Operations Management
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Lecture – 43
Sequencing Problems – I

[FL] Friends welcome to session 43 in our course on Operations Management and currently we are in the ninth week of discussion and in ninth week our focuses on production control. And as all of you are well aware by now that in production control we have four important functions that have to be met or achieved and in these four functions we have to focus on loading, we have to focus on scheduling sequencing and expediting.

So, we have to follow these four functions in order to have a smooth manufacturing of our products and we are able to meet the deadline or the due date. And in that context we must be having all the requisite planning ready with us that which particular component or a part is going to be processed on which particular machine.

And for how long and which work center is assigned to which particular product or a product line that decisions have to be taken well in advance so, that when the actual product is processed on the shop floor we are able to meet the deadlines which have been identified or which have been contracted with the customers. And in that context if you remember we started our discussion in this week with of with a focus on production control. And we focused on the four important functions of loading, scheduling, sequencing and expediting it try to understand what is the meaning of all these terms.

In our previous session if you remember we focused on loading which means that once we have received an order suppose for producing 1000 components. So, we have to allocate this 1000 components to the various workstations depending upon the sequence of operations that have to be done on these components.

Accordingly we have to load the workstations properly. So, that we are able to take advantage of the overall capacity of the workstations that is one important point that we have to focus on loading. Also if you remember we have seen that loading has to be in terms of manpower also in many cases where we have to allocate the manpower to the

respective workstations in order to ensure the smooth production of the product or the components for which we are doing our manufacturing activity.

In the next session, our focus was primarily on trying to understand maybe the second session only the next part of the second session our focus was primarily to understand that what do we mean by sequencing we have tried to explain it with the help of a queue in a bank where the customers are served one by one. The first customer who stands in the queue is served first followed by the next and the sequence continues like that.

So, we have tried to understand that if there are ten or 15 different products or a our components that are waiting to be processed on a particular machine in which priority or in what priority they may be sequenced to be processed on that machine. And in that context we have tried to understand that what can be the factors, what can be the criteria which must be kept in mind while deciding the appropriate priority sequencing.

And we have tried and we have seen 3 or 4 important priority criteria or priority methods which are used; the first one was the first come first serve then the earliest due date shortest processing time and the critical ratio of method. So, four different criteria we have seen and today we will try to understand the criteria with the help of certain numerical problems.

Also we will try to understand the problem of number of jobs being assigned to 2 machines and try to see how the jobs needs to be sequenced in order to optimize or minimize the total time spent for processing these two parts or in order to minimize the idle time of either of the machines our focus will primarily be to understand this type of simple problem in which maybe any jobs maybe 1, 2, 3, 4, 5 jobs have to be sequenced on machine 1 and a machine 2. So, that we are able to optimally you complete the process as well as minimize the time required for this overall activity or manufacturing activity.

So, with this we start today's session and on your screen we can see the title for today's session is sequencing problems. And today is the first session focusing on problems and in next session also we will try to look at certain advanced problems related to the problem of sequencing.

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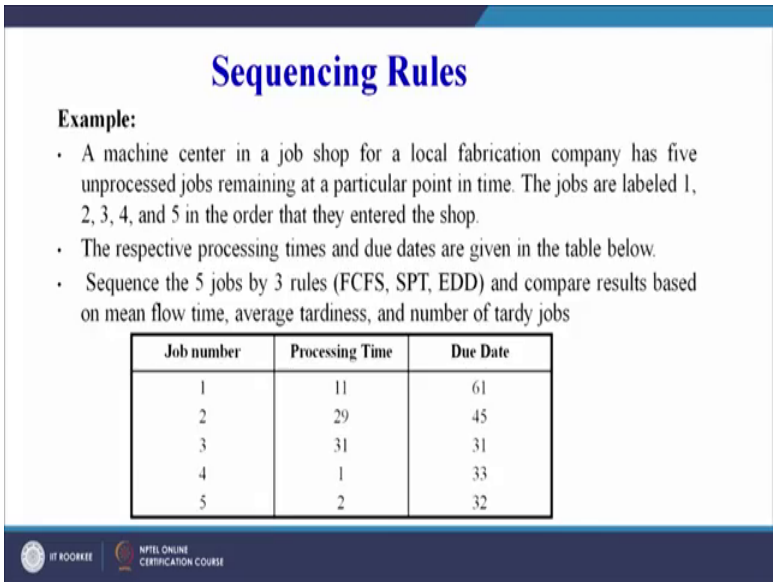

The first example on your screen you can see which is on sequencing rules. So, the example is a machine center in a job shop for a local fabrication company has 5 unprocessed jobs remaining at a particular point in time. So, there are 5 jobs which still need processing the jobs are labeled as 1, 2, 3, 4, 5 in the order of that they entered the shop.

Now this sentence is very important that is they are labeled as 1, 2, 3, 4, 5 token numbers are given based on the order in which they entered the shop. So, I request all learners to remember this sentence that 1, 2, 3, 4, 5 is based on the num the order in which they arrived at the shop.

So, basically these first two three sentences mean that there are 5 jobs that need to be processed and there is a machining center in a job shop type of fabrication company. So, there is one machine and there are 5 jobs that need to be processed. The respective processing times and due dates are given in the table.

So, we can see the table on your screen we have job numbers 1, 2, 3, 4, 5 in the order they arrived at the shop floor processing time is 11, 29, 31, 1 and 1 and due date is 61, 45, 31, 33 and 32. So, basically the date it is not exactly the date it is the time which is remaining or which is the due time for delivery which can be in number of hours. So, we can see the time is the due time by which the jobs must be completed.

