Operations Management
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Lecture - 38
PERT Problems-II

[FL] friends welcome to session 38 in our course on Operations Management and in the 8th week of our discussion our focus is on solving the PERT problems. So, as all of us have understood by now that CPM and PERT are two important networking tools that are used for project planning and control. So, we will see today that how we can solve a PERT problem.

In the last class or in the last session if you remember we have taken fairly simple problems in which there was a network the node numbers were already specified and the activities can be easily drawn with the help of the start node and the end node. Today our focus will be to look at a different type of a network in which there is a list of activities and then there is a precedent relationship or the immediate predecessors of the activities are given, the time is given we have to construct a network first and then we have to see that how we can do the calculations. I will say, I will show the problem state made and then maybe take a little bit break in order to see that how all of your able to draw the network. We have to do lot of practice and rough drafts have to be made may be by pencil we can draw number of times and try to see that whether all the constraints in terms of precedence relationship are satisfied.

Initially we can make use of we can be liberal with the use of dummy activities, but at later stage we have to minimise the use of dummy activities and still established the precedence relationship or the logic in the network. So, we will see that how the network has to be constructed.

Once the network is ready then the calculations are the simple calculations as we have seen in the previous class also. And if you remember I have emphasized an important point of criticality index again I am emphasising that in case of PERT because we have 3 different time estimates for each and every activity that is the optimistic time, the most likely time and the pessimistic time therefore, there is a variance involved in the completion of the project. Just to reiterate the point that I have already discussed in the

previous session that if we take the optimistic time estimate for each and every activity consisting of a network we will get one project completion time which is based on only the optimistic time of each activity.

On the opposite side suppose we take the pessimistic time of each activity and then we add this pessimistic times of all the activities on the critical path we will get the project duration for a network which will or for the project which will be entirely different from the optimistic time calculations. Therefore, we calculate a expected time based on a formula which all of you by now might have memorized that is t expected is equal to t pessimistic plus 4 times t most likely plus t optimistic divided by 6. So, we try to convert a PERT problems into CPM problem maybe that is an analogy that I am drawing by bringing down these 3 time estimate into a single time estimate which can be used for calculation of the project duration or to find out the longest path in the network.

So, the point here is that since there are 3 time estimates involved for each activity we have got a variation in the project completion time and how and what decisions we can take based on this network or what is the overall objective of drawing this network. So, the overall objective is to find out that which activity is dependent on which other activity or what other activity that is one thing already that is known to us in the form of the problem statement. But other important information that we deduce from a network is that when how maybe at what particular duration the activity will start when it will end when the two activities can start which are starting from the same node so that kind of information we can deduce.

But the most important information as far as may be networking is concerned is that we are able to focus on the critical path which is very very important. We have to focus on the critical path again and again I am emphasising the same thing that if we linger on the critical path if we delay any activity, if we are not able to complete the activities that are lying on the critical path as per the planned time duration of the normal time duration for that activities the project will get delayed and if the project will get delayed we may have to enquire the penalty cost or we may have to pay the penalty cost which none of the companies want to pay.

One thing is that it may lead to escalation of the project costs the other thing is the brand of the company also has to be may be is also tarnished or has to be taken for beating. So,

may be the brand also, the brand name also get some beating and then otherwise also you have to enter heavy penalty cost. So, why a company would like to overall delayed up delivery of the project of the overall delay the completion time of the project. None of the company wants that and there is the importance of the critical path that these activities that are lying on the critical path cannot be delayed otherwise the overall project duration will get delayed that is the summary that we get from a network. Other things also maybe secondary things also we can deduce that is what are the activities which we can rescheduled to non critical activities can be rescheduled as per the requirement. So, number of decisions can be taken based on the network.

Therefore whenever there is a variance, whenever there is a range in which the project may get completed we try to figure out the criticality index of each and every activity and by running the iterations maybe if it is a construction project thousands of activities may be involved. So, we cannot do manual calculation. So, we can run a program which can see based on take the random times for different activities to suppose there are 570 activities, each activity having 3 time durations that is the pessimistic time, the optimistic time and the most likely time.

