Operations Management
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Lecture - 35
Critical Path Method: Problems – II

[FL] friends. So, today we have the last session of week 7 that is session number 35 and in this week, we are focusing our attention on critical path method. So, by now, we know; what is the importance of critical path method in project scheduling, what is a critical path method, how to make a project network, what are nodes, what are the different types of activities, how the network and be represented such as activity on node and activity on arrow.

How the forward pass calculations can be done, how the backward pass calculations can be done and how we can find out the critical path. So, with this background information today we will try to further understand few networks and try to understand that; what is the importance of the critical path before, we go to the examples, today the importance of critical path, I want to highlight by pointing out that the activities that are lying on the critical path cannot be delayed or cannot be rescheduled otherwise the overall project duration will get delayed.

So, our focus must always be on the critical activities in a project network. So, that these activities are completed as per their scheduled duration or as per their scheduled time if these activities get delayed because of one reason or the other reason the overall project duration will also get delayed or extended and there are penalty cost associated with the delayed delivery of projects therefore, it is very very important that we focus on these activities which activities the critical activities. So, that we are able to complete the project in time.

Where ever little bit of flexibility is available with us for the noncritical activities we can reschedule them in order to make way for the critical activities sometimes it may so be possible that we have a resource constraints of a machine or manpower. So, what we can do; we can reschedule the noncritical activities and focus those resources or additional resources on critical activities. So, that our project duration is remaining intact or our

project is completed by the deadline or as per the time calculated from the critical path method.

Now, in the previous session we are calculated the 22 days are required for completing the project. Now, there were 5 activities on the critical path. Now these 5 activities must not be delayed otherwise the project duration will go beyond 22 days where as the other 7 activities have little flexibility that we have calculated in the form of total float.

So, we can reschedule those 7 activities in order to complete the project in 22 days. So, now, we will try to understand the problems maybe few other problems and how to draw the network and in the previous session if you remember the project network was the first stage of learning where the nodes were directly given.

So, you have to directly plot them on a piece of paper 1 2, 1 3, 1 4, 2 4, 3 5, maybe that way, it was easy to construct the network, but in most of the cases we may not get that kind of information. So, today, we will try to see that how to construct a network when node numbers are not given only the names of the activities are given or the alphabet alphabetic representation of the activities are given or is given.

Now, how do can construct the networks again using the forward and the backward pass we will calculate the early start early finish late start late finish for each activity and then we will see what is the critical path in the next problem, we will try to understand the use of dummy activities that why dummy activities are important and how they help us to satisfy the logic of the network or to fully comply with the precedence relationship of the network that we will try to understand and finally, we will try to see actual problem in which a problem statement is given we have to construct network and find out the critical path or the time required for the completion of the project.

Now, quickly we will go through this routine of this problems and I advise that all learners must focus on different types of problems based on critical path method there are number of good books available, you can focus on those groups those books and then find out different related problems to CPM and try to solve them on your own because this is something which you can gain through practice you cans look at different types of problem we are trying to highlight the most simplest problems in order to explain the concept of CPM, you can build on this concept and improve and hence or maybe in increase your knowledge related to the critical path method.

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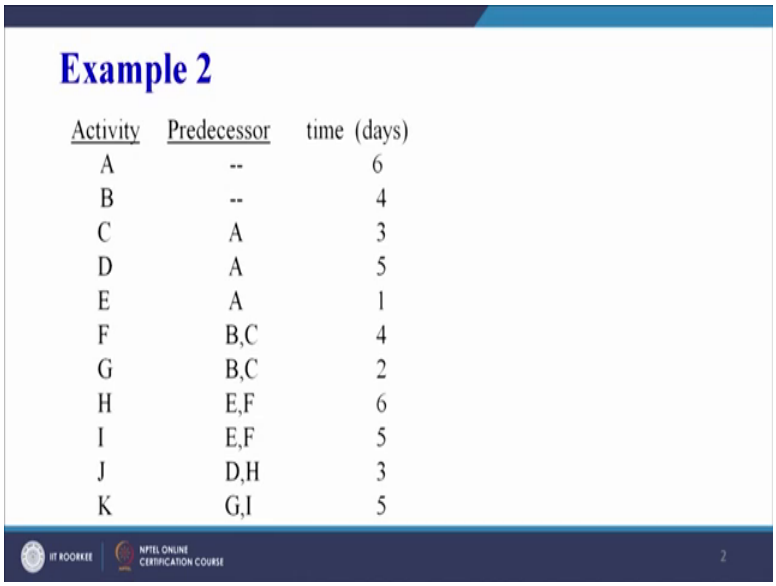

So, coming onto the very first problem in the example 1 that we have seen in the previous session, we have taken the node number as the activity names as 1 2, 1 3, but here activities are given and their predecessors are given in the form of alphabets only and the time required in days is also given.

