Operations Management
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Lecture - 37
PERT Problems-I

[FL] friends. So, we are currently in the 8th week of our discussion on the topic Operations Management, and today is 37 session in which we are going to cover the PERT problems.

In our 8th week our focus is on project scheduling or project management using the concept of PERT. In last week we focused on critical path method usually we call it is CPM and we try to solve different problems using the critical path method. We have seen that how to construct a network based on the information available with us. We have also seen what is the Fulkerson rule for numbering of nodes. We have seen that how are critical path can be calculated, how the early start late start at every node can be calculated, how early finish late finish early start late start can be calculated, how we undergo or how we perform a forward pass, how we perform a backward pass, what is a critical path, what is the importance of critical path. So, with all that background we are now trying to understand the problem where a element of probability is also introduced.

So, the overall objective of using a network is the same. In CPM also our objective is to find out that how much time this project will require which are the critical activities in the project on which we need to focus our energy as well as our attention then, how we can manage our resources, how we can allocate our resources in order to complete the project well within the stipulated time. So, same objectives we have in case of PERT also. In PERT also we construct a network we try to find out the critical path, we focus on the critical activities, we try to find out the probability of completion of the project because now here there is an element of probability.

Every activity as we have seen in the previous session has got 3 time estimates. We have got a pessimistic time for each activity which is the largest time estimate for that activity, we have a most likely time and we have optimistic time that is the minimum time that we feel, that we foresee that the activity may take. So, every activity has got 3 time estimates and overall project duration will also have a variance because some of you may say that

if we calculate the overall project duration based on the optimistic time of each activity we will get smaller project duration. If we calculate the project duration calculating the only considering the pessimistic time of each activity we will get a different project duration. If we focus on the most likely time of each activity and calculate accordingly we will get different time duration for the project.

So, therefore, there is a variance in the completion of the project time or the project completion time therefore, we calculate the variance for each activity the standard deviation is calculated and based on that we find out that what is the probability that the project will be completed within maybe 2 days of the estimated time or before time or after time.

Another term that we usually consider in case of PERT is the criticality index. In criticality index we run the simulations maybe number of times considering different time estimate for each activities, and then we try to figure out that what all are the activities that are going to lie on the critical path. Now, suppose we run a 1000 times simulation program and out of 1000 times there is an activity which is always falling on the critical path. So, we can say that the criticality index of that activity is one because 1000 times the simulation has been run and all 1000 times that that particular activity has fallen on the critical path.

So, one maybe to explain that I can just draw one diagram based on which you will be able to appreciate this concept of criticality index.

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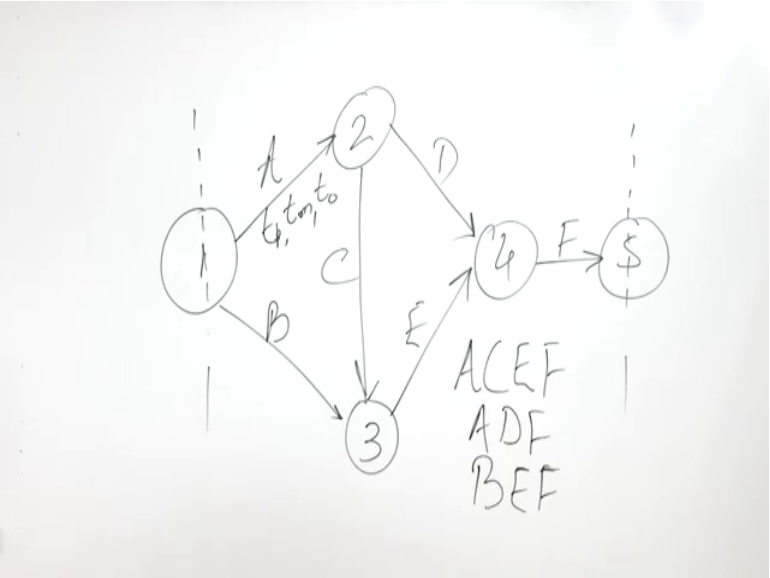

So, suppose we have a network like this, in this network if you see we can number the nodes as. So, if we calculate there are if we see that what are the number of paths available from the start note to the end node. You can see we start from here and this is the last node for this project network, and how many activities are there suppose we number the activities as A B C D E and F. So, we have on this project we have 5 activities, that have to be completed in order to ensure that the project is completed.