Now, the software can randomly pick and choose the times estimates for various activities and draw the critical path and then it can be seen that there may be a few activities which are always lying on the critical path. So, we as managers, as decision makers, as strategist have to focus on those critical activities that are falling on the critical path most of the time because if these activities get delayed our overall project duration will get delayed or our overall project duration will increase and the completion date will get delayed and that is what we think we must be able to avoid.

Therefore we these days we are taking help with taking the help of high end computational systems for running, these types of simulations to see that what are the activities that are lying on the critical path. And if you see if you see the previous session that is session number 37 we have drawn one network on the white board and seen that one activity is there that is always going to be critical for the completion of the project. There maybe 2 or 3 parts in that simple project, but one activity is very very very very critical and it will always lie on the critical path. So, as a project manager or as a project scheduler I must focus on that activity. So, that the overall project is completed within the defined stipulated time.

So, I think today again I have revised the important concept of critical path and we will see one simple problem today in which the activities are given the precedence relationship or the immediate predecessors for the activities are given the pessimistic most likely and finally, the optimistic time is given. And we have to draw a network based on the presidents relationship and then do the calculations to find out that what is the probability that a project will be completed within a specified time.

So, let us start the discussion now and see the problem. So, we have a problem statement here. So, I will read the problem statement for you.

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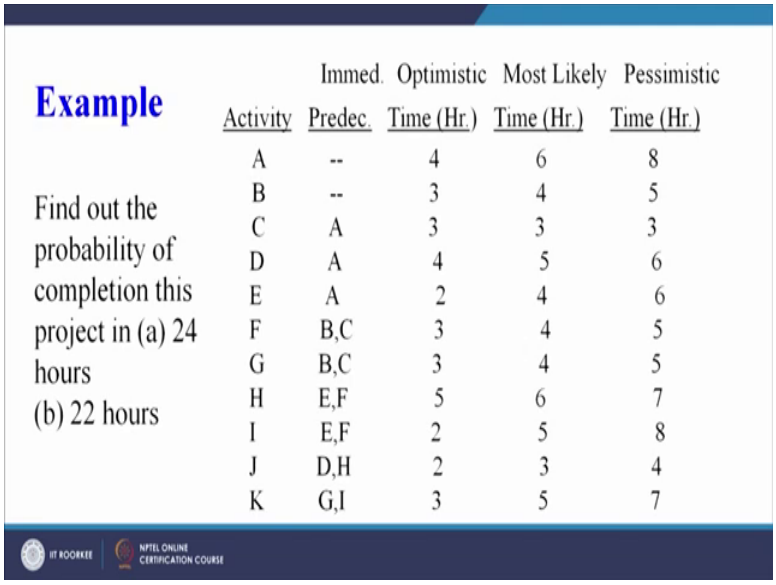

Find out the probability of completion of this project in A 24 hours in B 22 hours. So, we have two time durations mentioned here and we have to see that what is the probability that this project will be completed in 24 hours or what is the probability that this project will be completed in 22 hours.

The project statement is mentioned you can see we have here eleven activities A B C D E F G H I J K. So, 11 activities have got their immediate predecessor also. A and B has no predecessor I am reading it for you I wish that all of you has have a pencil and a white paper and you are trying to draw this network. So, we can see A and B can start from the same node because they do not have any predecessor or precedence relationship.

Now, A and B can start from the same node and you can see C and D both are dependent on A. So, wherever is ending from that node we have to start two activities that is activity C and activity D as well as activity E also, C D and E we can draw a network and we can do the practice of drawing this network that A B no pre precedence relationship. So, A and B can start from node 1. Now other 3 next 3 activity C D and E must start when A has been completed. So, they can start from A end node of activity A. Similarly F depends upon B and C. So, from second node we have drawn activity C where ever C is ending we can take the activity B to that node and from there activity F can start because F can only start once B and C are completed or have been completed.

Similarly, from the same node G can also start because G is also dependent upon B and C, and similarly H and I can be taken into account because both are dependent upon E and F activity. I am reading this with this notion that the learners who are less listening to this session will be drawing the network simultaneously. So, you can just do little bit of practice of drawing this network and if you are able to draw it successfully it is good for you and me as well as a teacher that whatever we are discussing. The learners are attending to that and they are able to draw the network or to convert this information into a network. So, based on whatever precedence relationship is mentioned on this side all of the learners must be able to draw a network. If you are not able to draw a network it is not that you will never be able to draw a network, it requires practice and we are in this short duration of 2 3 4 hours may not be able to take lot many problems, but what you can do you can take any good book on project management and try to practice maybe 8 to 10 different problems and try to construct a network.