So, for any CPM method you will get the information in this form the time is deterministic here activity if fix time it that is 6 days are required to complete activity A. So, once we have this type of relationship with pencil, we can try to construct satisfy the diff different relationships. Now pre dis precedence relationship those of you who are only attending this session maybe online H is dependent on E and F.

So, predecessor is E and F for the activity H which means that H cannot start until E and F has been completed. Similarly for activity A and activity B, there is no predecessor which means they are the first activities of the project and lastly activity K is dependent G and I which means that activity K cannot be completed un unless G I and have been completed. So, K is dependent on G and I, H is dependent on E and F, A and B are not dependent on another activity and the time in days is also specified.

So, let us see the network based on this information. So, the information here as we have seen is the predecessor are given for the various activities and based on that we can draw the network.

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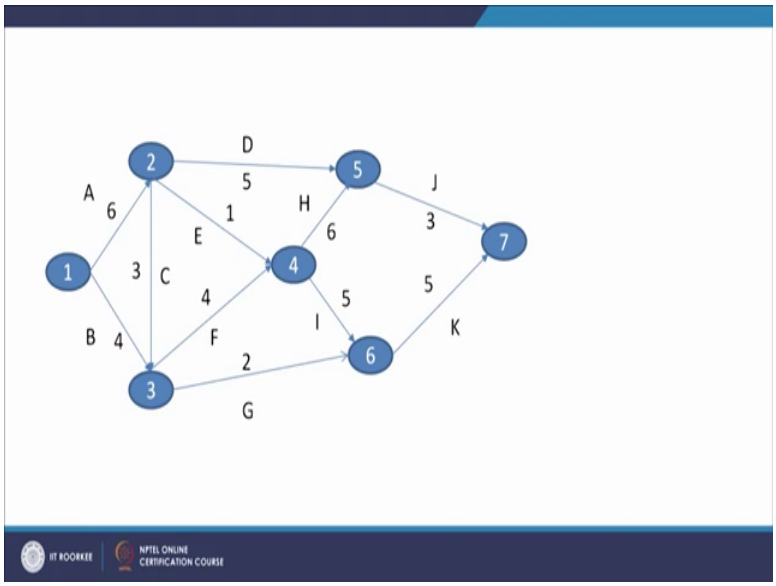

This is the network based on the information available in the previous slide.

So, we can see that J is dependent upon D and H which means J will only start after D and H have been completed. Similarly, K will only start after G and I has been completed. Similarly A is precedent or predecessor or is in precedence relationship with 3 activities, once A is completed D E and C. So, we can say C, E and D will start after A has been completed. So, similarly a network can be constructed based on the information and we can calculate the critical path. So, based on the information available, we will calculate the early start early finish late start late finish the as explained in the previous session and based on that information, we will get up float like this and we can calculate the slack which is a given here.

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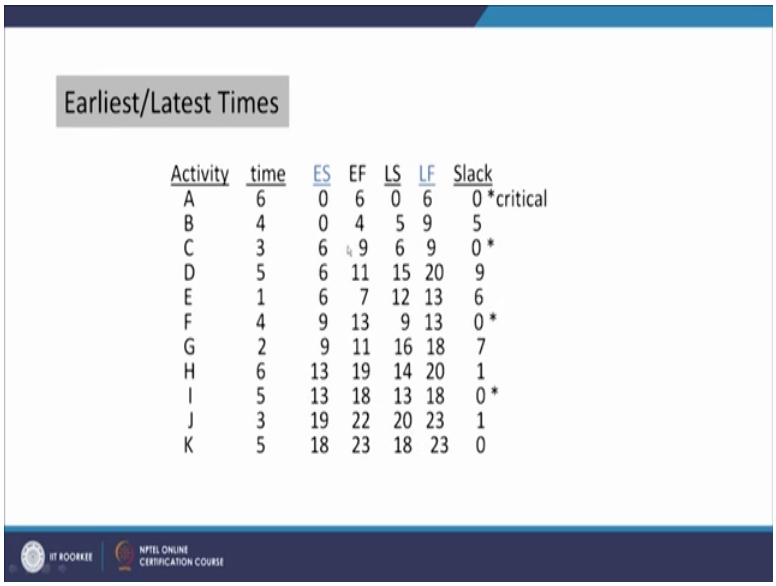

