Operations Management
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Lecture – 36
Program Evaluation and Review Technique (PERT)

[FL] Friends welcome to session 36 in our course on operations management. And today we are going to start the discussion for the 8h week we have already completed 7 weeks of discussion and we have covered different topics. I must say diverse topics related to operations management, starting from product design and development to layout selection to location selection prior to layout selection. Then we have seen the scheduling part using the project networks and the last week if you remember we were discussing an important topic that is the critical path method for finding out the project duration.

So, we were able to find out a critical path, which is the longest path in a network and it gives us the minimum time required to complete the project. As we have seen that if we are following the CPM method we have deterministic time estimates available with us; which means, that for every activity we have fixed time that is available we know, that this activity will require this much time for completion whereas, which may not be the actual scenario. In actual scenario there can be an element of probability that the activity may take a longer duration also or may be completed before the time that we have decided or calculated.

So, in research type of projects for example, we launch we plan to launch a space vehicle, we may not be having the exact idea that which activity may take how much time. Specifically, in research-based projects we are not aware or we are not sure with certainty that this research will take this much time it may take a longer time also. If we are successful or if we are lucky I must say with a project may be completed before the due deadline also.

So, there is an element of probability in terms of the time that an activity or a total project will take. Similarly, as I have already told in a overall project there will be number of activities that will sum up for the completion of the project. Now individual activities will have different time estimates some activities may take longer time, some activities may take shorter time and individual activity also sometimes may be completed

well in time may take longer than the expected time may be completed optimistically in a shorter time as compared to the deterministic time that we have fixed for that activity.

So, in nutshell I must address that in critical path method whatever problems we have seen the problems were with fixed time estimates. So, each activity or each task or each job was given a specified time. Now in case of pert that is program evaluation and review technique, we will have different time estimates for each activity to be more precise we will have 3 time estimates for each activity, one will be the most likely time the most likely time I can draw an analogy with the CPM method that that is the expected time for that activity that this is the time that the activity will take for completion.

But in many cases if everything goes wrong the activity duration may be delayed the activity may take take longer time as compared to the most likely time, and that time we will call as the pessimistic time. On the other hand, or on the contrary there can be a situation that; we are able to complete the activity in a shorter duration as compared to the most likely time we are lucky everything was in the favor and we were able to complete the project well in time or I must say before that time. So, that time is called as the optimistic time.

So, in case of program evaluation and review technique we will be having 3 time estimates for each activity that will be the optimistic time, the most likely time and the pessimistic time. Now some of you may be wondering that what is the difference between the CPM method and the pert method. So, one is that for every activity we have fixed time or deterministic time in case of CPM whereas, we have a probabilistic time in case of pert. We will have 3 time estimates for each activity in case of pert whereas, we will have single time estimate for each activity in case of CPM.

So, in nutshell we can summarize in single sentence or sentence that; CPM is deterministic in nature pert is probabilistic in nature. And the other difference is that CPM is usually applied the critical path method is usually applied in those projects where we know with certainty that this is going to be the time required for an individual activity. For example, construction of a house we see so many multi story towers or multi story buildings coming in. So, it is easy to figure out easy to predict that this is a 22-story building this much square fit area has to be covered.

So, with the experience the builders, the contractors, the real state agents know that this will require this much of time. Every story will require maybe this many months of construction activity. So, already that data is available for every step for every activity involved in construction sector the time can easily be deterministically known. So, one example of CPM can be the construction of a house whereas, in case of pert an element of probability is always attached. For example, a student joins for a PhD program there is a element of probability if he gets the results the project can be completed or the PhD dissertation can be completed or the PhD thesis can be completed in 3 years if he works for a longer time he may take 4 years time also.

Similarly, if you are doing a research for inventing a particular type of polymer you do not know when you will be successful in that invention process. So, there is an element of probability in these types of projects. So, research-based projects will be using pert and the construction sector-based project or those projects, where we have adequate know how already available with us will be using CPM type of project network.

So now we have understood the difference between CPM and pert and where which technique will be applied must be clear by now to all the learners. So, with this background we start our discussion for week 8 and today is the first session of week 8 and the first topic is program evaluation and review technique. We are just going to cover the introductory part of this topic, and we will try to see that how we can calculate the probability of completion of a project or we can find out that given a due date that; suppose we say the project has to be completed by 25th of september 2017.