Specifically with the precedence relationship and specifying the logic in the network that specific activities are dependent on their predecessor activities or proceeding activities. So, based on that information you must be able to draw a network. So, by now I think looking at the information available most of you might have been able to draw the network. So, once we are able to construct the network as is clear to all of you that in case of PERT we have 3 time estimates, we have times pessimistic time, most likely time and optimistic time.

So, let us take at least one example for activity E the optimistic time is 2 hours the most likely time is 4 hours and the pessimistic time is 6 hours. Similarly for activity K the optimistic time is 3 hours, the most likely time is 5 hours and the pessimistic time is 7

hours. So, I am not here again going to define the optimistic most likely and pessimistic time because the terms have already been defined in our earlier sessions when we started our discussion on PERT. So, this is the basic slide or basic questions that you will find in the question paper or in the industry when you have to sort out our problem.

In industries one more column will be added here that will be the description of a activity. If you are using PERT or CPM on industrial scale there will be another column that will be giving you the description of the activity. The description can be suppose we are constructing a house. So, A can be the purchase of land or it can be A can be suppose the land is already available as datum conditioner as a basic condition is already known. So, what we can do? A can be the construction or we can say the starting point for laying the foundation for the house.

Next can be, B can be erection of wall. So, there will be some precedence relationship among the activities, description is already known to you, the name of the activity is known the predecessor relationship is also known the time estimates because we are constructing a house let me ask you a question which type of network will be used for constructing a house, it will be a CPM or a PERT network. Just think over it. So, if you have been able to differentiate between CPM and PERT all of you must be able to answer that wherever the times are deterministically known, wherever we have prior experience of executing similar type of project times can be ascertained with certainty.

So, we know that the election of walls of this much height, with this much thickness will require this much time. So, time is known with certainty in those types of situations we will going we will be going to focus on CPM. But wherever there is a probability we are not sure about the time estimates in those cases we will go for PERT. So, if you see the example that I have taken that is the description is given for construction of a house all of you must be able to answer that it will take we will make use of a CPM method for doing the calculation because time can be estimated with fair degree of certainty or with fair degree of confidence.

So, but here today we are discussing a PERT problems. So, this is the basic structure of a problem that will be known to you what you have to calculate the 24 hours what is the probability of completing this project and in 22 hours what is the probability of completing this project. So, I think I have taken a fair amount of time to allow all of you

to just drop this network. So, if you have been able to construct the network, you can see this is the network that must be constructed.

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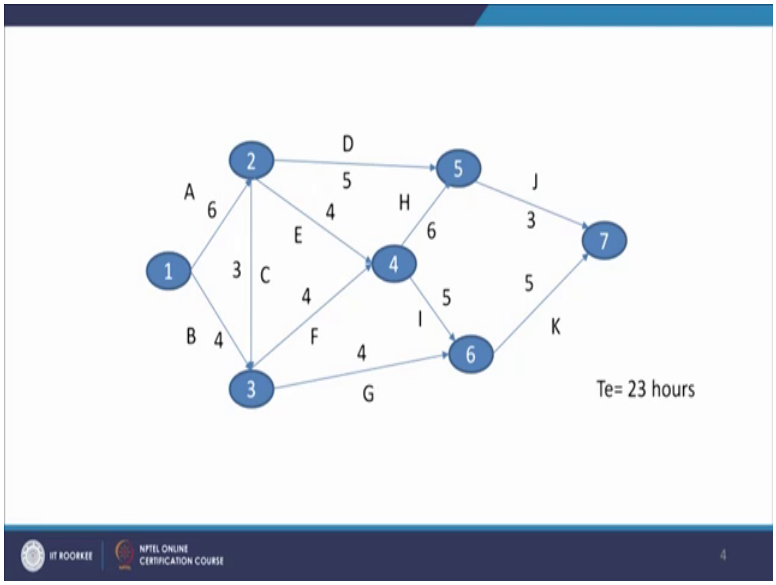

You can see the activity A is the predecessor for 3 activities, it is predecessor for D. So, you can see after node 2 D is starting, similarly it is predecessor for E. So, you can see from node two E is starting it is predecessor for C. So, you can see from node 2, C is also starting. So, C D and E all these are successors of activity A, and A and B have no predecessor. So, you can see A and B there is no predecessor for activity A and B. And B and C are predecessors for activity F and G which is known to you and for H and I hacked your E and F are the predecessors you can see for H and I, E and F are the predecessor.