Now, we can have time for each activity in case of PERT we will have 3 time estimates for each activity. For A we will have 3 time estimates one will be the pessimistic time, then the most likely time and then the optimistic time. So, we will have 3 time estimates for each activity similarly we will have for B C D and E as well as F, you can see 3 time estimates for each activity 5 different activities how many combinations are possible there are a number of combinations that are possible.

So, we can run a program and taking individual time for each activity and find out that what is the critical path and then we can find out that which activity is lying always on the critical path. And in this network if you see we can have we can just see the number of paths in the network we have one path A C; A C E and F we can write it as A C E and F. Another path can be A D and F, another path can be B E and F. So, I think these are the 3 paths that are available in this network if we start from A D and F, A C E and F, and B E and F. So, there are 3 paths only available in this network and you can see that activity

F is on all the paths. So, we can say that if we do the simulation using different time estimates for each activity if we done maybe 500 simulations activity F will always fall on the critical path. So, the criticality index for activity F can come in the simulation as one or the highest and the highest criticality index activity we have always to consider we have to always focus on that activity because any delay in this activity will delay the overall completion of the project.

So, whenever there is a element of probability involved in each and every activity of the network it is very difficult to calculate the overall project duration and that is why sometimes we take help of the cube computational tools to run the programs and try to see that which all activities are lying on the critical path, or which in other words we can say that what are the activities which have the maximum probability of falling on the critical path. So, that we focus only on those activities in order to ensure the completion of the project well within the stipulated time. So, this is just one example to emphasize the importance of these 3 time domain or these 3 time estimates that we take in case of PERT for each and every activity.

So, similar type of problem we will try to cover today in our discussion and try to figure out that how we can calculate the probability of the completion of a project within a fixed time domain. We can see that the probability will keep on changing as you keep on changing your expected duration of completion of project. So, first we will calculate the estimated duration of completion of project that will be based upon the estimated time for each activity.

If you remember in the last session we have seen that based on these 3 time estimates that is t pessimistic, t most likely and t optimistic. So, the where 3 time estimates t optimistic most likely and t pessimistically calculate unexpected time for each activity and based on the expected time for each activity as we do in deterministic time estimates in case of CPM we will calculate. What we are going to calculate? We are going to calculate the overall project duration and based on that we will then further find out the probability of completion of the project maybe 2 weeks prior to or 2 months prior to or 2 days prior to or 2 hours prior to whatever is the time unit we have taken completion of the project. For example, the estimated time for completion of the project is 24 days.

So, we will we can calculate as we have seen in the last session what is the probability of completion of the project within 22 days or what is the probability of completion of a project within 26 days. So, that probability we can very easily calculate because PERT as I have told earlier his probabilistic in nature. So, with this background and the emphasis on free time estimates and the element of probability involved in estimating the overall project duration I think the concept of criticality is very very important at that I have tried to explain today. Rest the mathematical part of the calculations are easy and can be done and by all the learners. And why do we need to do this type of calculation because always we need to focus on the critical activities to in order to ensure the timely completion of the project.

So, let us know run through our presentation for today and try to understand one simple problem related to PERT.

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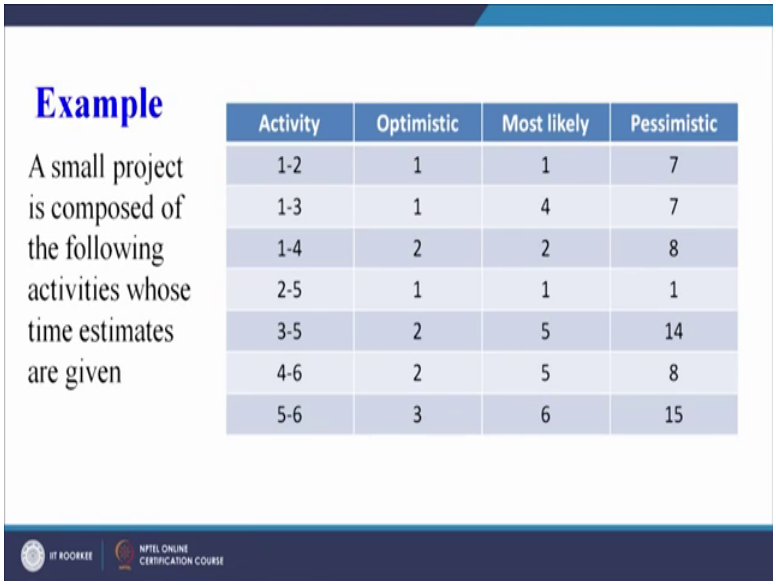