So, we can calculate based on the project network we will calculate what is the expected time for the completion of the project, what is the deadline that is 25th september 2017, what is the standard deviation for the activities on the critical path and thereby we can calculate that what is going to be the probability of this project to be completed by september 25th 2017. So, we will see that what is the formula related to that.

So, let us maybe this background if is clear to all the learners the other part that we are going to cover today will not be that difficult for you to just assimilate in your knowledge base. So, let us quickly start our discussion related to the topic that is program evaluation and review technique.

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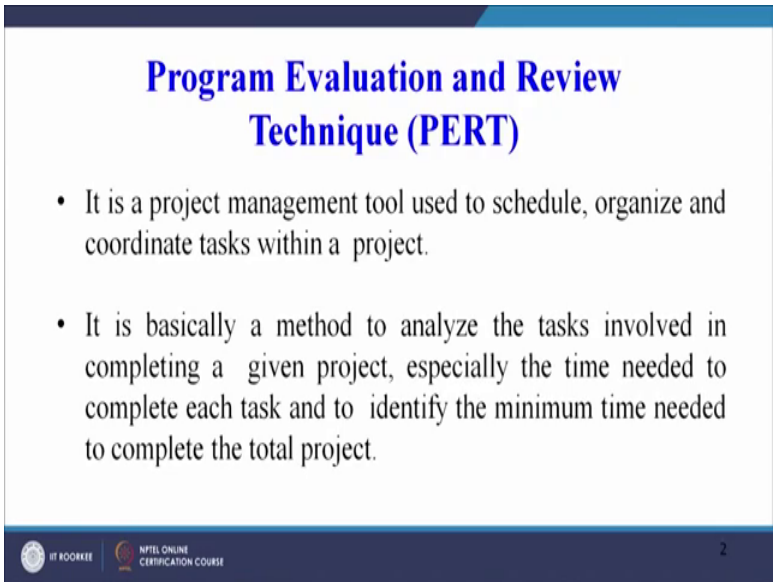

Now, it is a project management tool as same as critical path method and is used to schedule organize and coordinate tasks within a project.

Now, there are 3 things schedule organize and coordinate. So, it is not only related to time it is related to the other coordination activities also. We need to coordinate among the various activities which have to be completed in order to ensure that the project is completed well within the specified time. As we are going to find out the critical path all the activities that are lying on the critical path cannot be delayed even by a single day because it will lead to the extension of the project by one day.

Now, suppose we are focusing our attention on the critical path. The non-critical activities have the flexibility they have the slack that we can readjust them reschedule them and thereby we can release some of the resources by rescheduling these non-critical activities and those resources can be used to reinforce the critical activities. So, that the critical activities are completed as expected, as planned and the project is completed as per the due date.

So, it is not only related to time that we are going to use the pert network. We are going to use it for our coordination purpose also for our other organizational decision-making purposes also; that where how many people will be required and how many people can be laid off other decisions can also be taken based on the project network. So, pert is a very important project management tool, which is not only used to schedule the various

activity, but is also helpful in the overall organization and coordination among the various activities. It is basically a method to analyze the tasks involved in completion of a project or in completing a given project.

Especially the time needed to complete each task and to identify the minimum time needed to complete the total project. As as we have seen in CPM also we try to find out that what is the minimum time required for the completion of the project. Similarly, in pert also we will try to find out that what is the minimum time required for completion of the project. How we will find that? That we are going to understand today because in pert we will have 3 different time estimates, which will be pessimistic, optimistic and most likely time based on that we will calculate and try to find out the critical path, which will be the longest path in the network and will give us the minimum time required for the completion of the project.

