Operations Management
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Lecture - 40
Project Network: Crashing Problems

[FL] friends. So, finally, today we are at the end of eighth week of discussion and the last session today is focused on the problems are the numerical problems associated with crashing of the networks. Just to have a brief preview of what we have already covered we have seen the different types of network, we have seen critical path method, we have seen pert that is program evaluation and review technique, we have seen the difference between CPM and pert.

We have seen how to construct a pert network we have seen that there are probabilistic time estimates in case of pert and therefore, there is a variance associated with the project completion time, we have also calculated the probability of project completion time in context of the time duration of the project that is expected time duration that can be calculated from the critical path that if we extend the duration; what is the probability. If we reduce the duration, what is the probability of completion of the project network? In the last session, I think I have tried to explain the concept of crashing.

So, crashing is the process of reducing the duration of the network in case of exigencies or in case of special circumstances in many cases, we need to reduce the project duration because of number of issues the one of the issues can be the penalty clause the other issue can be that project being delayed because of some inevitable circumstances. So, if our overall project is getting delayed we try to bring the project on track by reducing the normal duration of the various activities we have seen that for each activity we must have 4 data points. Now as for the previous networks that we have drawn where our focus was primarily on time, we needed to have only one time estimate for an activity or only that time data for activity in case of pert we had 3 time estimates for an activity

But now we are focusing our attention on the time cost trade off of a project network. So, we require time also as well as we required the cost data also in order to do our analysis. So, from time point of view we require the normal time for each activity, we require the crash or the crash duration for each activity. Similarly, we require the cost involved for

completing the activity in normal duration and the cost involved for completing the activity in crash duration. So, now, for time cost trade off in case of network for every activity we require normal time crash time normal cost crash cost see also we require the indirect cost also for the project duration which I have shown is given in x per day your x per month or rupees x per year.

So, depending upon our time domain usually we see the project in terms of weeks or days. So, it can be rupees x per day or rupees x per week. So, that is the indirect cost in the previous session we have seen that the indirect cost and the direct cost add up to calculate the complete cost of the overall cost of the project and we have to find out the optimum duration of the project in terms of days or weeks for which our overall project cost is minimum and how we calculate the project cost by adding the direct cost and the indirect cost our focus today will be to see some of the problems that how we actually do the calculations in order to calculate the overall project cost.

So, first we will try to understand the concept of crashing with the help of a simple problem and then we will solve a problem in which all the data points are known to us and we try to reduce the duration of the project by crashing the various activities and must I tell you that this requires practice all learners must at least try to solve 5 problems related to the CPM pert and the time cost trade off in order to become maybe acquainted with the technique and then only you can use it in industry whenever you join a particular industry.

So, within the time allocated for this particular topic of project management I think we have been able to understand the concept of networks project networks how to construct a network how to do the calculations for the critical path and how to make use of the critical path. So, let us know quickly see the problem statement. So, as I have already told in previous class we have seen that for each and every activity there is a cost slope involved.

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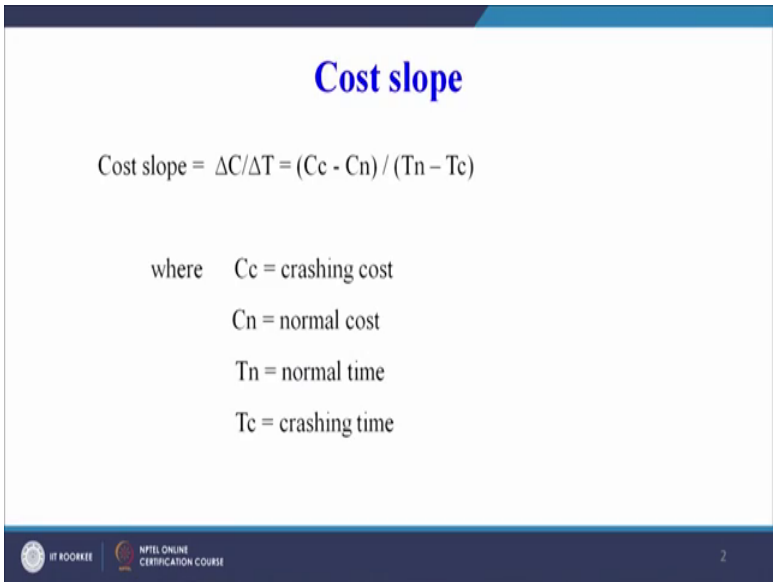

So, how we will calculate the cost slope you can see, it will be the number, it will be cost per unit time. Now per unit time can be in days or it can be in weeks. So, cost slope is delta C by delta T that is the crash cost minus the normal cost divided by the normal time minus the crash time.

So, this way we can calculate the slope and which activity, we will choose for crashing. Now, suppose, we have a project network and in the previous session, if you see, if the previous session, we have calculated one critical path and there were 5 activities on the critical path. Now out of the 5 which activity you will choose. So, first thing that everyone must remember is that in order to reduce the duration of the project, we have to focus on the critical path because if we focus on a not noncritical activities or a noncritical path.

