimum processing time. Now, let us focus on the second column machine 2 here we have to look for the largest value. So, you can see largest value is 5 here. So, is 3 greater than equal to 5, no so the first condition is not satisfied.

Now let us look at the second condition here we have to see the maximum value only that is 5, but here again we have to see the minimum value. So, in column 3 that is the processing times for various jobs on a machine 3 the minimum value is we can see 5 for 2 jobs. So, we have that value of 5 is the minimum. So, we can see that 5 is equal to 5 that is the maximum value for machine 2. So, it means that the condition is second condition is satisfied and this problem can be solved using the Johnson's algorithm.

(Refer Slide Time: 24:42)

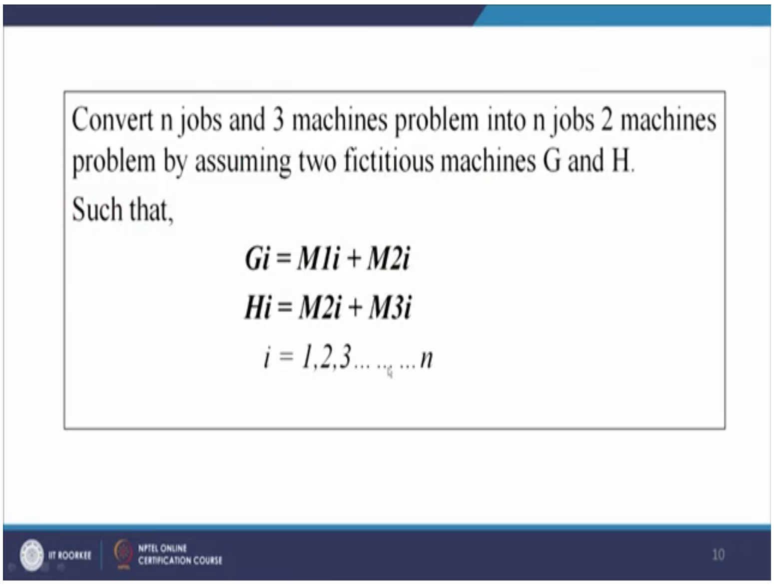


So, this is what I have already explained check further and patience to be satisfied first of all. So, the minimum processing time on M 1 is 3 for job A which we have seen here for job A the minimum processing time is 3 and the maximum processing time on machine 2 is for job D that is 5 maximum processing time on a machine 2 is 5 that is for job D and minimum processing time on M 3 is 5 for 2 jobs.

So, we can see that condition 1 minimum time on M 1 must be greater than or equal to the maximum time on M 2. So, minimum time on M 1 is 3 maximum time on M 2 is 5. So, 3 is not greater than equal to 5. So, condition 1 is not satisfied minimum time on M 3 is greater than or equal to maximum time on M 2. So, the minimum time on M 3 is 5 which is equal to maximum time on M 2 that is 5. So, condition 2 is satisfied.

So, as per our requirement either of the 2 conditions has to be satisfied or both the conditions have to be satisfied. So, at least one is satisfied therefore, the problem can be solved by n jobs and 3 machines algorithm. So, our condition is satisfied, now we will try to convert this problem into a 2 maybe n jobs and 2 machines problem how we have to do that we have to do it using this simple mathematics.

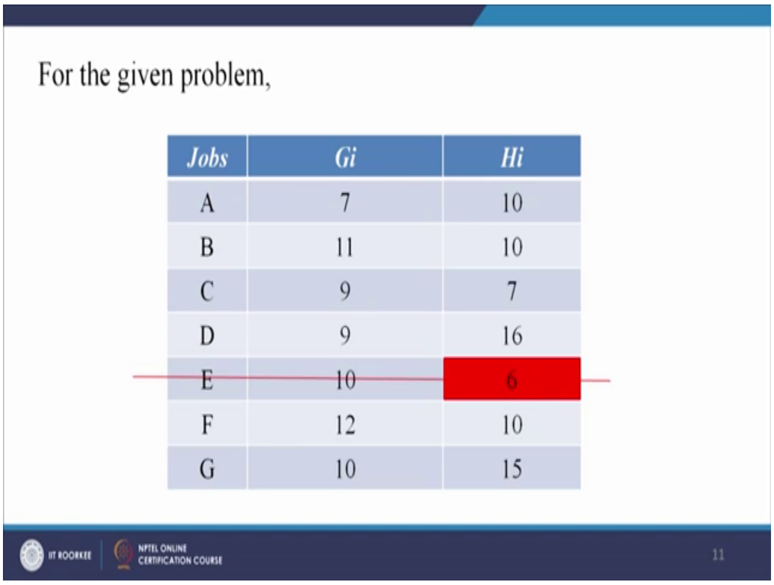
(Refer Slide Time: 26:17)



So, convert n jobs and 3 machines problem into n jobs and 2 machines problem by assuming 2 fictitious machines G and H. Now for G for first job what is the time processing time, processing time is the processing time required on machine 1 and processing time required on machine 2.