Now, what is the problem ? The problem is that we have to sequence these 5 jobs by three rules. Now three rules we have covered if you remember in the previous session that is first come first serve FCFS shortest processing time SPT earliest due date EDD and compare results based on mean flow time, average tardiness and number of tardy jobs. So, tardiness also we have covered in the previous session it is a positive value and gives a difference between the two important dates that is the completion date of the project or the job as well as the due date of the job. So, the difference between the two the positive difference between the two is considered as tardiness.

So, we know what is tardiness? We know when once we have a value for a particular job a positive value that we will call for that job we will called as a tardy job. So, if the value of tardiness is 0; so, the job is not tardy job. So, we have to calculate the tardiness for each job, we have to calculate the average tardiness value based on that we can see which are the tardy jobs and which are the due the jobs which are completed early or on due date and finally, we have to calculate the mean flow time.

Now, mean flow time basically is the time that a job spends on the shop floor by from the time of its arrival to the time when the processing is finished on that job the. For example, a job arrives in a particular shop floor on September 21 and it leaves the job leaves the shop floor after finishing on September 28. So, we can see the mean the flow time for that for that job particular job is the difference between September 28; when it is finished from the day when it arrived that is September 21 that is the flow time for that machine for that job.

Now, it may so, happen that the job is in the shop floor for 2 days and no processing has happened on that job. So, for 2 days it is in the shop, but no processing has taken place on that job. So, the processing has taken place for maybe 3 days or 5 days on that job. So, the total times spent by the job on the shop floor when the order has been received. And by the time the job is finished by the last machine which has to work on it. So, that basically we call as the flow time.

And our overall objective is to minimize the average flow time of the various job. So, that is very important if the flow time or the average flow time is less; it means our system our machines are very responsive and the jobs need not wait for a longer period for processing. So, we can see that if we are able to minimize the flow time; we are

trying to make our system more and more responsive. So, here the problem at hand is that we have to complete 5 jobs processing time is given for each of the job and the due time or the due date is also given.

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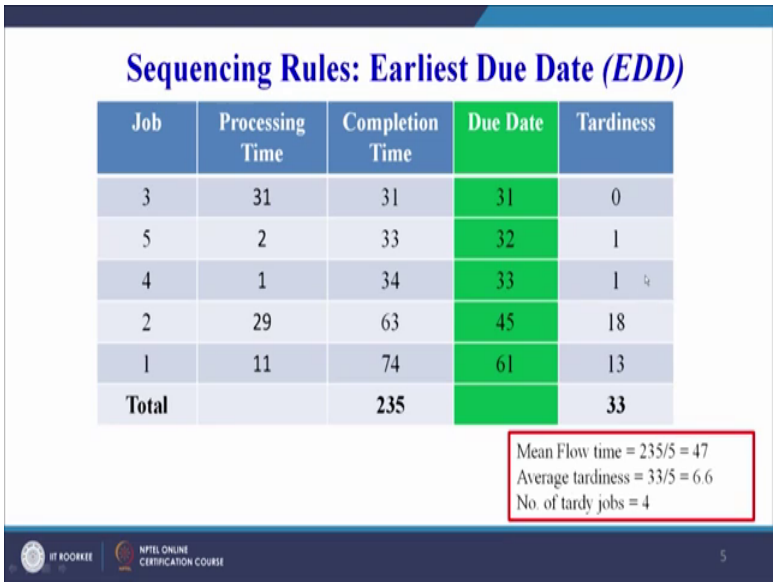

Now, let us see sequencing rules first rule that we are covering is first come first serve. So, in the problem statement if you remember we have seen the jobs are numbered based on the time they arrived at the shop floor. So, the first job arrived at the shop floor first and the fifth job came to the shop floor in the end. So, 1, 2, 3, 4, 5 is arranged in that sequence.

So, the first job will be served first because first come first serve rule we are using. So, the processing time is 11 days here. So, for the job one starts on a machine one completion time will be 11 and due date is 61. So, the tardiness value is 0.

Now, for job 2 which has to be sequenced because it arrived at the shop floor and second in number. So, it will start the machine will start or the machining center will start processing after 11 days the processing time is 29 days because for the first 11 days or the 11 hours suppose we take the time as hours.

So, the job one is being processed on the machining center. So, 11 the time unit 11 is spent their 29 units of time are spent for job 2. So, we add 29 plus 11 completion time for

job 2 is 40 time units due date is 45 time unit. So, we can see it is not a tardy job tardiness value is 0

Similarly, then third job will be sequenced processing time is 31 completion time will be it starts after 40 time units 40 plus 31 is 71 time units, but the due date is 31 time unit. So, we can see the tardiness is 40 similarly we can sequence all the 5 jobs one after the another and calculate the start time the finish time and based on the due date we can calculate the value of tardiness as we have seen in the previous session.

And therefore, finally, we can calculate the mean flow time which is the completion based on the completion time 268 divided by 5 jobs average is 53.6 average tardiness value 40 plus 39 plus 40 two that is 121 divided by 5 which is 24.2 and then the number of tardy jobs in this case is 3.

So, that is we can say the general calculations for the first come first serve rule. Now here we can see the second rule that is a shortest processing time. So, we have arranged all the jobs based on the shortest processing time in the increasing order. So, job four requires one time unit job 5 requires two time units and the maximum time required for processing is by job 3 that is 31 time units.