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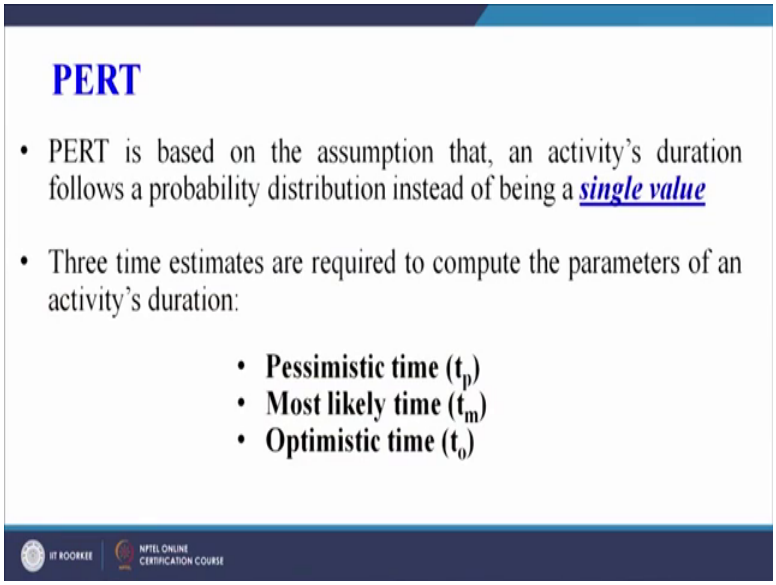

Now, pert is based on the assumption that an activities duration follows a probability distribution instead of being a single value. So, on your screen you can see we have highlighted the term single value. Now single value basically is deterministic time which we use in the CPM method. So, pert is basically based on the assumption that the activities duration follows a probability distribution, which means that if you do the same activity again and again you will not be able to complete it in the same given time. Sometimes you will be completing it before the average value sometimes you may be

delayed from the average value. So, you may be able to complete the activity after the average time taken.

So, there is a probability that the activity may be completed in the average time it may be completed before the average time it may be completed after the average time. So, therefore, there is a probability of completion of activity in case of pert is specifically in terms of the time of completion of an activity with respect to the duration of the overall project. So, because the overall project will have some overall project duration time and individual activity also has a probability that it may be completed early it may be delayed or it may be taking an average time.

So, 3-time estimates are required as I have already highlighted to compute the parameters of an activities duration. Now an activity duration can have 3 values or this variable can take 3 values. First value is the pessimistic time which is the longest time the activity may require for completion. Then the most likely time depicted by tm which is we can say a kind of an average value which can be used for calculating the expected time for completion of an activity. And finally, the optimistic time that everything goes well we are able to complete the activity well in given time or maybe well before the most likely time.

So, most likely time is a central value on one side we have the pessimistic time on other side we have the optimistic time. And these 3-time estimates will be used for the calculation of the expected time for the completion of the activity. So, these 3-time estimates that we are using here, many times the learners are a bit confused that these are for the project or for the activity. Now these 3-time estimates are for each and every individual activity or task which has to be completed for the completion of the project. For example, a project has 10 different activities. So, we will have 10 different pessimistic times, 10 different most likely times and 10 different optimistic times.

So, for every activity we will have a pessimistic optimistic and the most likely time. And then for the project we will calculate the project variance and the project standard deviation, but for every activity we will have these 3 time estimates. I think all these 3 points I have all explained.

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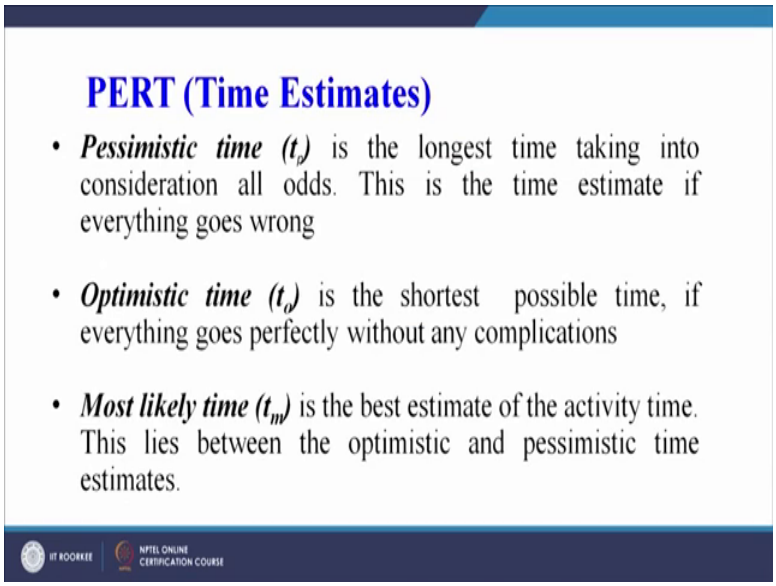