Similarly, for fictitious machine H what is the processing time processing time required is the processing time required on machine 2 plus the processing time required on machine 3.

(Refer Slide Time: 26:52)

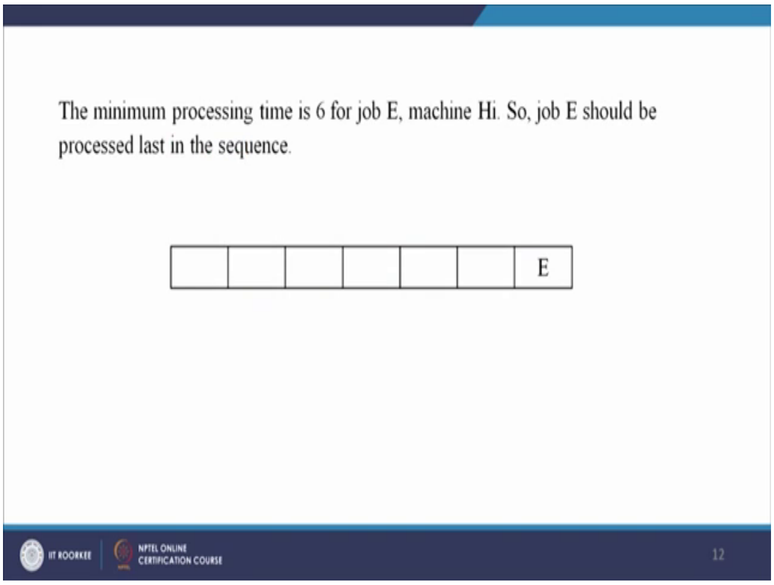


Jobs	Gi	Hi
A	7	10
B	11	10
C	9	7
D	9	16
~~E~~	~~10~~	~~6~~
F	12	10
G	10	15

So, here we can see we have now done the summation of first and second column for machine G and second and third column for machine H and we get these values. Now this is a simple problem here we can see we have 7 jobs processing times on a machine G and processing times for machine H.

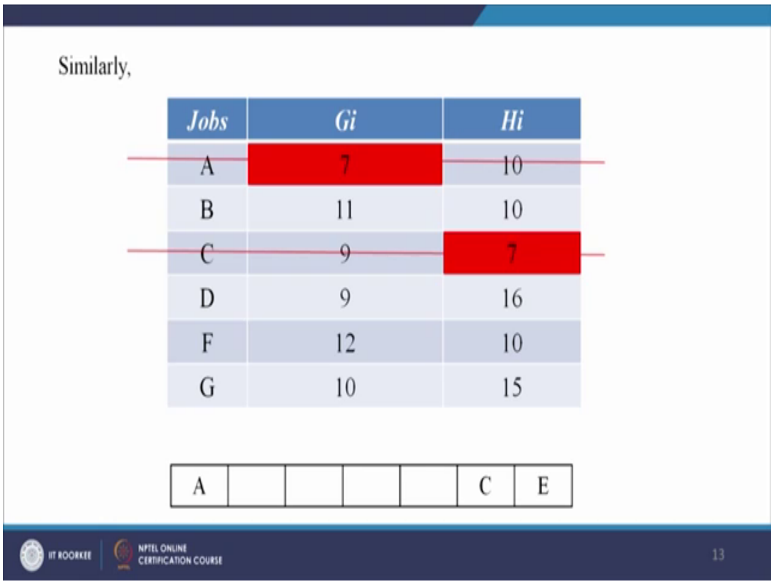
Now, let us see what is the minimum value here as per Johnson's algorithm, minimum value is 6 here, but that is in the second column that is for machine H. So, we will schedule it towards the end and we will strike off this machine E. So, first we will strike off this machine.