So, I think we have been able to construct the network successfully and now we can calculate the, what we have to calculate? The critical path. So, what we need to do? We need to calculate as we have seen in case of CPM first we need to see that what is the expected time for each activity.

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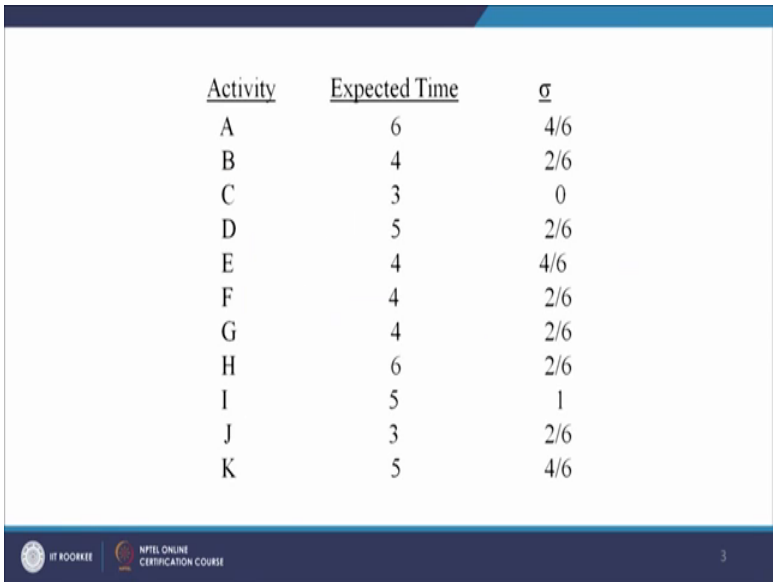

So, we can calculate the expected time for each activity by using the formula which we I have already said in today's session also t p plus 4 times t m plus t o divided by 6; t o is the optimistic time, t p is a pessimistic time, t m is the most likely time. So, using this formula we can calculate the expected time and we can calculate the standard deviation and variance for each and every activity. So, we will quick and first calculate the various and accordingly calculate the standard deviation which is simple mathematics. So, we for can calculate for each activity the expected time and the standard deviation and variance.

Network already we know and then we can do the calculations we can perform a forward, pass we can perform a backward pass for each node we can calculate the early start time, the late start time and from there we can calculate that where which are the activities for which there is no slack available which cannot be rescheduled or for which the slack is 0, we can calculate the critical path. The other method can be we can list down all the paths that are there or that exist in the network from the starting node as I have already told in one of our previous sessions that most of the students commits this mistake that they leave the network incomplete. So, for a complete network we have to ensure that there is single start node and there is a single completing completion node or the last nodes.

So in our network if you see the start node is node number 1 and the ending node is node number 7. So, within 1 and 7 between 1 and 7 we can see that what are the paths that are

available. So, you can see 1 2 5 and 7 is one path, 1 2 4 5 and 7 is another path, 1 2 4 6 and 7 is another path, then 1 3 4 5 and 7 is another path, then 1 3 4 6 and 7 is another path. So, we can write down all the paths and do the calculation, calculate the number of days required for the completion of each path and the maximum value will give us the longest path and which will be the time required for the completion of the project or the minimum time required for the completion of the project. So, that we also we can calculate.

So, here we can see our expected duration for the project is coming out to be 23 hours. So, let us see that how we can arrive at this value. We can arrive at this value if we see the path 1 2 5 and 7 it is 6 days plus 5 days plus 3 days that is 6 plus 5 11 plus 3 14. Similarly 1 2 4 5 and 7, 6 plus 4 10 plus 6 16 plus 3, 19, so 1 2 4 6 and 7 6 plus 4 10 plus 5 10 plus 5 plus 5, 15 plus 5 20, so if you see 1 2 3 4 5 7 6 plus 3, 9 plus 4, 13 plus 6, 19 plus 3, 22.