So, we can see here late start minus early start 5 minus 0; 5 is given here. Similarly, here we can see late start minus early start 20 minus 19; 1 is given. So, in this way we will see that for which activity is slack is 0, those activities will fall on the critical path. So, I think I will try to see here A, C, F, I and K are on the critical path.

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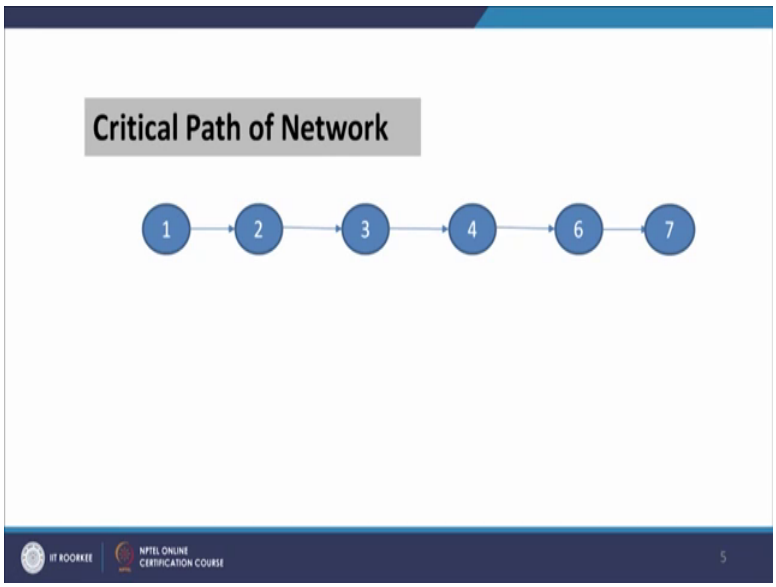

So, let us see. So, 1, 2, 3, 4, 6, 7 are the nodes of the critical path. So, let us see now the network 1, 2, 4, 5 and 7 is the critical path. So, we can see 1 2 A, 2 4 E, 4 5 and 5 7.

So, this is the critical path that is 6 plus 1; 7 plus 6; 13 plus 3; 16. So, maybe this is the critical path as per 1, 2, 4, 5 and 7 is the critical path 1, 2, 3, 4, 6 and 7; sorry, 1, 2, 3, 4, 6 and 7 are the critical path again I will go. So, that there is no confusion 1, 2, 3, 4, 6 and 7 is the critical path.

So, let us calculate now 1, 2, 3, 4, 6 and 7. So, 1 to 2; 6 days, 2 to 3, 3 days or maybe it is days only as far as I remember, yes it is days. So, 1 to 6 days; 2-3; 3 days, 6 plus 3; 9 plus 1, 2, 3, 4, 4 days so, 6 plus 3; 9 plus 4; 13 and then 5 days 13 plus 5; 18 and K that is 6, 2, 7, 5 days 23.

So, the longest path maybe in this network will be of 23 days duration and that will be the minimum duration required to complete the project and if the pro if any of these activities that are on the critical path that is activity A, activity C, activity F, activity I and activity K; all these activities which are on the critical path if these activities get delayed the project network will.

Now, the project duration will get delayed. So, this is the critical path 1, 2, 3, 4, 6 and 7 that is 1 2 A, 2 3 C, 3 4 F, 4 6 I and 6 7 K. So, we can identify the activities that are lying on the critical path and focus on those activities. So, that the overall project duration is achieved.

Now, this is example 3 similar type of example A B C D E F G H.