So, similarly we will see first we will sequence job number 4 it will require one time unit completion time it starts on day 0. So, the completion time is one time unit due date is 33 time unit. So, the tardiness is 0 in this case now the job 5 will start after 1 time unit it will require 2 time unit. So, 2 plus 1 3 completion time is 3 time units due date is 32. So, tardiness is again 0.

So, in this way step by step we will calculate and finally, we see that 4 jobs have a tardiness value of 0 only job three has a tardiness value of 43 and the completion time is 135 time units accordingly we can calculate the mean flow time the average tardiness value and the number of tardy jobs in this case.

And finally, the third rule that we are going to follow is earliest due date. So, earliest due date as we know here we can arrange it in increasing order. So, the earliest due date is for job number 3; the latest due date is for job number 1 and accordingly the processing time is given for each job. So, for job number three processing time is 31 time units completion time is 31 because this is a first job to be sequenced due date is also 31. So,

we can say the tardiness value is 0 here the next sequenced job is job number 5 processing time is 2 unit it will start after 31 time units. So, 31 plus 2 is 33 and the due date is 32. So, we can say tardiness value is 1.

So, similarly we can calculate the difference between the due date and the completion date and calculate the tardiness for each and every job. And the average tardiness value we can calculate mean flow time we can calculate and the number of tardy jobs in this case is 4.

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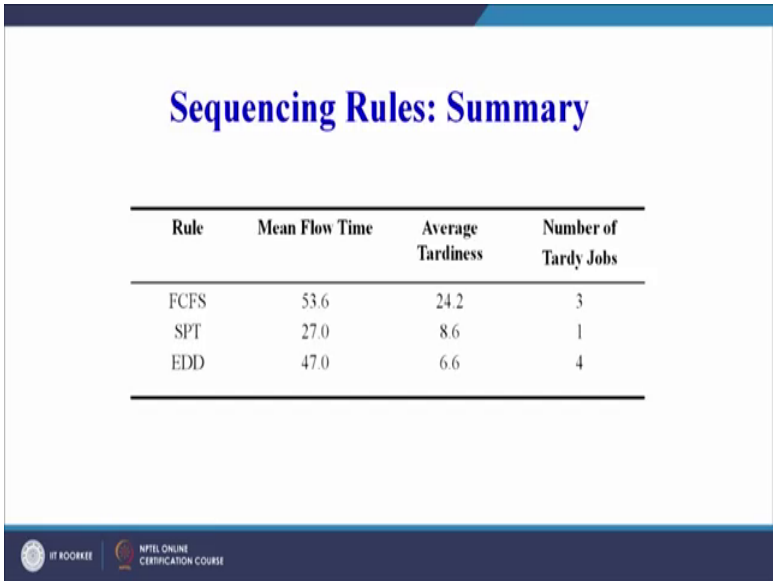

. So, we can see this is a summary the first come first serve method mean flow time is 53.6 time units earliest due date is 47 time units. Similarly the number of tardy jobs if we see is maximum in case of earliest due date and minimum in case of shortest processing time the average tardiness is 8.6 in case of shortest processing time.

So, we can see we have taken three different rules for sequencing the jobs on a single workstation and try to calculate the response criteria or we can say try to calculate the measures based on which we can compare. So, these measurements or calculated values are of mean flow time average tardiness and the number of tardy jobs.

In certain scenarios it may so, happen that we cannot afford to delay the delivery of our jobs. So, we will focus on the number of tardy jobs in that scenario that there should be no job which is delayed. So, in this case in each of the three rules whatever rule we

follow at least one job is delayed. So, we can focus on maybe shortest processing time rule because only one job is delayed here in case of earliest due date 4 jobs are tardy jobs and in case of first come first serve 3 jobs are tardy jobs.

So, if our due date is very very crucial very very important and religiously we have to follow the due date; we can go for shortest processing time the sequencing rule because only one job is tardy in this case. And also if we want to see the responsiveness of the system the shortest processing time the mean flow time is less or may be the minimum out of the 3 sequencing rules.

So, if you can see mean flow time is 27 time units in case of shortest processing time. So, the system is also more responsive number of tardy jobs is also less moreover the average tardiness value is also reasonable and is less than first come first serve basis. So, we can say that based on these values we can it will these values will help us to find out that which sequencing rule we must follow for our jobs so, that we are able to meet our deadlines.

Now, the next stage of problems is a sequencing problem where we can have different number of machines different number of jobs. So, no machine can process more than one job at a time.

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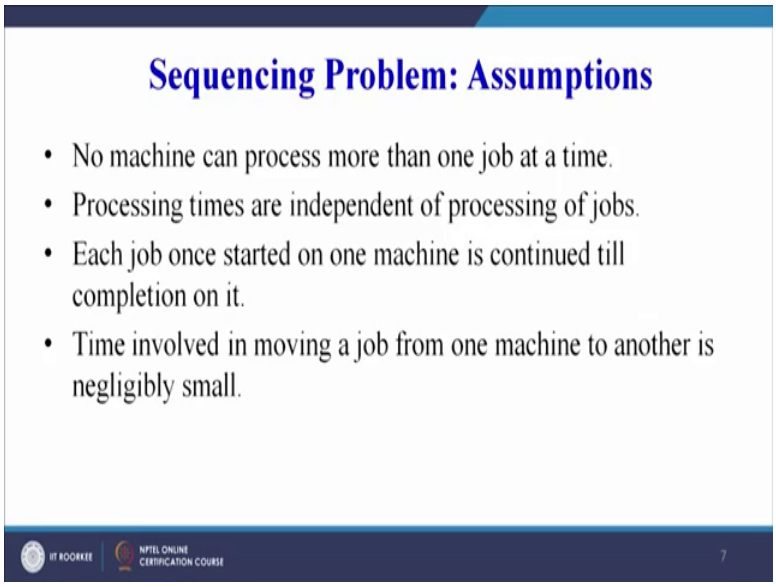