The overall project duration will not reduced why because the longest path is the critical path even if we reduce the duration of the noncritical activity still our project duration will remain same, therefore, our first focus must always be to focus on the critical path. Now, suppose on the critical path we have 5 different activities which one we will choose

So, this cost slope will help us to identify that which activity we must choose. Now first and foremost, we must know that the crashing is possible for that activity or not many times it may so happens that the crash duration is equal to the normal duration which means that you cannot crash that activity you cannot reduce the duration of that activity

even by pumping in more resources even by spending more money on that activity you cannot reduce it is the process which requires that much of time. So, cannot reduce that duration. So, there is no crashing possible for that activity. So, your cost slope; you cannot calculate because this is your Tn is equal to Tc that is your normal duration is equal to the crash duration cost then becomes immaterial.

So, out of the 5 activities suppose our 2 activities crashing is not possible because the normal duration is equal to the crash duration now for the remaining 3 activities we have to see the cost slope that the we will choose that activity for which the cost slope is maximum or minimum, you can just see we want to spend the minimum money for the project completion. So, we will go where the slope is minimum. So, we will focus goes the cost slope if you see delta, see is in the numerator. So, we will try to minimize it. So, we will focus on the activity for which the slope is less.

So, now as per this slide for every activity we must have 4 data points that is the crashing cost normal cost normal time and crashing time let us take first example most simple network we have tried to construct. So, that everybody is able to understand the source is also given Bernard W chapter 8 crashing introduction to management science.

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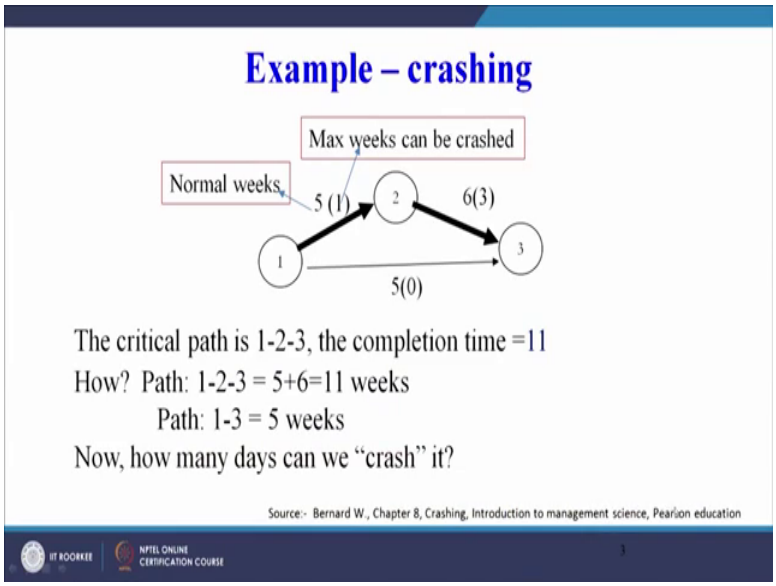

The Pearson education book; now you can see most simple network on your screen there are 3 nodes only 1, 2 and 3 and there are 3 activities. So, this activity is representing node 1 and 2 the start node is one and end node is 2 normal time is 5 weeks maximum

weeks that can be crushed is only one week. So, this activity cannot be crashed beyond 4 weeks. So, minimum time required for the completion of activity 1; 2 is 4 weeks, it can only be crashed by 1 week from its normal duration of 5 weeks. So, similarly activity second that is 2 3 6 weeks is the normal duration can be crashed by 3 week.

So, minimum duration for activity 2, 3 is 3 weeks cannot be crashed beyond that point. Similarly activity 1, 3, 5 weeks is the duration normal duration cannot be crashed at all because this is 0. So, there is no crashing possible for activity 5. Now, let us see if we want to reduce the duration of this project we want to release the resources from this project and put these resources is some other project we want to reduce the duration from the critical path if we see what is the critical path add the normal durations for all the activity. So, 5 plus 6 that is 11 weeks are required for the completion of this project whereas, the other path only 5 weeks; 1, 2, 3, 5. So, the longest path is 6 plus 5; 11. So, the critical path is 1, 2, 3, the completion time is 11 weeks.

Now, we have to crash the activities our focus will primarily be on the critical path non critical path activities 1, 2 and 3 are there. So, what we can do we can crash activity 6 by 3 weeks and we can crash activity 5 by 1 week. So, it becomes 4 weeks and it becomes 3 weeks. So, it will be 7 days that the project can be completed. So, path 1, 2 and 3 is 11 weeks path 1 and 3 is 5 weeks. Now how many days we can we crashed this. So, if we will focus on this duration crash duration possible for activity 1, 2; 1 week of crashing is possible for activity 2-3; 3 weeks of crashing is possible.

So, what we can do we can completed in 4 days, we can completed in 3 days. So, we can complete the project in 7 days; 4 plus 3; 7. So, here we can see the maximum time that can be crashed for is one that is for activity 1, 2 and 3 for activity 2 3. So, 1 plus 3; 4 weeks the project can be crashed in path one 3 no crashing is possible.