Friends let us take an example on your screen you have one example in which it is a small project that is composed of the following activities whose time estimates are given. So, you can see the easiest way of drawing any network is that we know the nodes the start node and end node for every activity. So, this is a easiest way as in case of critical path method also. We have seen this way of representing our network.

Also we have seen that there are 2 major ways of representing any network, one is activity is on the aero, another one is activity is on the node. So, here in this case one 2

me this is 1 2 is the activity, the 1 and 2 represent the node that 1 is the start node, 2 is the end node for the first activity. Similarly 1 3 is another activity starting at 1 and finishing at node 3. This is one easiest representation because when we have the presidents relationship given we have to first construct the network satisfying all the presidents relationships. So, that additional job is added for constructing the network and sometimes we have to use the dummy activities also in order to ensure the logic of presidents relationships. But this is a simplest way of representing networks and it is easy to construct the network as in this case.

So, how many activities are there we can count? You can see that there are 7 activities in this project. So, we have the optimistic time, the most likely time and the pessimistic time for each and every activity and these 3 time estimates we are going to make use of when we are going to calculate the estimated time for each activity. And if you remember in the previous session we have taken that estimated time t e is equal to t o plus 4 times t m plus t p divided by 6 where t o is the optimistic time, t m is the most likely time and t p is the pessimistic time, so t o plus 4 times t m plus t p divided by 6. So, these 3 time estimates we are going to convert into the estimated time for each and every activity and based on that we will calculate the critical path for the activity and then we will see that what is the probability of completion of the project within that stipulated or calculated time which we have calculated from the critical path.

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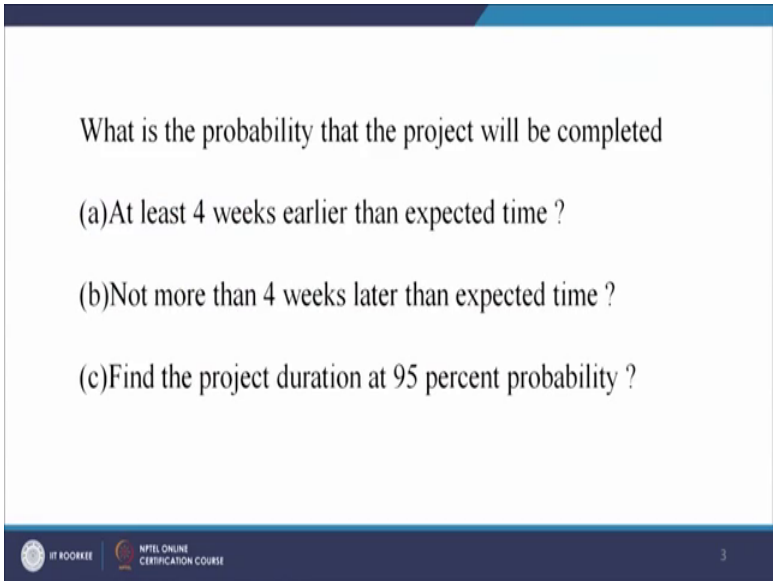

So, we will have a problem at hand now, now what is the problem. One is the information that is available with us in order to explain it in the most simplified manners we have taken a network in which the node numbers are already given the activities are being represented by the start node and end node.

So, there is no problem, but so ever in constructing the network it is easy to construct the networks. Similarly the 3 time estimates are also given. So, based on that what is desired, but we need to calculate. So, we need to calculate 3 things here. What is the probability that the project will be completed under 3 different conditions. So, first is at least 4 weeks earlier than the expected time. So, friends from where we will get the expected time, expected time as I have already told you we have 3 time estimates from these 3 time estimates for each activity we will calculate the expected time.