This is assumption of our sequencing problem processing times are independent of processing of jobs; each job once started on one machine is continued till completion not of on that machine. We cannot change may be cannot stop the process in between suppose a job has to be processed on a particular machine for maybe 5 hours. So, we cannot stop it after 2 hours and then give a gap and then again start after some time and then finish it for another 3 hours that is not possible time involved in moving a job from one machine to another is negligibly small.

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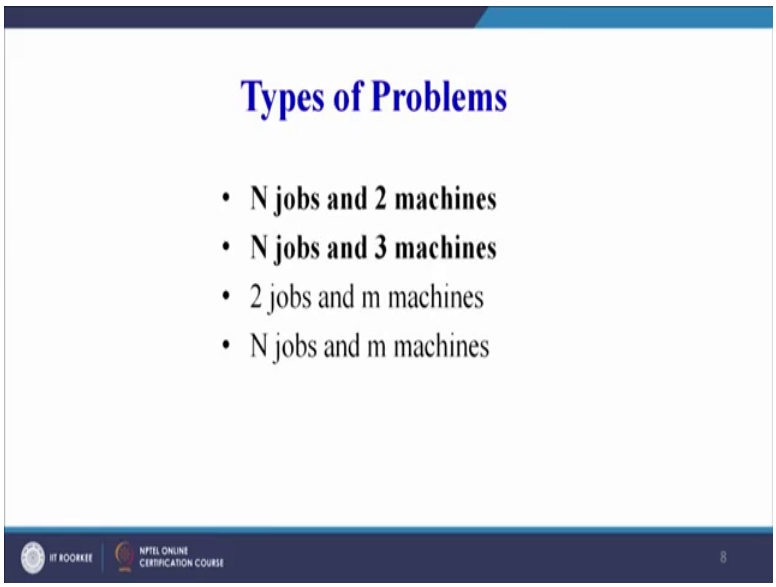

So, these are certain assumptions based on which we are we can try to solve the sequencing problems.

Now different types of problems can be considered when we are doing the sequencing of jobs on machines or on the machining centers or the work centers. Now the problem can be that we have N jobs for example, in the previous case that we have taken or the previous example that we have taken there were 5 jobs. So, N jobs and there are two machines; in our previous case if you see we had only one machining center or one work center.

Here the problem can be N number of jobs to be sequenced on 2 machines then N number of jobs can be sequenced on 3 machines; then 2 jobs and M number of machines are there then N jobs and M number of machines that is a very very general problem N number of jobs which is a variable and m number of machines which is again variable.

So, we will try we have try to highlight two particular scenarios here within the limited time that is available with us. We will try to address these two sequencing issues in our sessions. So, today now onwards I will cover the first issue that is N jobs have to be sequenced on 2 machines and in the next session our focus will be N jobs to be sequenced on 3 machines.

Now, let us see now N jobs on 2 machines how to solve this problem? So, we will try to explain it from two points of view; first one is the steps that need to be followed in a general problem and then taking a data and trying to solve that problem.

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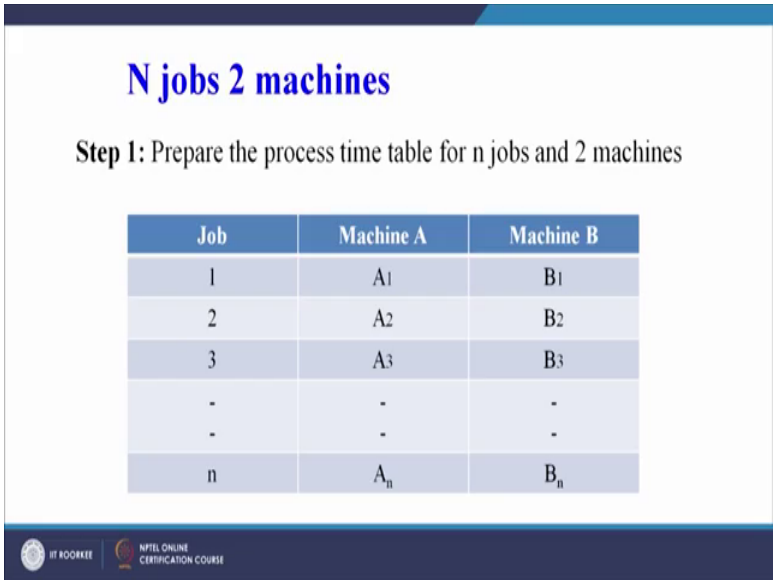

So, the step one is first we have to prepare the process time table for N jobs on two machines. Now we can see jobs we have to arrange in a tabular form n number of jobs we can take 4 jobs eight jobs twelve jobs whatever the number of jobs then machine A as we have taking n jobs and 2 machines.