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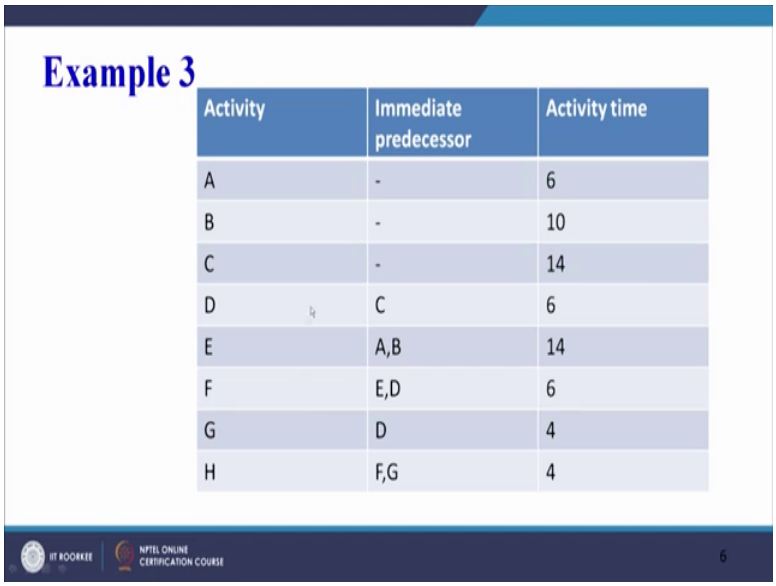

Activities and the immediate predecessor are given C A B. So, H can only start after completion of F and G. Similarly F can only start after E and D have been completed the activity time for each activity is also given to let us quickly see this is the type of network that has to be constructed and it explains the use of dummy activities is here 3 to 2 is the dummy activity.

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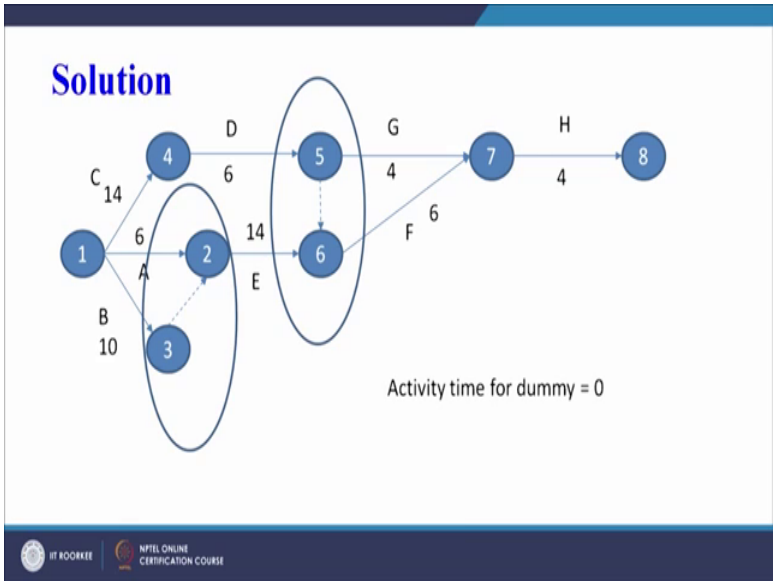

Similarly 5 to 6 is the dummy activity. Now why this dummy activities have been used, now you see E is dependent on A and it is also dependent on B.

So, as per our rules for drawing the network no 2 activities can have the same start node and the finish node or the end nodes. So, we cannot connect B directly from A to sorry from node 1 to node 2. So, we cannot make node 1 and 2 as the starting points of activity A starting and end points of activity A as well as starting and end point of activity B some of you may say that we 1 and 2 we can connect also activity B like this. So, it is against the rules of construction of project network. So, therefore, we are using a dummy activity here as E is dependent on both A and B. Now let us see our information we can see activity E is dependent on both A and B.

So, that has been depicted with the help of this dummy activity there is another dummy activity we are using here the situation is slightly different you can see activity G is dependent on D. So, as per the table available with us G has a immediate predecessor D. So, G can only start once D has been completed. So, G can only start once D has been

completed, but F is also dependent on D and E. So, that also, we can see F is dependent; F is dependent on activity E and activity D F is dependent on both.