So, quickly we can read that what are the 3 time estimates in case of pert. First one is the pessimistic time depicted by t with subscript p, it is the longest time taking into consideration all the odds away all the odds in the activity that everything is going wrong or maybe it is being delayed everything related to the activity is delayed. So, this is a time estimate if everything goes wrong.

So, we are taking much more time as compared to an average time that must have been taken for the completion of that activity. So, that time estimate we usually call as the pessimistic time. Then opposite to the pessimistic time we have the optimistic time. Now optimistic time is the shortest possible time if everything goes perfectly without any complications.

So, in the pessimistic time we have lot of complications involved therefore, the project sorry the time duration for that activity gets delayed whereas, in optimistic time there is no complication as such without any complication everything goes on well and we are able to complete the activity well within time maybe even less than the most likely time. And the most likely time is the best time estimate of the activity time this lies between the pessimistic time and the optimistic time estimates.

So, basically for every activity now the engineer may not be giving a single time estimate with these experience or her experience an engineer will specify the 3 time

estimates for each activity. Now these 3-time estimates will be the pessimistic time the most likely time and the pessimistic time the optimistic, pessimistic and most likely time.

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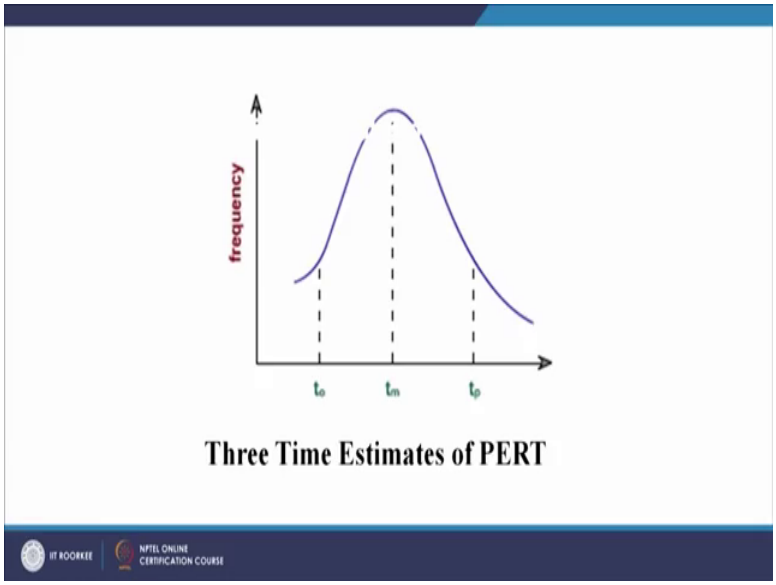

Now, this graph is a very simple graph which is showing the 3 time estimates of pert which we have seen in the previous slide. And on y axis we have the frequency and on x axis we have the time estimate.

So, we can see the optimistic time the frequency is lower similarly for the pessimistic time the frequency is again lower, but the most likely time has got the maximum frequency. So, this means or this shows that suppose we conduct a particular activity of a project 100 times. I think I have been able to make it clear that in the overall project we pick one activity and that activity we perform 100 times.

We will get maybe maximum times the time required for conducting or for organizing that activity or for completing that activity will be maybe 60 times out of 100. And maybe 25 times we may complete it before the most likely time and then maybe remaining maybe 15 times we may complete it after the most likely time. So, if we fix the time estimates. So, most likely time we will have the maximum frequency and pessimistic and optimistic time we will have lesser frequency.

So, out of 100 60 times we are completing it at an average value of most likely and 25 times as per my example in optimistic and 15 times in pessimistic time estimate. So, we

can see that the time is following a distribution or the time required for completion of an activity is following a distribution here whereas, in case of CPM it was one single time estimate which was used for the calculation of the critical path. So, that is one may be major difference between the CPM and the pert technique.