So, 2 machines machine A and machine B or it can be machine 1 and machine 2. So, we are two different machines and A 1, A 2, A 3 to A n and B 1, B 2, B 3 to B n represents the processing time that is required for job 1 on machine 1 and machine 2 or machine A and machine B. So, this A 1, A 2, A 3 and B 1, B 2, B 3 represent the processing times.

Now, what do we have in this problem? We have number of jobs we have 2 different machines and we have the processing time required for each job on machine 1 and

machine 2 identify the given technical order. So, that can be basically the sequence that has to be followed for example, the simplest can be that first the operation has to be done on a machine 1 followed by the operation on machine 2.

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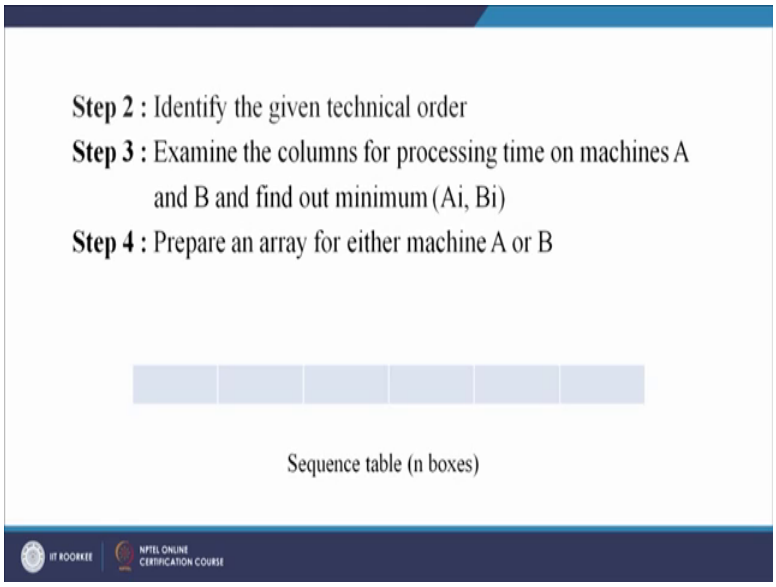

Examine the columns for processing time on machines A and B and find out the minimum that is A i and B i. Now we have two problems we can see again these are the two columns machine A and machine B and these represent the processing times for different jobs on machine 1 or machine A and processing times on machine 2 for the different jobs. For example, job 1 may require A 1 as the machining time or process time on machine A and B 1 as the machining or the processing time on machine B.

Now, we have to look at these two columns and try to figure out the minimum processing time in column for machine A and in column for machine B. So, that we can call as A i and B i; so, in this column first we have to identify the minimum value or the minimum processing time and then here also we have to check what is the minimum processing time.

Now, suppose examine the columns for processing time on machines A and B and find out the minimum values A i and B i certainly we will get one value A i which can be common also make a 2 machines may 2 jobs may have the same time. So, we have to look at that time which is the minimum time in column for machine A and minimum time for column for machine B and this time is the processing time for various jobs.

Now, prepare an array for either machine A or B this type of an array we can produce I think it is clear on the screen; however, we have 5 jobs here or maybe 10 jobs here. So, accordingly we can prepare an array like this n boxes because we are talking of n jobs to be scheduled or sequenced on 2 different machines. So, for either of the machine we can make this type of array.

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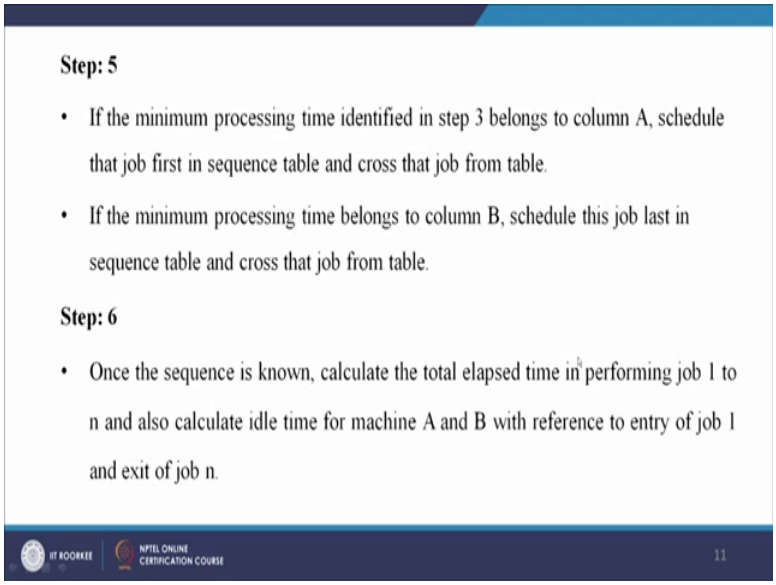

Now, the important thing is we know the minimum processing time on machine A, we know the minimum processing time on machine B for the various jobs. If the minimum processing time identified in step three belongs to column a schedule that job first in sequence table and crossed that job from the table. So, if for example, our minimum processing time is for machine A in column 1 or the column for processing times for machine A we will select that particular job for which we have the minimum processing time and we will schedule that job on machine 1 and cut that from the table also from the machine 2 also.

Then if the minimum processing time belongs to the column B that is for machine B; the value of minimum processing time we can find out. We have to look at all the values and try to figure out which is the minimum processing time; it can be in column A or it can be in column B. So, if it is in for machine column A then we have to schedule it from the left hand side. Suppose the minimum time comes out to be in the processing times for machine B; so, we have to schedule it from the end that is from the right hand side.