Now, let us see; what are the alternatives available with us for drawing this network this is the network. Now what we can do? We cannot have G and F as starting from the same node why because G is not dependent on E now suppose we dont put this dummy activity here, then F will also start from here. So, that is against the rule of drawings the network node 2 activities that is activity G and activity F can start from the same node 5 and end at the same node 7.

So, therefore, we have to start them separately what can be the other option available with us activity D directly comes to activity 2 node 6. So, week again from here we cannot have 2 activities G and F starting from the same node 6 and ending at the same node 7 and it will also make E as a predecessor of G which we do not want because E is not a predecessor G only D is the predecessor of g, but E and D both are predecessor of F therefore, this necessitates the use of the dummy activity, but the activity time for dummy is always taken as a 0 which means the dummy activities are incorporated into the network are put or used in the network in order to establish the precedence relationship among the various activities, it does not take any resource, it does not take any time.

But only is put into the network in order to establish the relationship. So, we can see maybe forward and backward pass immediately or very quickly the calculations which are not a being affected by the use of the dummy activities. So, let us see these 2 scenario where dummy activities have been used we see a dummy activity here between node 5 and node 6 and dummy activity between node 3 and node 2.

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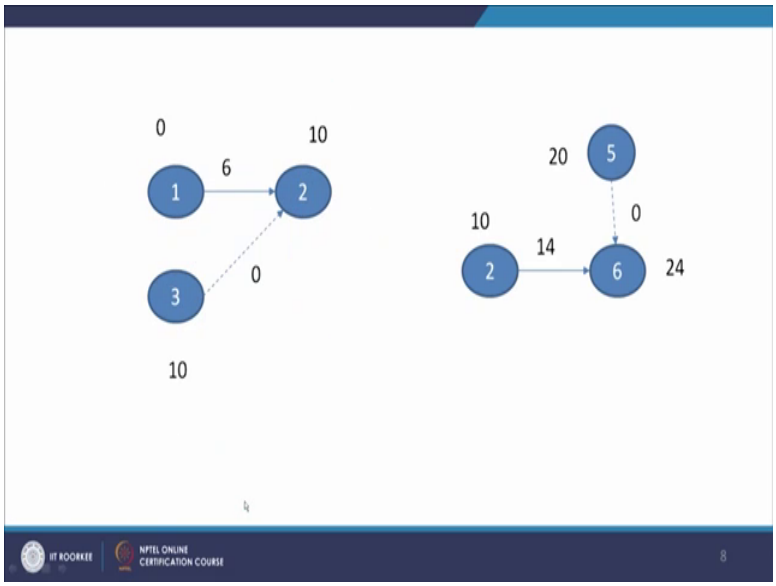

Now, this is the situation 1 2 and 3 is as dummy activity is here. So, where we can see it is taking 0 time. So, we have to take the maximum value while performing the forward pass.

So, 6 plus 0 and 10 plus 0 so, 6 plus 0 is 6; 10 plus 0 is 10. So, we have to take the maximum value in the forward pass. So, we will take the early start of the next activity that starts from the node 2 as 10. Similarly here also 10 plus 14 is 24 and 20 plus 0 is 20. So, we take the maximum of 20 and 24 that is 24 that is all activities that are starting from node 6 can start at the earliest on after day 24.

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Activity	Activity time	Earliest time		Latest time		Slack time
		Es	Ef	Ls	Lf	
A	6	0	6	4	10	4
B	10	0	10	0	10	0
C	14	0	14	6	20	0
D	6	14	20	20	26	6
E	14	10	24	10	24	0
F	6	24	30	24	30	0
G	4	20	24	26	30	6
H	4	30	34	30	34	0

Similarly, for this network as we have seen earlier we can calculate the activity time this is all already sorry not we will not calculate this, this is already given to us in the problem statement we will calculate the earliest time the latest time the slack and accordingly we will find out the critical path.