So, now we will have 7 time estimates only because our project consists of 7 activities. So, for each activity we will calculate the expected time. How we will calculate? All of you know the formula. for calculating the expected time. Now, once we have calculated the expected time for each activity now for 7 activities we have 7 expected time, now we will see what are the different paths in the networks based on that we will calculate the critical path. And once we know the critical path we can very easily find out or we will be very easily able to establish that what is the expected time. So, our first job will be to calculate the expected time now the first question is that what is the probability that the project will be completed at least 4 weeks prior to or earlier than the expected time that we have calculated from the critical path.

Second problem, not more than 4 weeks later than the expected time. So, we have expected time. We wish to calculate that what is the probability that the project will be complete 4 weeks before the expected times. What is the probability that the project will be completed at least 4 weeks after the expected time? And the third problem at hand is that find the project duration at 95 percent probability, that how many weeks will or how many hours it will required in order to in this case our problem is in weeks. So, here we are dealing with week sorry I initially mentioned hour. So, we have to see that how many weeks are required in order to ensure that 95 percent probability is there for completion of the project.

So, we can see that now we can calculate different things related to the project duration and why means another thing that comes to my mind is what is so important about the project duration. Mathematically it is easy all of you have understood that we can we have a piece of information a tabular form of data available with us based on that we draw a network, based on the network we do some arithmetic or mathematical calculations and we find out that what is the critical path. Then in case of PERT as I have drawn the diagram this network and shown that there are a number of permutations and combinations possible, there is variance, there is standard deviation related to the completion of the project because here we are not having deterministic time estimates for each and every activity we have probabilistic time for each and every activity. So, therefore, we need to calculate that what is the maybe probability of completion of the project.

So, mathematical whatever I have stated is absolutely correct, but as a decision maker as a strategist how I can make use of all this mathematical calculations that is the most important thing. Once I know the critical path first and foremost thing is all my energy is must be focused on that critical path or the activities that are lying on the critical paths. The second most important decision is that when you are in business you will be focused on both the ends, you have to deal with your raw material suppliers with your vendors on the other side you have your customers, you have your dealers, you have your retailers, so you are pressed from both sides.

So, in overall business cycle we must know that what is going to be our project duration. And once we know that we can negotiate, we can do hard negotiations with our vendors, we can be very very confident with our customers with our dealers with our warehouse managers because we know that as per my calculation as per my mathematics the project must be completed there is 100 percent probability that the project will be completed within this many number of weeks or this many number of months.

So, it is not just a number it is basically going to help us in formulating our strategy related to management of our overall operations within the organisation. So, we must not just see that some mathematical calculations we are doing and how these are going to affect the operations. So, we must address and we must understand that this is not purely mathematics that is this is going to help us in our decision making related to the operations management.

So, here I have tried to emphasize on the importance of these probabilities that is what is the probability of completing the project 4 weeks before the expected time, what is the probability of completion of the project 4 weeks after the expected time or estimated time. Then if we want to ensure that I have to be 100 percent sure that the project has to be completed what is the duration for ensuring maybe 98, 99, 100 percent completion of the project. So, that number of weeks, number of months we can also calculate mathematically.

So, here we have a problem at hand where we have 3 different calculations to be done. So, first and foremost when you have a project data available with you, you have to construct a network.

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Activity	Expected Time	σ
1-2	2	1
1-3	4	1
1-4	3	1
2-5	1	0
3-5	6	2
4-6	5	1
5-6	7	2

So, here, this is first stage because as we have seen each and every activity in case of PERT will have 3 time estimate, based on those 3 time estimates we will calculate the expected time for each and every activity based on the formulation of formula that we have seen in the previous session. So, that formula is being used here that is t p plus 4 times t m plus t o divided by 6. So, based on that we calculate the expected time for each activity and similarly we calculate the standard deviation for each activity. So, this is the table is showing you expected time and standard deviation for each activity.