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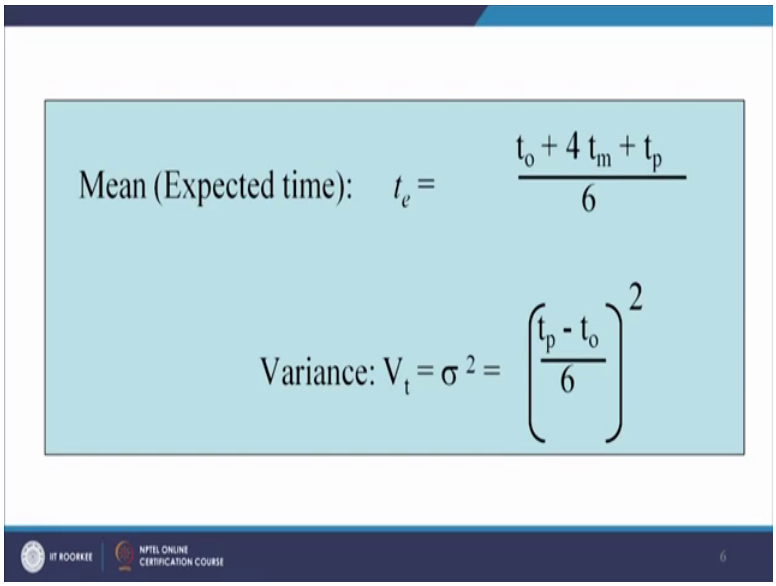

Now, we will try to understand that how to perform the calculations. Now one method can be that in case of pert we have 3-time estimates. So, we can calculate the project duration by only considering the most likely time and we can say ok. This is going to be my project duration, but things may not go as we have assumed. So, that assumption part has to be minimized and scientific logic part has to be maximized. So, that we are able to complete the project as per the deadline or as per the schedule.

So, in case of only using most likely time we may not be able to find out the exact time required for the completion of the project. And therefore, we use another value which is going to give us a better estimate of our project duration and that we usually call as the expected time denoted byte. And that is dependent upon all the 3-time estimates that we have seen till now that is the optimistic time the most likely time and the pessimistic time.

So, the expected time is calculated based on these 3-time estimates. And then expected time is used as we use the deterministic time in case of CPM to calculate the critical path. So, the formula for expected time is given on your screen you can see te is equal to

to that is the optimistic time plus 4 times the most likely time plus the pessimistic time divided by 6. So, for every activity we have to calculate the expected time in terms of te, which is a single time estimate now for that particular activity. And then we calculate the variance that is given by sigma square is equal to tp minus to that is the pessimistic time minus the optimistic time divided by 6 as a and then the whole square of this value.

So, we can calculate the variance for each activity, we can calculate the expected time for each activity. So, in nutshell in order to summarize or in order to explain to a layman without going much into the mathematics involved in that; we can say that we are trying to convert a probabilistic type of problem into a deterministic type of problem that is CPM.

So, we the 3-time estimates that we had for each and every activity of the project we are bringing these 3 time estimates into one-time estimate, which will be further used for our calculation of the overall project duration. Also, we are calculating the variance for each activity and then adding up the variance of the activities that will lie on the critical path. We can calculate the overall project variance also and try to figure out that what is the probability of completion of the project as per the schedule date.

So, these are the 2 values that we will calculate for each and every activity. Now for calculating the critical path we have already seen a problem in the last session or in the last week we have seen in CPM different types of problems at least 2 3 problems. We have seen there in which we have calculated the critical path here also we will have to calculate the critical path and identify our critical activities as well as the non-critical activities.