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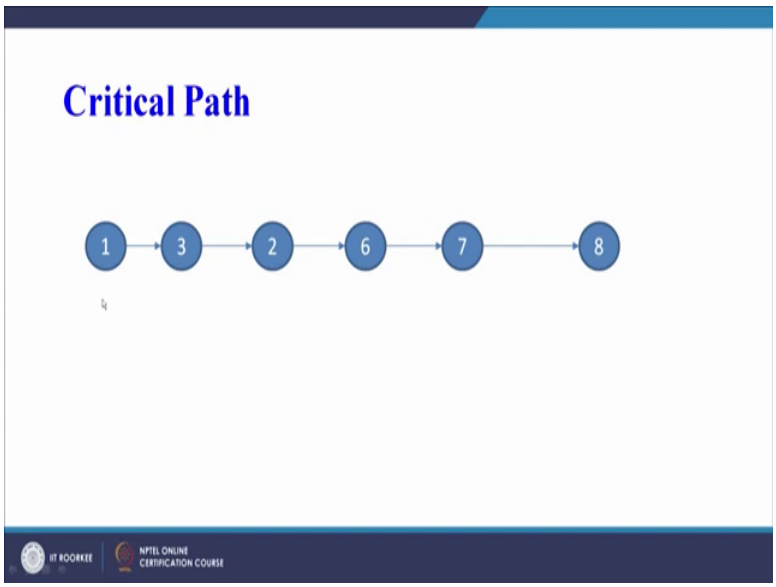

So, in this particular network the critical path is 1, 3, 2, 6, 7 and 8 which means that there are 1, 2, 3, 4, 5 activities on the critical path now coming onto the last example that we can take today a publisher has a contract.

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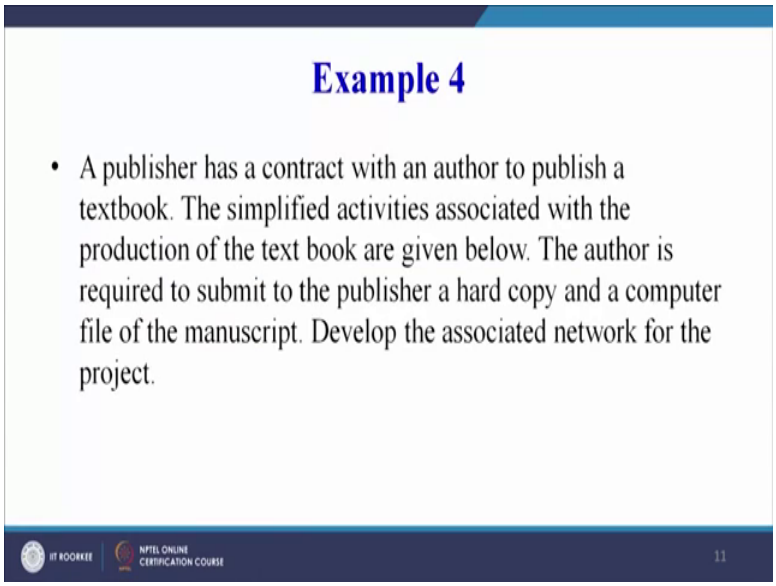

With an author to publish a text book the simplified activities associated with the production of a text book given below maybe in the next slide.

We will see what are the various activities required for publishing a book the author is required to submit to the publisher hardcopy and a computer file of the article of the book that he wants to publish develop their associated network for the project. Now here only the broad guideline is given we need to first identify that what are the various activities required for publishing the book what else we require on the based on the previous experience, we require how much time will be required for each activity that must be deterministically known to us that we must be confident that publishing or maybe printing of the book will take this much time only.

So, we require the list of activities the time required for each activity and then the precedence relationship among the various activities for example, the printing of book or the final printing of book is not going to happen until the proofreading step has been completed. So, there is a sequence of steps that have to be completed in order to ensure the completion of the project and that comes from precedence relationship among the various activities that have to be completed in order to ensure the completion of the project and project in this case is the publish publication or publishing of a book. So, whatever is the problem statement based on that the information required for making a project network.