And why standard deviation is being calculated? Because if you remember in the previous session we have scenes in order to calculate the Z statistics, we require the

standard deviation, we require the expected time of project completion and we require the expected time plus we also require that the time for which we want the project to be compared that what is the probability that the project will be completed within 28 weeks or 30 weeks as in our case we have to first calculate that what is the expected time for the completion of this project.

So, first and foremost we have to calculate the expected time for each activity, standard deviation for each activity, then we have to network we have to construct this is a most simple form of the network as the node numbers are already known to you 1 2, 1 3, 1 4, 3 5, 5 6.

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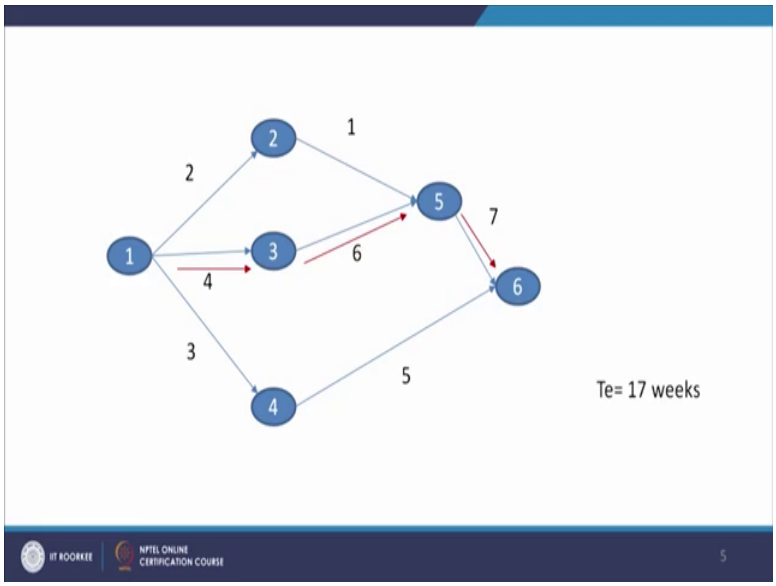

So, this is we can calculate and then we can very simply see is like the easiest method of finding out the critical path if we do not want to go into the calculations of early start, early finish, late start, late finish, we can calculate for the different path what is the time required, so 1 to 5 and 6 2 plus 1 3 plus 7 10, 1 3 5 and 6 6 plus 4 10 plus 7, 17; 1 4 6 3 plus 5, 8. So, we see 1 3 5 and 6 4 plus 6 plus 7, that is 17 is a longest path, 1 3 5 and 6 does becomes the critical path and expected time for completion of this project is 17 weeks.

So, first thing we have already done what we have done first we have calculated the estimated time or expected time for each activity, then we have construct to the network

based on the expected time for each activity we have calculated the critical path and our critical path has been highlighted here 1 3 5 and 6 and the time is 17 weeks.

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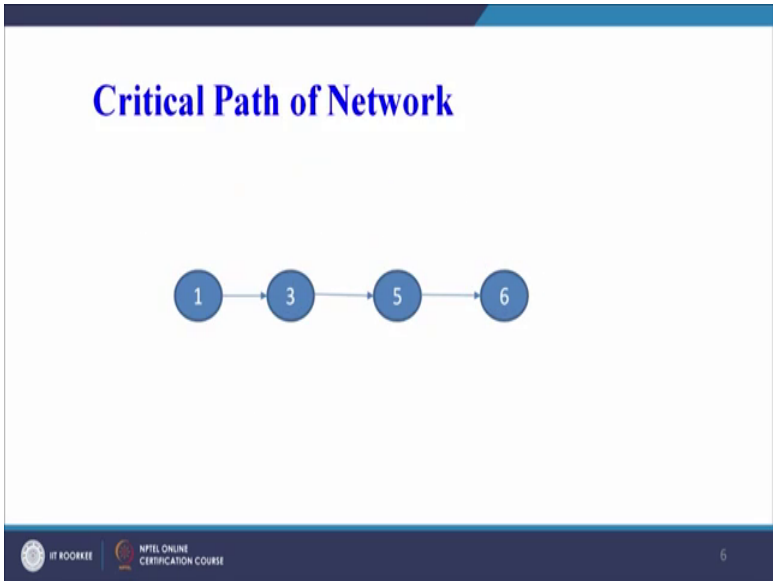

So, this is our critical path 1 3 5 and 6.