So, if we calculate for each and every path we can find out that which path is giving us the maximum duration. So, maximum duration can be in number of hours, it can be number of days, it can be number of weeks, it can be number of months whatever is the time unit that we have selected for constructing our network. So, in our case our critical path is coming out to be 1 2 3 4 6 and 7, so activities A C F I K.

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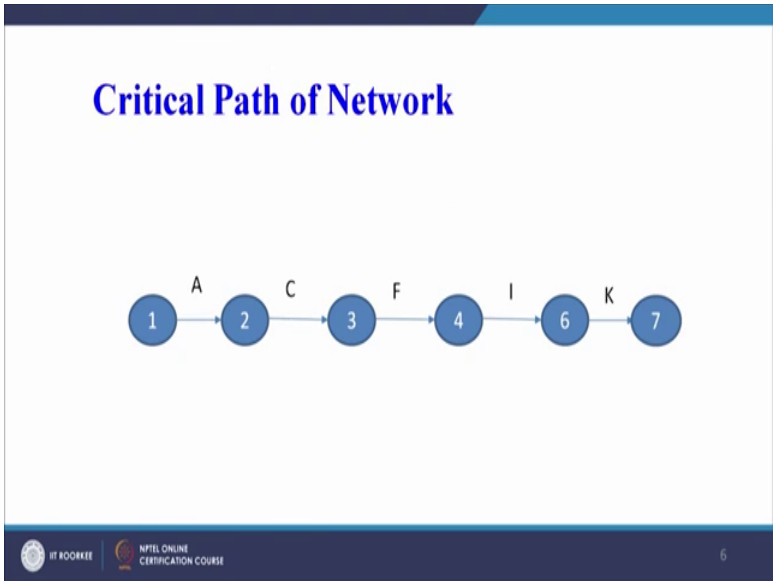

So, I will again go back to the network A C F I and K. So, we can see activity A 6 days, activity C 3 days, 6 plus 3, 9, F is 4 days. So, it is 6 plus 3 9 plus 4 13, activity I is 5 days, 13 plus 5, 18 plus K is 5 days 23. So, this is A C F I K is the longest path. So, we can see A C F I and K is giving us the critical path that is 23 hours sorry I think I have said days. So, our current project is in hours. So, it is 23 hours, 6 plus 3 plus 4 plus 5 plus 5 activities A C F I and K is our critical path in this network.

And similarly if we do the calculation in the tabular form early start, early finish for each and every activity we can see activity A, slack 0 C, slack is 0 F Slack is 0 H, sorry I slack is 0 and K slack is 0. So, A C F I and K is our critical path so that path for which our slack is 0.

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Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0 *critical
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	10	9	13	4
F	9	13	9	13	0 *
G	9	13	14	18	5
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	23	18	23	0 *

This is a critical path. So, now, as we have seen this formulation in the previous class also we have to calculate that what is the probability of completion of this project in 24 hour, what is the probability of completion in 22 hours.

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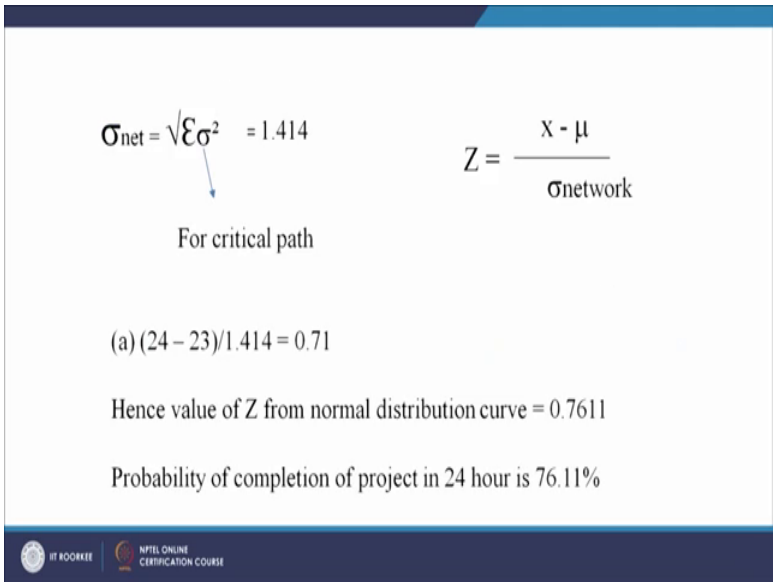