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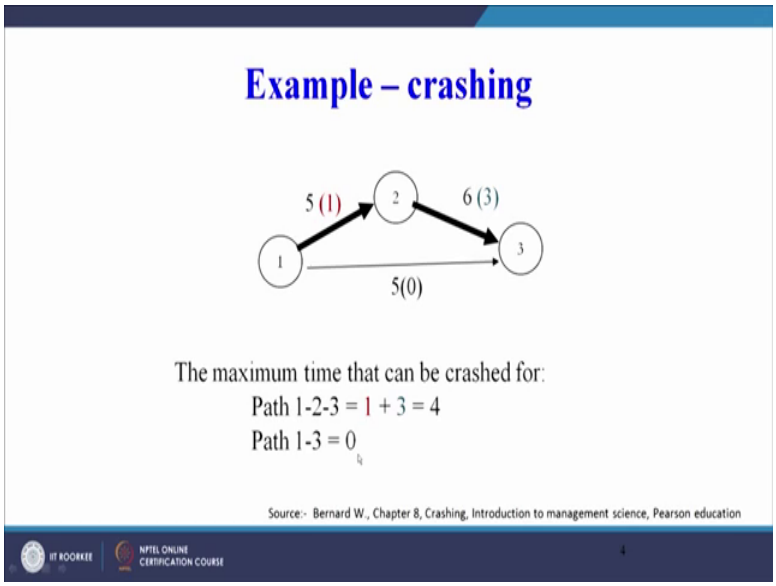

So, it is given 0. So, we can see that we can crash the project by 4 weeks.

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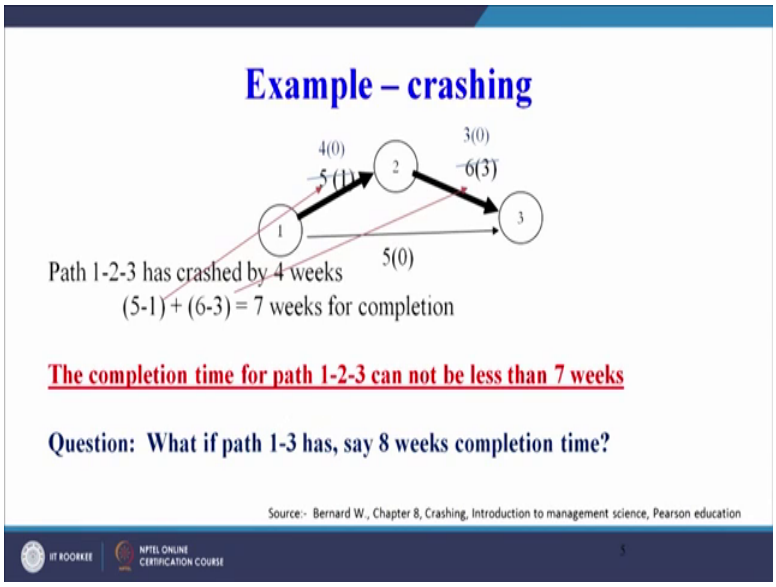

And therefore, the project can now be completed in 7 weeks now here we can see path 1, 2, 3 has been crashed by 4 weeks. So, now, it is 4 no further crashing possible 3 weeks no further crashing possible. So, 4 plus 3; 7 and from this path 1, 3, 5 weeks still our 1, 2, 3 remains the critical path. So, 7 weeks for the completion of the project.

So, the completion time for path 1, 2, 3 cannot be less than 7 weeks why because now no crashing is possible for activity 1, 2 similarly no crashing is possible for activity 2, 3 and

no crashing was always not possible for path 1, 3 or the activity 1 3. So, the project cannot be completed in 7 weeks, but here we have not considered the concept of cost we have just seen the time only that whether the activities can be crashed or they cannot be crashed they up basic concept of crashing has been explained here, but when we are reducing the duration of the project from 11 weeks to 7 weeks we are reducing the overall project duration by 4 weeks and these 4 weeks reduction in the overall project duration will cost us handsomely.

So, we will have to spend additional resources additional man path we have to spend in nutshell additional cost to accomplish this objective of reducing the project duration from 11 days; 11 weeks sorry 2; 7 weeks. So, we are able to maybe shorten the duration by 4 weeks, but at a cost, but what we are saving we are saving 4 weeks and analogous to that we are saving the indirect cost which is on weekly basis, suppose, if you remember in the previous session I have drawn 2 or 3 graphs to explain the concept of crashing and if you remember the indirect cost increase linearly with the increase in the project duration.

So, for 11 weeks we will multiply the weekly cost by 11 in order to calculate the total cost of the project, but now in the change circumstances we are now been able to complete the project in 7 weeks only. So, we will multiply the weekly cost by 7 only in this case and we will not multiply it by 11 because now we have been able to reduce our project duration from 11 to 7. So, indirect cost per week will be indirect cost per week that is suppose x multiplied by 7 plus the direct cost now direct cost for 11 weeks will be less because the cost corresponding to the normal duration is less the cost maybe analogous to our corresponding to the crash duration is more.

So, we are spending more money in the direct cost we are spending now less money in the indirect cost and the total cost will be made up of the direct cost plus the indirect cost. So, accordingly we will see that whether our overall project cost is reducing or it is not reducing it may be many cases be increasing also, but still we are able to release certain resources from this project for some other maybe more important project and then our other project may be running well in time.

So, we are what we have been able to achieve we have been able to reduce the duration of our project by slightly spending some more money, but our other projects where these

resources have been committed are running well in time. So, this type of calculations we have to do in industry in order to meet our deadlines for the various projects now there is a question from mathematical calculation point of view what if the path one 3 has see 8 weeks completion time very logical question here activity 1 3 we have taken as 5 weeks without any crashing possible.

But suppose this is 8 now what is happening we have reduce the duration of activity 2-3; 2-3 weeks, we have reduce the duration of activity 1, 2 to 4 weeks. So, our critical path in this network is 1, 2 and 3 that is 7 weeks, but suppose this is 8 weeks what we will do then our path one 3 will become a critical path and the project will require 8 weeks for completion because the other path is for 7 weeks.