So, if the minimum processing time belongs to the machine B schedule this job last in the sequence table and cross that job from the table. Now once we have done this step has to be done again and again crossing the jobs which have been scheduled, and we can then fill our sequence table and that sequence we have to follow and then calculate further that what is the total time elapsed for completing these n number of jobs in these 2 machines. So, sequence we can find out using these steps; further calculations we can do and find out that what is the total elapsed time, what is the idle time for machine A, what is the idle time for machine B ?

So, we will try to understand this with the help of an example.

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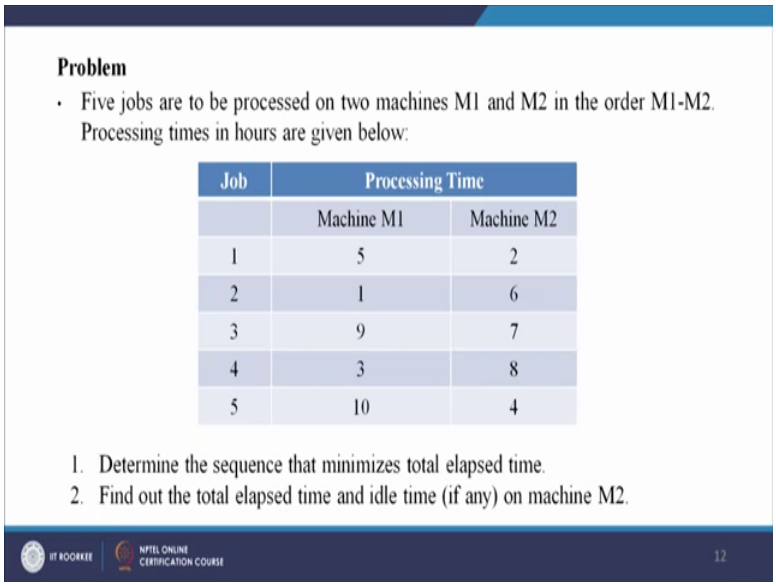

Now here we can see 5 jobs are to be processed on two machines M 1 and M 2 and in the order M 1 M 2. So, all the jobs first will go to M 1 and then after completing or completion on M 1, the job will move to M 2.

Processing times in hours are given below we can see the table gives the processing time job one on machine 1 it requires 5 hours and on machine 2 it requires 2 hours. So, what we have to do? The table is given there are 5 jobs which you have to be sequenced on 2 different machines the processing time for each job on each machine is given and what we have to find out we have to find the sequence that minimizes the total elapsed time and find out the total elapsed time and idle time on a machine 2. So, first is we have to

find out the sequence, after that we have to find out the total elapsed time and then we have to see that which are the idle jobs.

Now, here we can see the minimum processing time we can find in this column in a machine 1; it is 1. So, that is for job 2 and in this table we will put since it is for machine 1. So, we will start from this end and we will schedule or sequence job 2 first.

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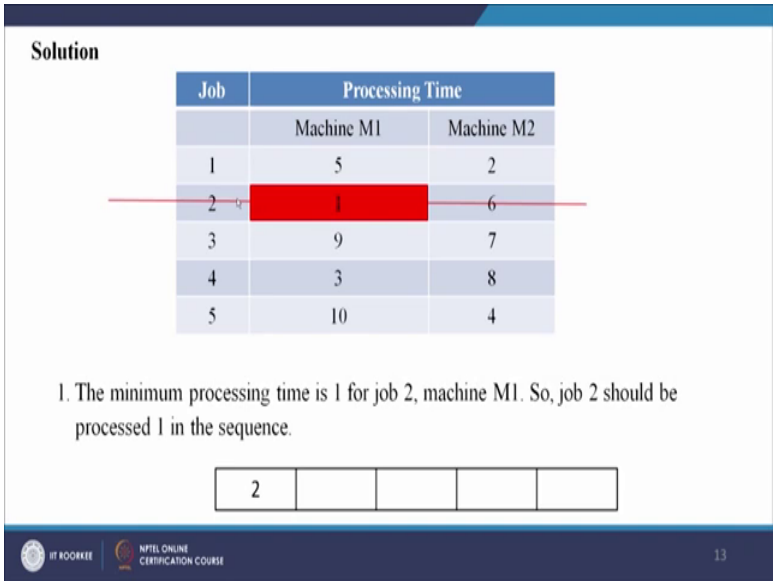

So, we can see job 2 minimum time processing time is 1 hour. So, we will schedule and we will cross this line here.

So, this is cut now then again we will look that which is the minimum processing time. So, now, you can see the table the minimum processing time now is 2. So, we will schedule now job 1 and it is in column 2; that is for machine 2. So, job 1 will be sequenced from the right hand side if I go back. And maybe just we can see this if the minimum processing time belongs to column b schedule this job last in the sequence table and cross their job. So, we have to see a schedule this job last in the sequence table since it is in the second column or for machine 2.