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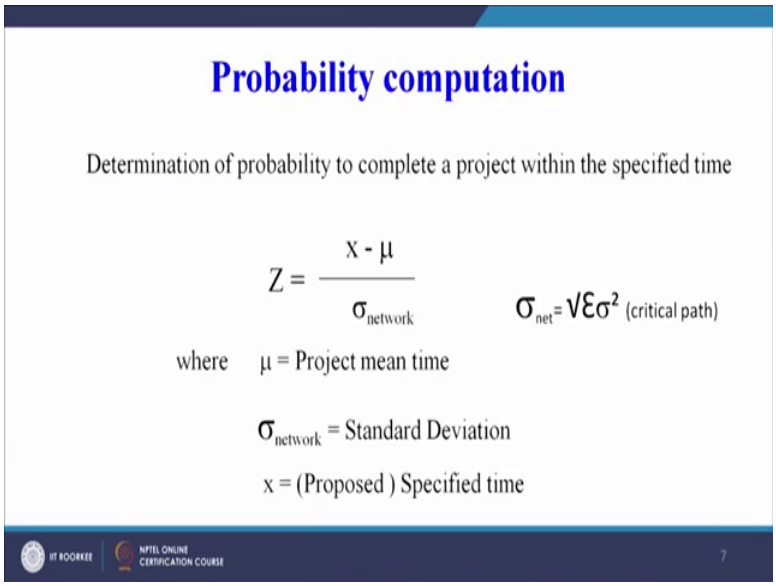

Now, we can mathematically find out the probability to complete a project within the specified time. Now what is the difference in case of CPM we had defined time estimates for each activity and we were able to find out the early start and late start for each and every activity and from they are calculating the slag we were able to find out the critical path, but here since there is probability involved in each and every activity. So, the overall project duration is also probabilistic in nature therefore, we can determine the probability to complete a project within the specified time using this mathematical formula on your screen. We usually calculate the value Z which is equal to x minus mu divided by the standard deviation.

Now, x is what x is the proposed or the specified time now it can be if we fix up a date we can calculate how many days. So, suppose we say a first of October. So, we can calculate first of October from today how many days are there, then we can calculate mu which is the project mean time which will also be a number in terms of number of days. So, it can be the project mean time calculated from the te values. So now, in a pert network instead of 3 time estimates we will use one time estimate that is te that is the expected time. Now this based on this expected time we will get one value that is being given as mu that is the project mean time.

So, using te we have calculated mu that is the project mean time which is which has been found out from the critical path of the network. We know the date by which we want the

project to be completed. So, the difference between the 2 the deadline the time required for the completion of the project and it is divided by the standard deviation we will get one value. The value is given as the Z statistic. And that Z statistics a will be found out from the tabular data which is already available with us. And corresponding to the z value we can see that what is the probability of completion of a project within the specified time.

So, mathematically we have to find out the z value and from that z value just we have to look at the table which is available in almost all books related to operations management related to CPM and pert from there we from z we can correlate what is the probability of completion of the project. So, this is simple statistics we will try to use this formula in our calculations and find out and learn that how the values come from the network and how we use them using this formula. Sigma is you can say summation of the variants for each activity and then square root of that will be giving us the standard deviation.

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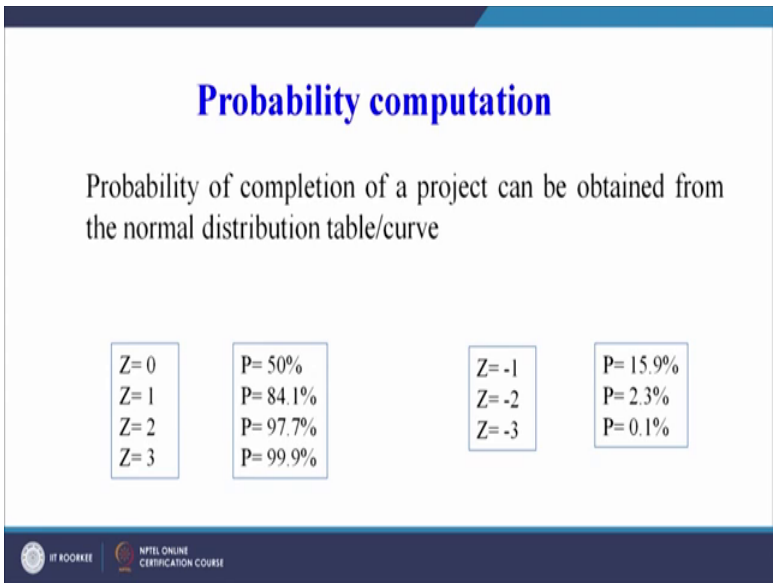

Now, probability how we will calculate this data is available Z equal to 0 probability 50 percent and accordingly we can see based on the value of Z. We can get the probability we will use it in our calculations when we solve the problems for pert.