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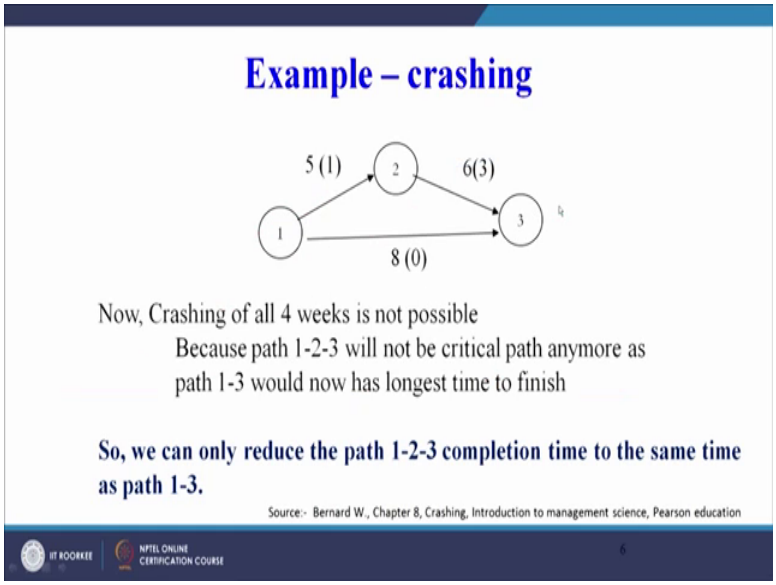

So, this is now the other problem 5 weeks one week crashing possible 6 weeks 3 weeks crashing possible and 8 weeks no crashing possible to crashing of all 4 weeks is not possible because path 1, 2, 3 will not be critical path anymore as path 1 3 would now has the longest time to finish. So, if we crash all these duration 6 we bring 2 3 weeks by crashing it by 3 weeks 5, we bring to 4 weeks by crashing it by one weeks.

So, 4 plus 3; 7 still our project will be completed in 8 weeks only because activity 1-3, then becomes the most critical activity involving 8 weeks. So, path one 3 would now has the longest time to finish. So, we can only reduce the path 1, 2, 3 completion time to the same time as path one and 3. So, we can bring this path 1, 2 and 3-2; 8 weeks first.

Now, we have 2 critical path here path 1, 2 and 3; 8 weeks after crashing activity 1, 2 and 2 3 and path 1-3 again of 8 weeks. Now further crashing is also possible, but what we need to do now we need to reduce the duration of both the critical path 1, 2 and 3 and path 1-3; both these we need to reduce by one more day.

So, we will do a trade off we will see how much money, we have to further spent to reduce this duration and how much money we are saving from the indirect cost because whenever we reduce the project duration by one day our indirect cost is saved at the cost of direct cost that we are spending in order to reduce the duration. So, direct cost increases when we reduce the duration of the project indirect cost reduces when we are reducing.

So, direct cost will increase when we reduce the project duration indirect cost will reduce when we reduce the project duration. So, accordingly we have to do a trade off that the overall project cost that is the addition of the direct and the indirect cost is reduced.

Now, here we can see the total time for path 1, 2 and 3 is equal to the path one and 3 that we have seen that if we reduce the path 1, 2, 3, 2, 8 weeks path 1, 3 is also 8 weeks. So, now, we have to critical path that is 8 weeks duration. So, if the cost for path 1, 2 and path 2-3 is the same then pick any path randomly and crash. Now, suppose we want to further reduce the duration to 7 weeks in 5 we have the flexibility.

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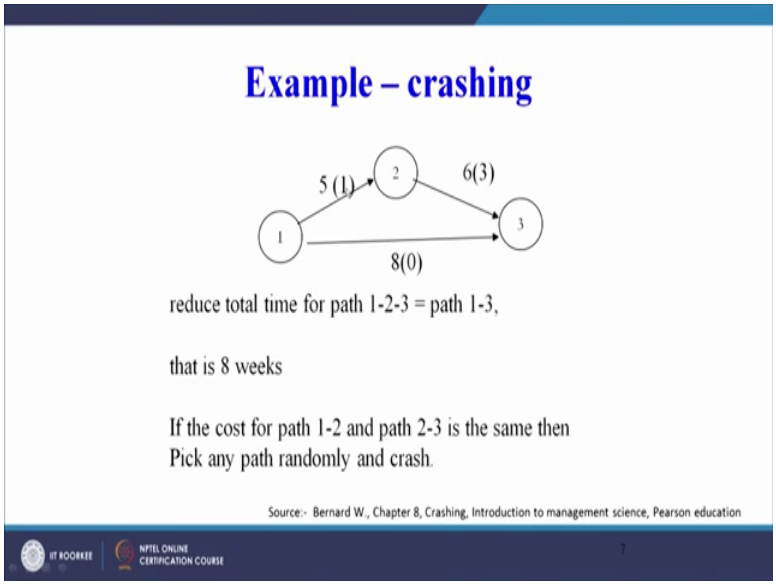

Here also activity 2-3 we have a flexibility because we have to reduce it by further one day suppose whatever 3 days re reduction we have done we have done here. So, one day is possible with five. So, we will focus on the path five, but in this particular network the point that is to be addressed from this slide is that if we have the option to choose from activity 1, 2 and 2-3 we will choose that activity for which the cost slope is minimum.