So, here up to this we have seen now 2 is the minimum time or so, 2 is the minimum time. So, we will sequence it the job 1; the minimum time is 2 are the job 1 is corresponding to that minimum time/

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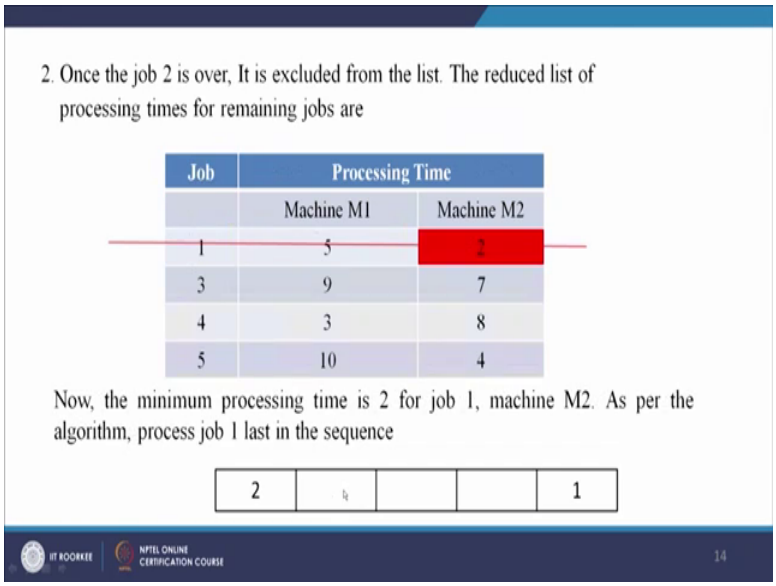

So, we will sequence the job one here and initially we have already seen that the minimum time initially was for job 2 which we have sequenced in the beginning of the sequence table we have started with that sequence. Now we can see we have already scheduled two jobs we have scheduled job 2 we have scheduled or sequenced job 1.

Now three are remaining let us see the minimum time minimum time is 3 hours here and the job is job 4. So, we will since it is in the first column we will put the value 4 here because that is a second job to be schedule starting from the left hand side.

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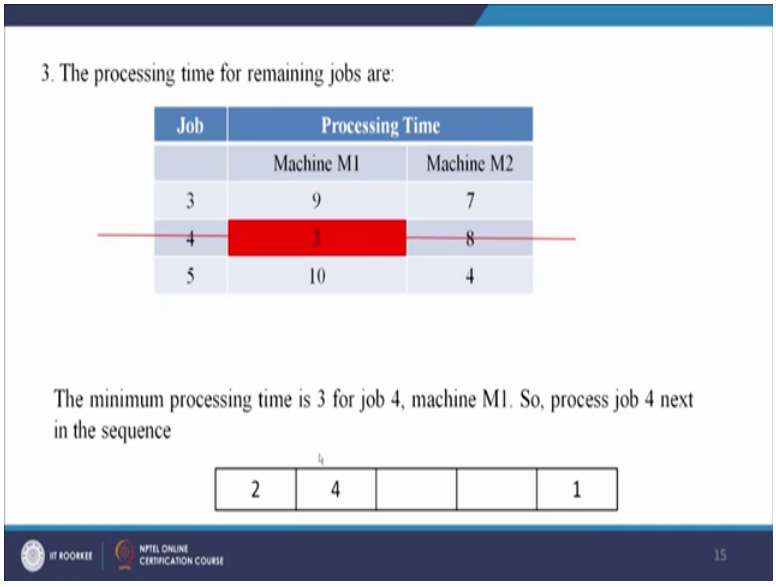

So, this is 2 and 4 and now we can see we have to cross this and finally, see the minimum is 4 processing time corresponding to job number 5. So, 5 will be scheduled from the end side.

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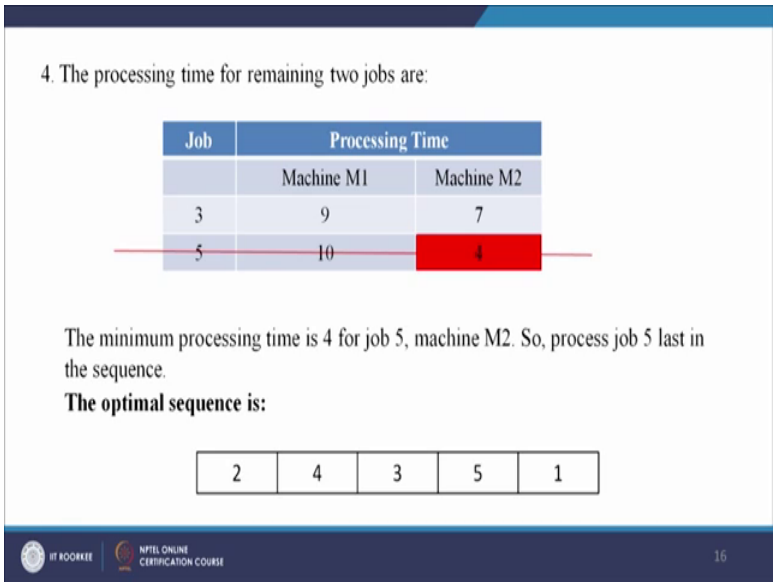

5 is schedule now the minimum value is 7 only 3 is remaining; so, we will schedule 3 also.

So, the first step or the first job that was at our hand was to find out the sequence of jobs. So, sequence of jobs is 2 4 3 5 and 1; so, the technical order we know that first all jobs have to be processed on machine 1 and then they have to be processed on machine 2.