So, using the standard formulation on your screen you can see first we have to calculate the Z static, Z statistics, this is Z statistics x is going to be our expected duration, mu is giving us the duration that we have calculated from the network, x is what is the problem statement that whether it is 24 or it is 22 or it is 28 hours or it is 30 hours. So, whatever is given to us for which we want to check the probability x is going to give us that value. Mu is what we have calculated from the network and this is a standard deviation of the network that is for the critical path we will calculate that what is the summation of the standard deviation for the activities lying on the critical path.

So, in this problem this sigma square, square root of this summation of sigma square sigma net is coming out to 1.414. So, we have to check for 24 hours. So, 24 hours minus our expected duration is coming from the critical path that we have drawn the network and done the calculation, so 24 minus 23 whole divided by our standard deviation of the network. So, it is 1.414 and that network I mean to say the critical path of the network that is coming out to be 0.71. So, the first we have to calculate the Z statistic and from there the value of Z when we see in the table we see corresponding to Z equal to 0.71, we get a value of 0.7611 which means the probability of completion of a project in 24 days, sorry 24 hours is 76.11 percent.

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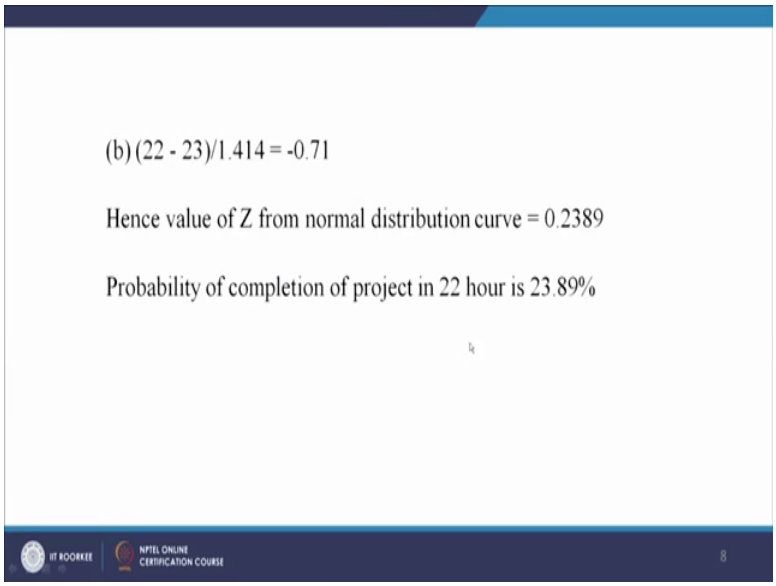

So, similarly just we have to change the value of x which is now 22 hours in our next case. So, when we put this value x is equal to 22 we get a value of minus 0.71 which is giving us 23.89 percent probability which shows that if we extend the duration by one day our expected time is 23 days that we have calculated from our network using the critical path method means the using the calculations based on the critical path. We can see that the duration when is being extended from 23 we are our probability is higher. If you are reducing the duration from 23 to 22 the probability of completion of the project is less.

So, this way we can judge, we can calculate, we can estimate, the probability of completion of a project by just find tweaking our expected duration that is we can see that our as per our network we are getting 23 hours for completing the project; what if we try to increase the duration what is the increase in the probability or what is a change in the probability or if we try to reduce the duration what is a change in the probability and accordingly we can take our decision.

So, with this I think most of you must have been able to know understand and maybe draw the networks and do the basic calculations based on the networks and at least see that what is the critical path and how we can make use of this information of the critical path for a specific network.

So, our focus till today has been on time only because in most of the networks you see we have time based on that time we have done the calculations and figured out, what we have found out? We have found out the project completion time. In our next session we will focus on another important that is the cost because every project will involve some cost and we will focus on the cost calculations related to the project networks.

Thank you.