But in this specific case the activities cannot be crashed beyond a particular point that is project duration will be 8 only because activity 1-3; we cannot further crash it is our limitation because there is no crashing possible here. So, the project duration remains at 8 weeks there is possibility to bring this 1, 2, 3 path to 7 days, but it is of no use because we cannot crash activity 1-3 because 8 weeks are definitely required for the completion of activity 1-3.

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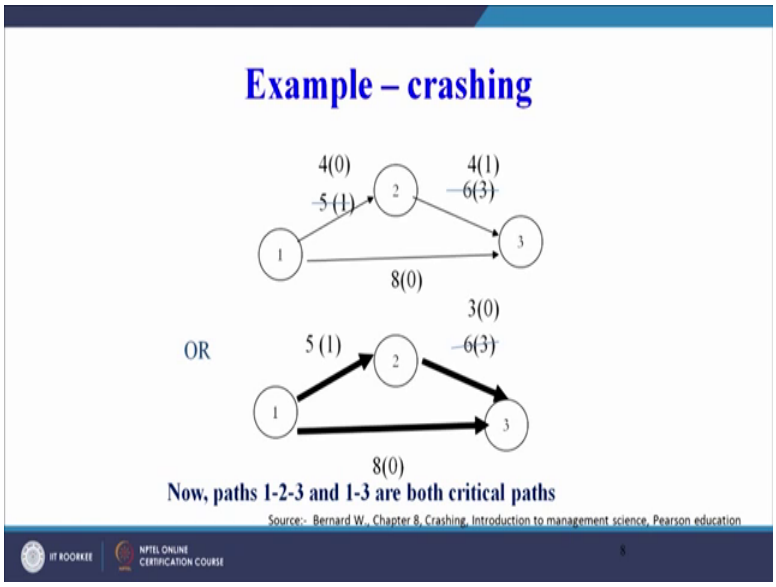

Now, here we can see now path 1, 2, 3 and 1-3 are both critical again that is shown that we have reduced the project duration by crashing activity 2-3 by 3 days. So, now, this is 3 and this is 5 still we have the liberty to crash activity 1, 2 by 1 day still we can do that, but there is no use why because activity 8 cannot be crashed. So, the minimum project time required for completion of this project is 8 weeks. So, we can reduce it by one more week, but there is no use because still the project duration remains 8 weeks only.

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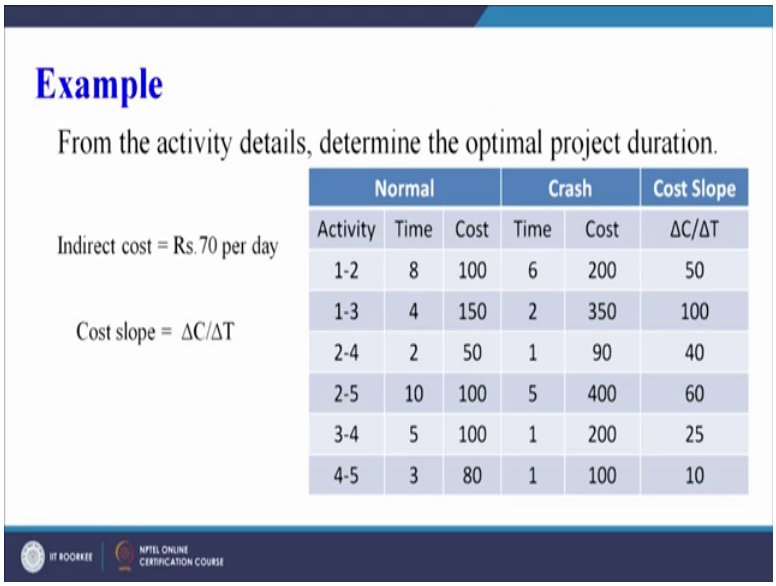

	Normal			Crash		Cost Slope
	Activity	Time	Cost	Time	Cost	ΔC/ΔT
	1-2	8	100	6	200	50
	1-3	4	150	2	350	100
	2-4	2	50	1	90	40
	2-5	10	100	5	400	60
	3-4	5	100	1	200	25
	4-5	3	80	1	100	10

Now, let us take one more example here and in this example we can see that we have 4 data points for each activity now activity is 1, 2 normal time is 8 crash time is 6 that is it is in days.

Now, the cost also is given the normal cost is rupees 100 and the crash cost is rupees 200. So, very easily you can calculate the cost slope that is the cost will be in the numerator 200 minus 100 divided by 8 minus 6. So, you are hundred divided by 2 is equal to 50. So, very easily we can calculate the cost slope and this will help us in making a decision that which activity we must focus on and the activity which has the minimum cost slope we will sorry focus on that activity first. So, for direct cost calculation point of view we have here 1, 2, 3, 4, 5 and 6. So, we have 6 activities and for each activity we have the normal time we have the crash time we are the normal cost we have the crash cost.

So, 4 data points are available for each activity and from these 4 data points we have calculated the cost slope this is our first stage of calculation and for calculating the overall project cost we require indirect cost also and the indirect cost is given here as rupees 70 per day. So, we have normal time crash time normal cost crash cost and the indirect cost data available with us indirect cost is available in the form of rupees 70 per day and the normal and crash data is available in terms of every activity.