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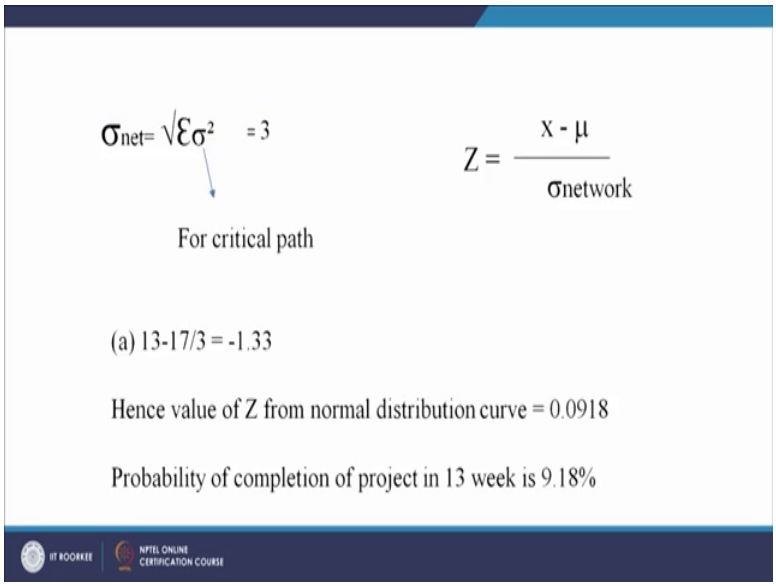

Now, this formulation we have seen in the previous class Z statistic is equal to x minus mu divided by the standard deviation, x and mu you know. So, we calculate the sigma net which is a summation of maybe the standard deviation of the activities that are lying on the critical path. So, for a first problem was that we have to calculate that 4 weeks

prior to the completion or prior to the expected time. So, 4 weeks prior to the expected time of project completion of project completion as per our critical path we have seen the expected time is 17 weeks. So, 4 weeks prior to that is 13. So, 13 minus 17 divided by the standard deviation of the project of the activities submission of standard deviation of the activities lying on the critical path which comes out to be 3, so 13 minus 17 divided by 3. So, the Z statistic, that is Z x minus mu divided by standard deviation.

So, mu is our expected time, x is our variables which we have taken 4 weeks prior to the expected time that is 17th. So, 13 minus 17 divided by 3 comes out in minus 1.33. So, hence the value of Z from the distribution curve from the data table is 0.0918 percent. So, the probability of completion of project in 13 weeks is 9.18 percent only. So, we have calculated that expected time is 17 weeks, we are calculating the probability for completion of the project in 13 weeks which is 9.18 percent only.

Now, suppose we have to calculate 4 weeks after the expected time, expected time remains same that is 17, 4 weeks after mean 17 plus 4, 21 weeks, so 21 minus 17 divided by 3, it comes out to be 1.33.

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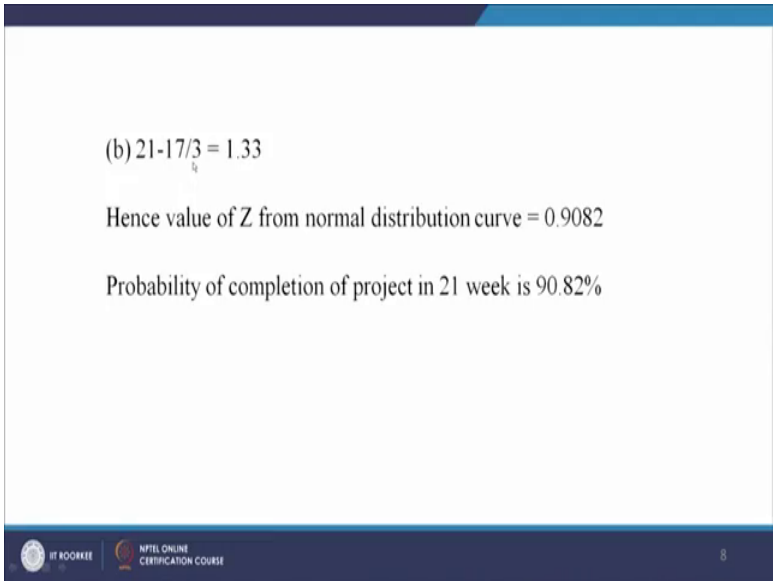

And I must say that is divided by 3 is for overall 21 minus 17, it is not 21 minus 17 divided by 3, this is 21 minus 17 that is equal to 4 divided by 3 which comes out to be 1.33, as you can see in this formulation here x minus mu divided by the standard deviation.