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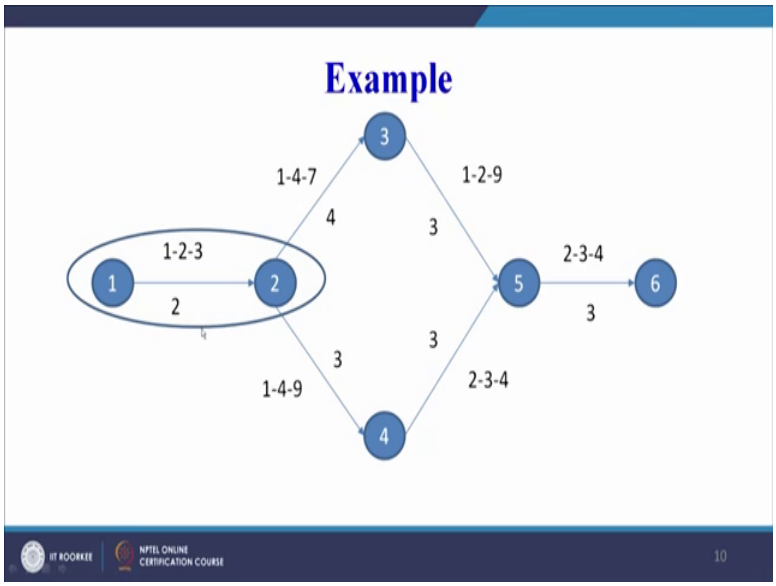

Now, this is one simple example just to end today's session, how to draw the network is known to all of the learners now here we see there are 1 2 3 4 5 and 6, 6 nodes are there 1 2 3 4 5 6 activities we have here. So, you can see activity 1 2 there are 3-time estimates. The pessimistic time is 1 the most likely time is 2 and the pessimistic time is 3 and we can calculate for this activity that what is the expected time using a formula.

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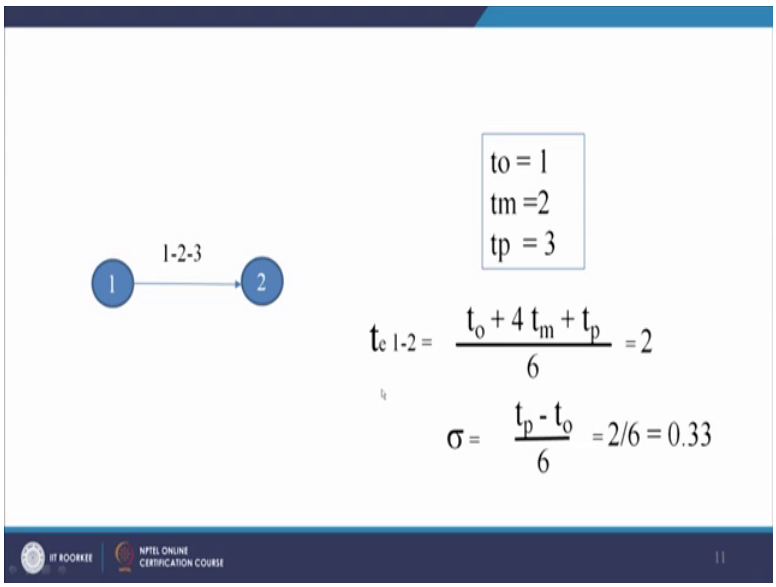

Now, expected time for activity 1 2 is the optimistic time plus 4 times the most likely time plus pessimistic time divided by 6 it comes out to be 2. And the standard deviation

can be calculated by the pessimistic time minus the optimistic time divided by 6; that is 0.33. So, we have calculated the standard deviation for one particular activity we have calculated the expected time based on the 3-probabilistic time estimates for one activity.

Similar, calculations will be done for all the other activities then we will calculate the standard deviation summing up all the activities that lie on the critical path and finally, we can calculate the Z statistics and then from the table we can see that what is the probability of completion of a project within the specified time.

So, with this we conclude today's session in next session we will discuss the problems related to the pert technique of project scheduling.

Thank you.