Now this is the first calculation that we require to do second is as we have tried to put the node numbers here it is very easy to draw the network.

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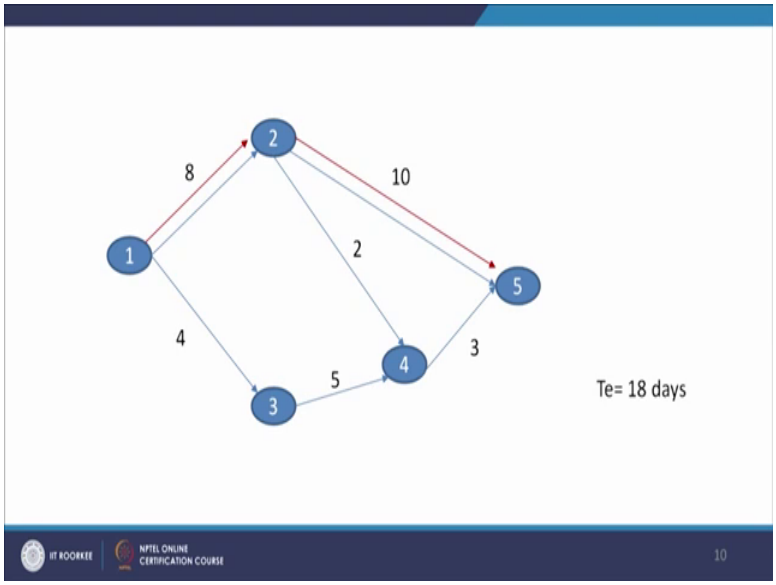

Now, this is our network 1, 2, 5 is one path which is the critical path as per our calculation then 1, 2, 4, 5 is another path 1, 3, 4, 5 is another path. So, we can see here that our critical path is 10 plus 8 18. So, week see here 10 plus 8; 18 is a critical path, we have 2 other paths in this network that is 1, 2, 4 and 5 that is 8 plus 2 10 plus 3 thirteen and we have another path 1, 3, 4 and 5 that is 5 plus 4; 9 plus 3; 12.

So, we are 3 paths from the start node to the end node 18 thirteen and 12. So, if we want to reduce the duration from 18 days what must be our focus our focus will be the critical path. So, there are only 2 activities on the critical path that is activity 1, 2 and 2-5 now we have to reduce this duration in order to reduce this duration we have to look at the cost slope of these 2 activities now let us look at the cost slope. So, for 1, 2 it is rupees 50 per day and the other activity is 2-5.

So, for activity 2-5, it is rupees 60 per day. So, 1, 2 is rupees 50 per day and 2-5 is rupees 60 per day. So, what will be our focus our focus will be the minimum slope that is rupees 50. So, activity 1, 2 can be crashed first in order to reduce the duration of the project moreover if we reduce the duration of the project by one day we are spending 50 rupees more because direct cost is increasing, but we are saving 70 rupees per day as the indirect cost.

So, what we will do we will go for forward and do the crashing.

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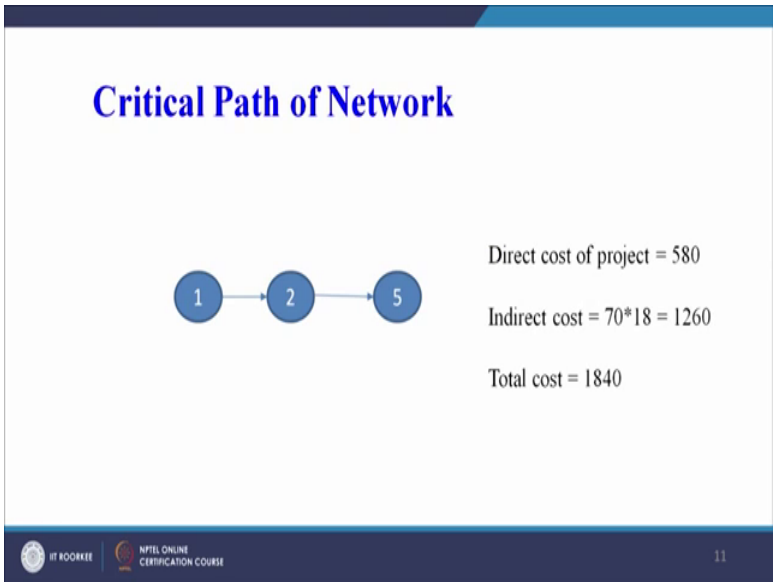

So, direct cost of the project by adding the direct cost of each activity is rupees 580 indirect cost because we have seen here 10 plus 8; 18 is the critical path. So, 18 days are required. So, 18 into rupees 70 per day; so, 1260 rupees of 580 plus 1260 the total cost of the project as per the critical path is rupees 18 40 now we can reduce it as I have already explained.