So, the x minus mu value has to be divided by the standard deviation. So, here 21 minus 17 4 divided by 3, 1.33. Hence the value of Z from normal distribution curve is 0.908 corresponding to the Z value of 1.33. So, probability of completion of project in 21 is 90.82. So, you can see that once we are trying to shorten the duration from 17 weeks to 13 weeks we are getting 9.18 probability only, but when we are extending the project duration from 17 weeks to 21 weeks we are getting a higher probability of completion of the project that is 90.82.

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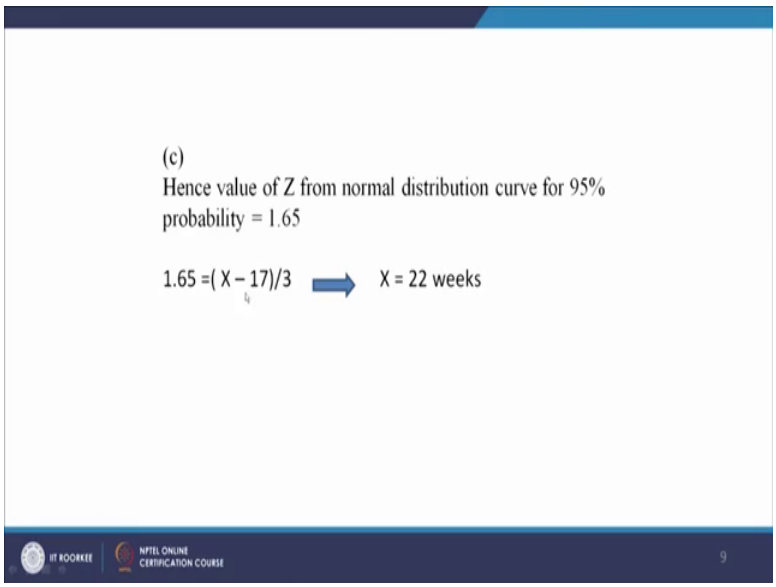

And the last problem is we have to calculate that, what is the duration, what is the expected duration of the project so that 95 percent probability is there for the completion of the project. So, for 95 percent from the normal distribution curve we can calculate Z value must be 1.65. So, in the previous to A and B problems we were actually calculating Z and then seeing in the table that corresponding to that Z value that we have calculated what is the probability. Now, here probability is given to us we are doing the back calculation based on 95 percent probability we are doing the reverse calculation what is the value of Z.

So, in this case that is known to us the expected time that we have calculated from the critical path, 17 is known to us standard deviation, for the activities on the critical path is known to have summation of standard deviation. So, we have out of the 4 things Z, x, mu, and the standard deviation; in this case Z is known to us we have to calculate the

value of x because mu and standard deviation is already known to us. So, from here we can very easily calculate x, x minus mu that is expected time 17 days divided by 3 here it is correctly represented bracket is there, x minus 17 divided by standard deviation equal to corresponding to 95 percent probability that Z value is 1.65. And from here we can calculate the x is equal to 22 weeks. That there is a probability that if we are given 22 weeks we will be able to complete the project 95 percent probability is there that a project will be completed in 22 weeks.

So, you see that when we are reducing the project duration we are seeing what is change in probability, when we are giving more time than the expected time the probability of completion is increasing. And further x is increasing, so last calculation that we have seen for x is equal to 21 days we have seen the probabilities 90, approximately 90 percent. But here we are seeing that is further one additional week we spend the probability increases to 95 percent. So, PERT can give us a probabilistic estimate of completion of the project and this can be helpful for the managers of an organisation to plan their operations accordingly.

So, with this become to the end of today's sessions. In next session we will try to address a different problem based on PERT.

Thank you.