(Refer Slide Time: 27:30)



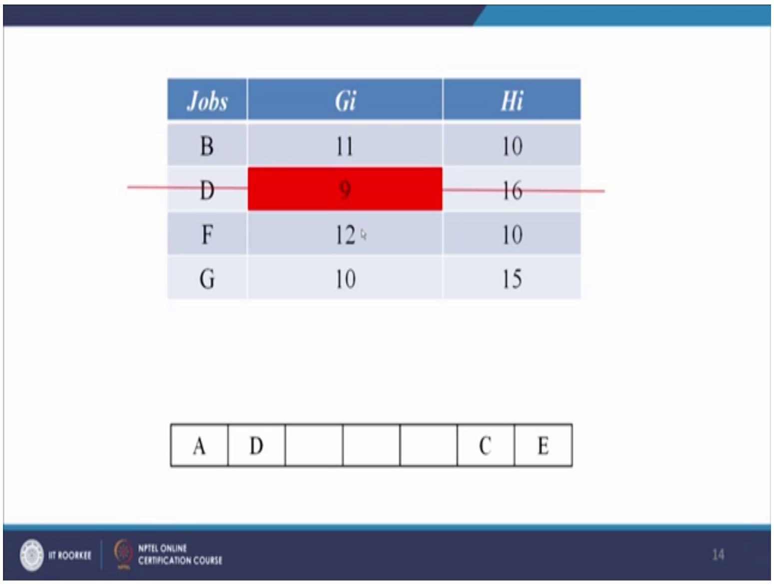
And we will schedule it at the end.

(Refer Slide Time: 27:34)



Now, again we will see whatever data is available with us now we see that here we have 2 minimum values that is for machine G job A require 7 hours and for machine H job C requires 7 hour. So, here we will see A and C both A is in column first. So, we will schedule it in the beginning and C is in column H that is the second column for second machine we will schedule it in the end.

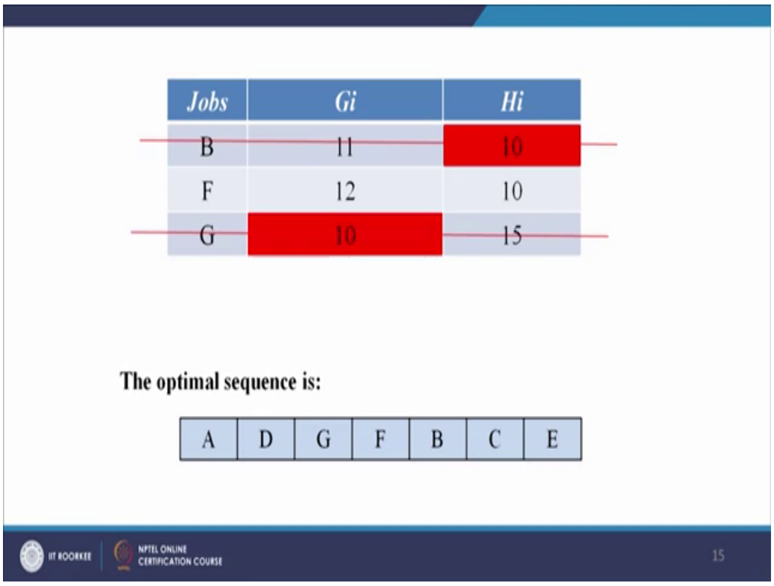
(Refer Slide Time: 28:11)



So, we will strike off these 2 jobs, now coming on to the next minimum value is 9.

So, there is no other 9 value so we it is in column 1. So, we will schedule D in the beginning for towards the start.

(Refer Slide Time: 28:22)



And strike off this and now we have 2 different values 10 and 10. So, we have to schedule these jobs G is in column 1 minimum value 10. So, we will schedule G in the beginning and H 10 value same, but it is in the second column we will schedule it towards the end so, we will strike off the 2.

(Refer Slide Time: 28:43)

Minimum total processing time to complete all the 7 jobs through all the 3 machines

Optimal Sequence	Machine M1		Machine M2		Machine M3		Idle Time M2	Idle Time M3
	In	Out	In	Out	In	Out		
A	0	3	3	7	7	13	3	7
D	3	7	7	12	13	24	-	-
G	7	14	14	17	24	36	2	-
F	14	22	22	26	36	42	5	-
B	22	30	30	33	42	49	4	-
C	30	37	37	39	49	54	4	-
E	37	46	46	47	54	59	7	-

And this way we can calculate the sequence of the jobs.