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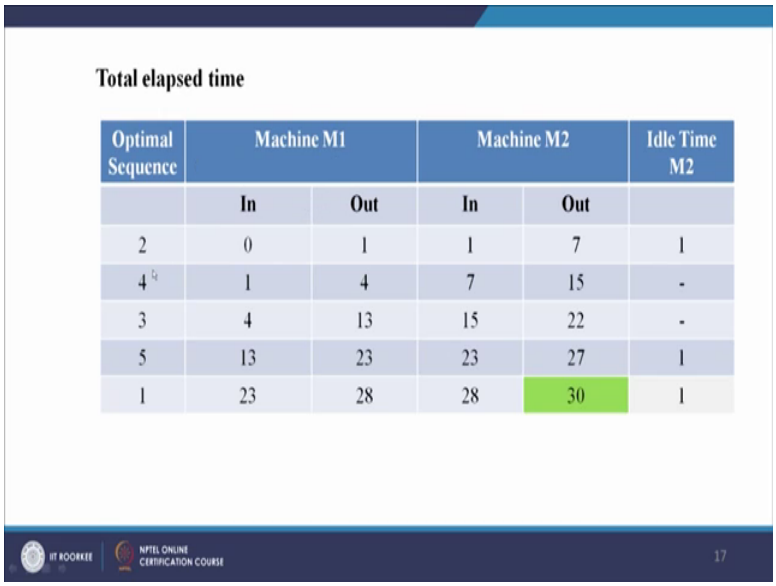

So, similarly we can see that once we have decided on the sequence we will see that machine 1 job 1 job 2 goes to If you see sorry.

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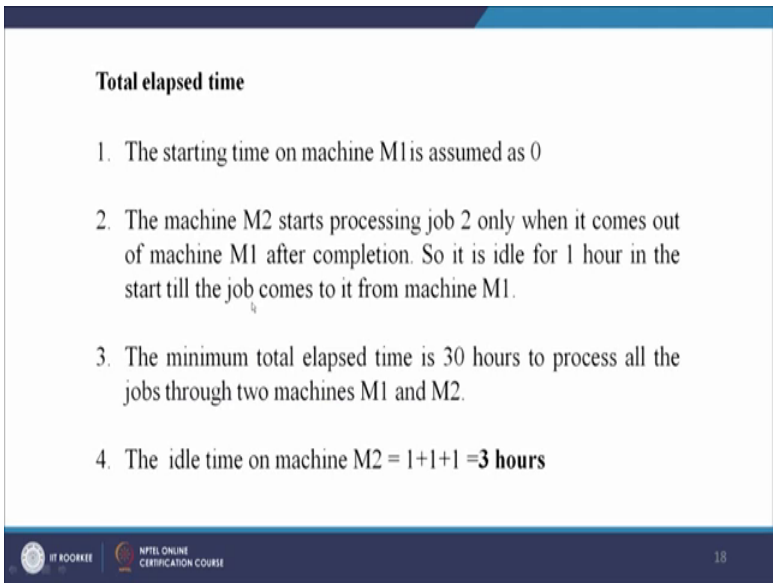

If you see the sequence optimal sequence 2 4 3 5 and 1; so, first job 2 has to be sequenced. So, job 2 it will be in at time 0 out at time one because it required 1 hour duration only then it will go to machine 2 when it will go to machine 2 1 hour of processing already has been done on a machine 1. So, it goes to machine 2 after 1 hour and on machine 2 if you see the table again it will require 6 hours; so, it will be out at 7.

So, the idle time on machine 2 is 1 here because we are we are starting at time 0. And there is no shock that can directly be scheduled or sequenced on a machine 2 why? Because the technical order has to be followed. Then the next in the optimal sequence is job 4 and accordingly we will see that it will go to machine 1; after 1 hour why? Because in the first 1 hour machine 1 was busy with job 2.

So, it goes after 1 hour whatever is the time required processing time for job 4; we will add that to 1 hour we get it will be out at 4 here and then it will enter machine 2 at 7; why it will enter at 7? Because machine 2 is already busy with job 2 because it requires 6 hours duration.

So, similarly we can sequence the all the different jobs that is 3, 5 and 1 and accordingly calculate the idle time on a machine 2. As we have seen in the very beginning the machine was idle because job number 2 was being worked upon or was being processed on a machine 1 and machine 2 was waiting for the job 2 to arrive and to start the processing.

Similarly, in we can see that in job number 5 also machine 2 will be idle for 1 hour and similarly for job 1; it will be idle for one and total elapsed time we can see is 30 hour. So, 5 different jobs when they have to go to two different machines we have been able to sequence them with the help of a standard procedure so, that the total elapse your time is minimized.

So, we can see the starting time on a machine 1 is assumed as 0. The machine M 2 starts processing job 2 only when it comes out of machine 1 after completion. So, it is idle for 1 hour in the start till the job comes to it from machine M 1. The minimum total elapsed time is 30 hours to process all the jobs through 2 machines M 1 and M 2 and the idle time on a machine M 2 is in three scenarios we have seen it is idle for 1 hour is a especially when in the beginning of the processing then for 5 job number 5 and job number 1. So, 1 plus 1 plus 1 3 hours it is having an idle time that is machine 2.

So, with this I think we will conclude the today's session. We have seen that when 5 jobs have to be sequenced on one workstation, how the sequencing rules can be used we have seen three sequencing rules first come first serve, shortest processing time and earliest due date and tried to compare the performance of the three sequencing rules.

Then we switched to the sequencing problem where we have n number of jobs and 2 machines and we have seen as a procedure which can help us to sequence these 5 job oh sorry n number of jobs on 2 machines so, that our idle time is minimized as well as our total elapsed time or total completion time for this n number of jobs is minimum.

So, in next session our focus will be as was highlighted today; we will try to solve additional problems on sequencing. And in the last session we will focus on the scheduling problems or the master production schedule.

Thank you.