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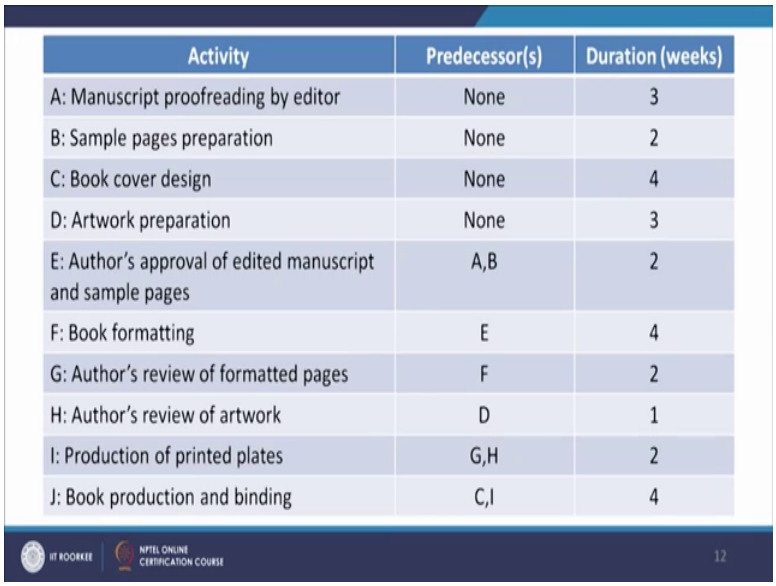

Now, we have a case at hand we have a project at hand we have to first divide the project into the individual activities. So, we can see what are the individual activities manuscript proofreading by the editor sample pages preparation book cover design artwork preparation the diagrams the figures the pictorial representation the schematics the flow charts that is a artwork preparation authors approval of edited manuscript and sample pages book formatting authors review of formatted pages authors review of artworks production of printed plates and book production and binding. So, these are the various steps involved for publication of a book.

Now, they are the predecessor relationship that is manuscript proofreading by editor and sample pages preparation they do not have any predecessor they can start on any given day book cover design also has no predecessor can start on any given day artwork preparation also has no predecessor, but production of printing plates or printed plates H G and H as the predecessor which means that the production of printed plates can only start when authors review of the formatted pages is complete that is activity G is complete and the activity H is complete that is authors review of the artwork is complete which means the author has given a green signal based on the review of the formatted pages as well as review of the artwork.

So, he has read all the text he has understood or he has tried to finalize the artwork whatever a diagram figures are going to go into the books then only the production of the

printed plates will start. So, that the final publication or production of the book or printing of the book can take place and that will depend upon the production of the printed plates. So, which means G can only be done until I has been completed.

So, if I completed then only G can take place similarly it depends upon C also that is book cover design. So, cover design is required production of printed plates is required then only the book production and binding can take place.

So, we have a set of activities available with us we have predecessors available for each activity and we have duration here we can see the duration is in terms of weeks where is in previous examples we have seen that the duration was marked as days. So, we can have our time in terms of days we can have time in terms of weeks and in many long projects maybe one year which may be 2 years or 3 years project our time duration can only be in tow or can also be in terms of months also.

So, in this case for production of a book our duration is in terms of week. So, we have tried to take a specific problem a specific project tried to divide it into its individual activities tried to establish up residence relationship among the various activities tried to deterministically put the time required for each and every activity and once this information has been compiled we can make a network based on the information and this is the network available here we can see activity A to last activity J we have been able to plot in this network.

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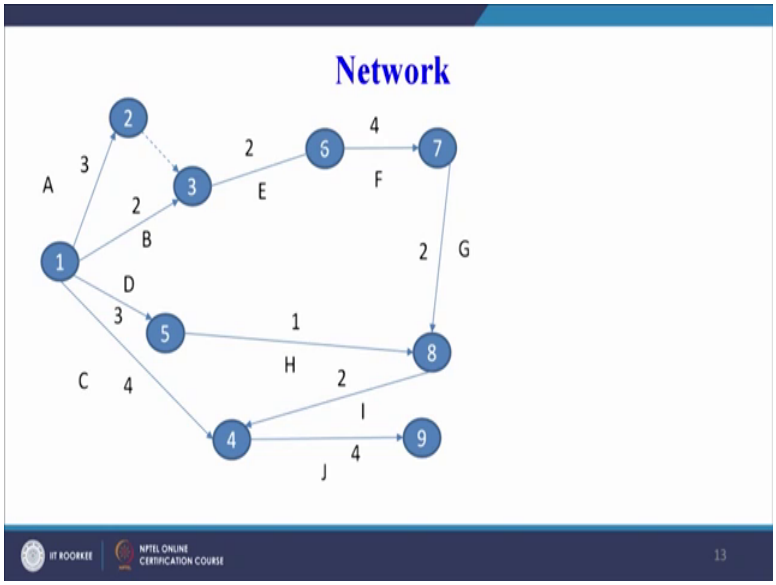