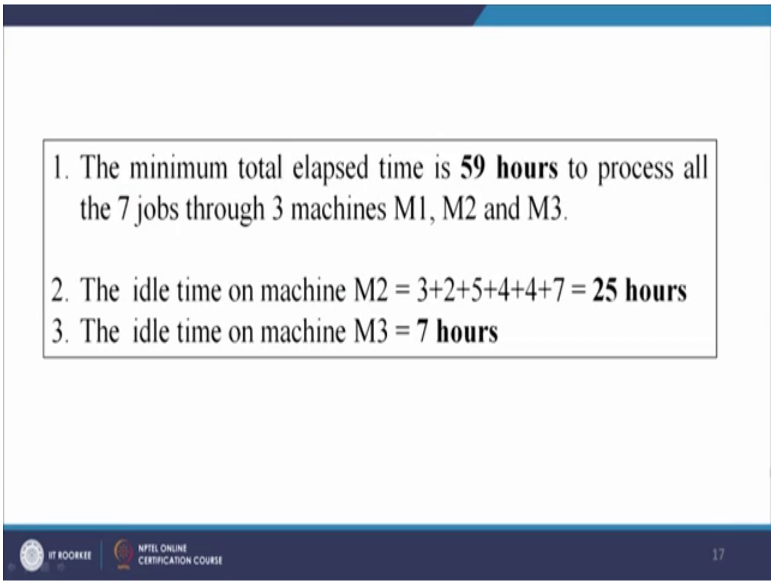
Now, once the sequence of the jobs is finalized we this optimal sequence we get this output ADG FBC E. So, once this is first column on your screen you can see if this is finalized column first column optimal sequence if this is finalized then we will start we know the order of the order is all jobs I have to first go to machine 1 then to machine 2 and then to machine 3 sequence already we know that what is the sequence to be followed very easily we can see that job a which is a first job to be scheduled enters machine 1 at time 0 it whatever time it requires, it requires 3 hours.

So, it is out at 3 hours and at machine 2 it enters at 3 hours just immediately as we have seen we have assumed that the transfer between the machines takes minimal time or maybe we can say negligible time it enters machine to after 3 hours and is out from machine to after 7 hours and enters machine 3 after 7 hours and gets out after processing for 6 hours in 13 hours.

So, that is our C, we can see a machine A the working of sorry job A not machine A the working on job A in 3 machines and similarly then D if scheduled when it will start on machine 1 after 3 hours because for first 3 hour job A is sequenced on machine 1 and similarly we will make all the calculations sequence the jobs as per our optimal sequence ADG FBG CE and finally, we can calculate the idle time here.

We can see idle time for machine 2 is 3 hours only why 3 hours because our job A took 3 hours on a machine 1 by that time it was idle and then finally, the job A went to machine 2 after 3 hours and therefore, it started it is after that it started it is functioning. So, this way we can do that what is the earliest time them is the job will go to a particular machine and from this data we can very easily calculate the idle time as well as a total elapsed time for completing processing of these 7 jobs in the optimal sequence on these 3 machines M 1, M 2 and M 3.

(Refer Slide Time: 31:18)



So, the minimum total elapsed time is 59 hours to process all the 7 jobs through 3 machines M 1, M 2 and M 3 the idle time on a machine, M 2 we can calculate as we have calculated here step by step we can calculate this time and here we can see that how let us take one value how 5 is the idle time for machine 2. So, we focus on machine 2. So, in machine 2 here we can see that 22 hours after 22 hours machine 2 is starting to process F whereas, the component that has left earlier that is component G was completed after 17 hour.

So, we can see that for 22 minus 17 for 5 hours the machine 2 was idle why it was idle because the processing of machine 2 on component G was over after 17 hours and because of the sequence that we are following we got this value. Similarly, how this number 7 is coming here we will see that machine 2 the component was started to process that 46 hours after 46 hour whereas, machine 2 was has finished the work assigned to it after 39 hour. So, this 46 minus 39, 7 hours idle time was there between the processing of component C or the job C and job E on machine 2, similarly for machine 3 also we can calculate the idle time.

So, we then we can add the idle time for each machine to calculate the total idle time. So, the idle time for machine 2 we can add up 3 plus 2, 5 plus 5, 10 18 plus 7 25 hours and idle time on machine 3 is 7 hours. So, friends with this I think we have learnt at least 2 simple methods of solving the up sequencing problems.

The very first we have seen the priority sequencing, what are the rules for priority sequencing and how we can solve the problems on a single workstation and in the previous two session today and the previous session we have covered how to solve the problem of n jobs on 2 machines and today how to solve a problem of n jobs on 3 machines and how to convert a n jobs 3 machines problem into n jobs 2 machines problem.

And finally, how to use a step by step procedure to sequence the various job how to calculate the idle time for each and every machine and how to calculate the total elapsed time. So, with this we close our discussion on sequencing and this is the fourth session in this week and in the last session we our target or our attention will be on master production schedule.

Thank you.