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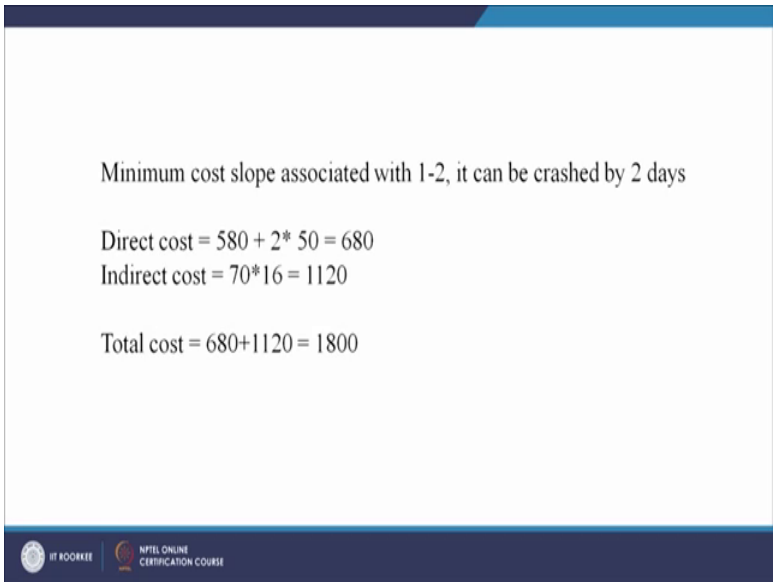

So, minimum cost slope is associated with activity 1, 2 which is rupees 50 per day, it can be crashed by 2 days.

So, if we crash it by 2 days, we have to spend 50 into 2 that is 100 rupees more, but we are saving 40 rupees how because now the indirect cost we have to multiplied by 16 days only instead of 18 days which was over critical duration or the maybe the overall duration of the project under normal circumstances under crashing we are reducing the overall project duration by 2 days.

So, 16 has to multiplied by 70. So, we can see indirect cost is rupees 70 per day, but now the project will be completed in 16 days. So, see we are multiplying it by 16 direct cost was 580 plus now the direct cost is increasing because we are crashing activity 1, 2 by 2 days and the cost of crashing is rupees 50 per day. So, 50 into 2; so, it becomes 680 plus the indirect cost is 1120.

So, we added; so, we get a value of 1800 here. So, our project cost total cost associated with 18 days of project one 8 days of project was rupees 1840, but now we have done the project cost calculation by reducing the duration of 2 days for of activity one two. So, our overall project cost is coming out as 18 hundred. So, we are able to save some money and the duration of the project also is getting shot and so, in both cases, we are at an advantages position.

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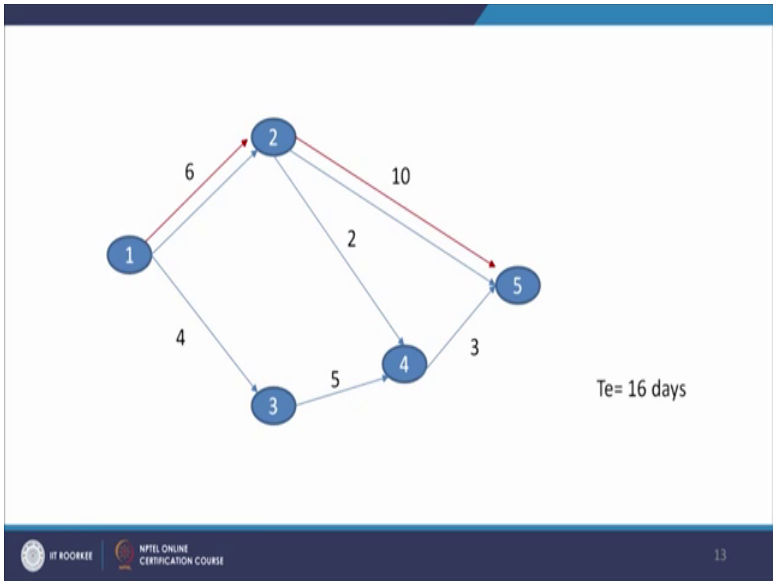

Now the next further crashing can be; so, we have seen that the now the overall project is can be completed in 16 days.

Now, further crashing is possible whatever was possible with 1-2, we have done that now for 2-5 also crashing is possible and our other networks are now 6 plus 2; 8 and 3; 11 and 5 plus 4; 9 plus 3; 12 and 10 plus 1; 16. So, there are 3 path; one is requiring 16 days another is requiring 11 days another one is requiring 12 days. So, we can go now to 12 days and see that what is the cost involved.

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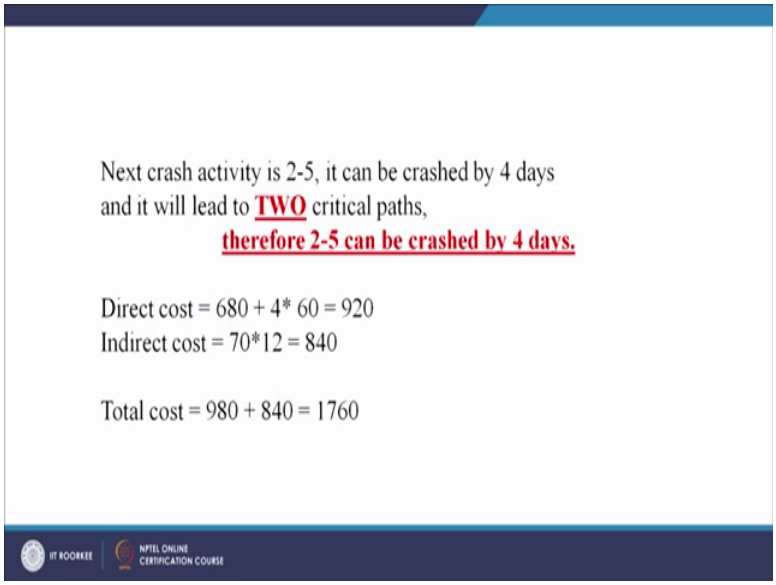

So, for next crash activity 2-5, it can be crash by 4 days although possibility is therefore, 5 days, but the other critical path is also then there is no point in crashing from 16 by 5 days and bringing the project duration to 11 because we know that there is another path which is for 12 days. So, let us first go to 12 days duration only.