So, we can see if our first activity is activity A and the last activity is activity J. So, a to J we have been able to represent in this network and based on this network based on the forward pass and the backward pass calculations based on the early start early finish late start late finish activities or for each and activity the times that is early start late start early finish late finish times for each activity we can calculate the slack and based on the slack we can find out the critical path and once we know the critical path we can see that which are the activities which must not be delayed in order to achieve the launch date of the book because it may involve coming up of the various dignitaries for the launch of the book the date is already finalized.

So, we have to focus on the critical activities that the function which has been kept for the launch of the book is going to be organised on the specific date only otherwise if the critical activities get delayed the launch of the book will also get delayed and then the overall project may be termed as a failure from the project management or project scheduling point of view may the book may be an excellent book, but from project management point of view we may say that we have not been able to plan our production properly otherwise the book production must have been completed by the launch date.

So, that is the, we can say importance of CPM as a project management tools. So, here based on the calculations we can see this is the critical path.

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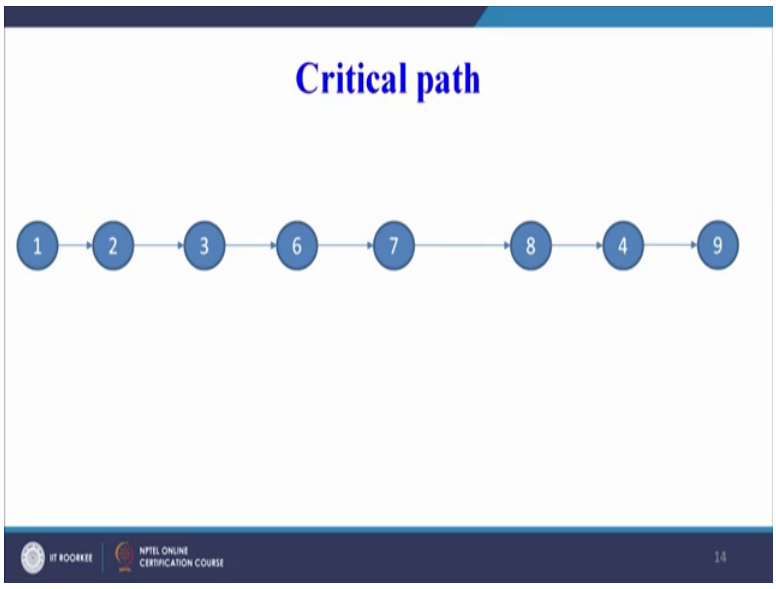

That is 1, 2, 3, 6, 7, 8, 4 and 9. So, this is the critical path in the network also we can see 1, 2, 3, 6, 7, 8, 4, 9, 1, 2, 3, 6, 7, 8, 4 and 9. So, this is our critical path and we can calculate the number of days required for completion of this project.

So, I think we have tried to understand the concept of identifying a problem listing the various activities related to the completion of the project setting that time for each and every activity setting the precedence relationship among the various activities constructing a network based on a specific set of rules numbering the nodes accordingly doing the calculations for early start early finish late start late finish time for each activities based on that finding out the critical path and then focusing our energies on the critical path in order to achieve the project deadline.

So, this is the summary of the critical path method and in the next week we will start our discussion on pert which is slightly different from CPM and we will focus on the calculations try to understand the problems that we have seen here and I must address and emphasize and reiterate it again that all learners must try to do solve number of problems related to CPM. So, that you become an expert in solving the problems.

So, this we practice will make you perfect in solving problems related to CPM in whatever short duration we got we have tried to highlight the importance of the CPM try to show one or 2 sample calculations for forward pass and backward pass and try to introduce the concept of CPM as a tool for project management. Similarly, we will try to emphasize the role of pert as a management tool for operations or for a management of projects.

Thank you.