So, we can crash it by 4 days and it will lead to 2 critical path, then because the other path is also requiring 12 days completion time. So, if we crash it by 4 days. So, direct cost earlier we have seen it is 680 plus 4 into 60 because the crashing cost for activity for a 2-5 is rupees 60 per day to 60 into 4 overall cost direct cost is 920 indirect cost. Now because the project will now be complete after crashing in 12 days.

So, 70 is the indirect cost per day 70 into 12 it is 840. So, if we add this up we are getting 1760 which is till lesser than the starting cost of 1840. So, now, our overall project

duration is 12 base and we are spending less money for the completion of the project because we are saving money on account of reducing the project duration in terms of the indirect cost although the direct costs are increasing, but the indirect costs are reducing.

So, we are able to save the money now what happens beyond this point. Now we have 3 paths 12; 11 and 12. Now, in order to reduce further we need to focus on 2 paths of 12 days each. So, we have to reduce the activities on both these paths in order to bring the activity or the project duration to 11 days, now our focus is are 2 paths. Now if we see these are the 2 critical paths; 6 plus 6; 12, 5 plus 4; 9 plus 3; 12 and 6 plus 2; 8 plus 3; 11.

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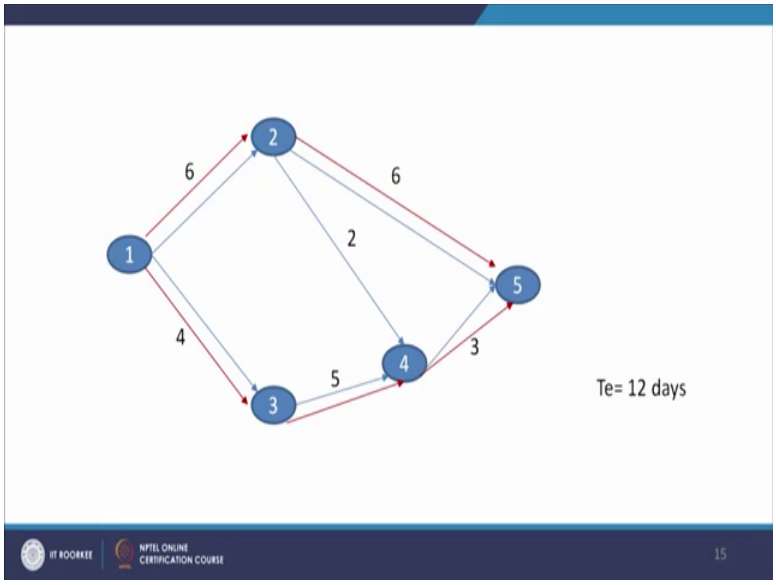

So, we are focusing known these 2 paths. So, we can see this; we cannot crash further because already, we have crashed it by 2 days if you see our activity 1, 2 initial was normal duration was 8 days crash duration in 6 days already we have crash it to 6 days. So, not possible our activity 25; 10 days we can crash it to 5 days duration 4 days already we have crush. So, we can crash it further by one day, but it requires rupees 60 per day.

So, we can further reduce this activity by one day. So, this path will become 11, this is already 11. So, in order to reduce this path 1, 3, 4 and 5 by 1 day, we have to look at this 3 activities 1, 3, 3, 4 and 4, 5 and if we see the minimum slope is coming out for activity 4 5 that is rupees ten. So, we will focus on 4 5 that is we can crash it and whether the crashing is possible we can see yes crashing is possible normal is 3 days crashes one day. So, we can reduce it by 2 days, but should we go for it.

So, if you see that 10 rupees spent here for reducing one day 60 rupees spent here for reducing one day 70 rupees spent for reducing the duration by one day at what we are saving 70 rupees in the indirect cost. So, 70 is equal to 70 our overall project cost remains same. So, we may not like to go for crashing any further because it is not going to help us to further reduce the duration. So, in this way we can do the crashing step by step by step by step we can make a table and put crashing one which activities we are focusing what is the cost slope for that activity what is the direct cost what is the indirect cost and what is the total cost.

So, we can make a tabular from and step by step by step we can do the crashing, but what is the point up till which we have to do the crashing we have to go to that point up to which our total project cost is reducing beyond a particular point whatever savings we are getting in terms of indirect cost by reducing the project duration our direct cost that we are spending on the activities for doing them quickly maybe more than the indirect cost.

So, our indirect cost savings may not compensate for the direct cost that we are spending for reducing duration of the activities. So, we have to settled down we have to trade off at a point up to which our overall project duration is giving us the minimum total cost or minimum total project cost and this will help us in figuring out that how we can plan the various activities.

So, that the total project cost is minimum. So, with this we come to the end of our discussion for week 8 and we will start a new topic in our discussion in week 9 in case of any problems anything that you are you have not been able to comprehend in CPM and pert you can write to me on the discussion board and we will try to answer to your queries to the best of our capabilities.

Thank you.