Operations Management
Dr. Inderdeep Singh
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 34
Critical Path Method: Problems – 1

[FL] friends welcome to session 34, in our course on operations management. And currently we are discussing the project scheduling part in which we are trying to understand the various network diagrams that are used for planning the time domain of a project.

And if you remember in this particular week we are focusing on critical path method which is one of the important techniques for planning related to activities which are having a deterministic type of time estimates. I think I have used 2 3 different words deterministic and time estimates. So, the basic point is that suppose we are constructing a house and we hire a contractor for making this house he or she will be able to guide us that for this much square feet of area making 3 bed rooms, double storey will require this much of time. Foundation will require maybe x time, erection of valves will require suppose y amount of time, flooring will require this much time, construction of room will require roof will require sorry this much time. So, we have deterministically determined time estimates for critical path method. So, we know that how much time is going to be required for a particular activity.

I think this aspect I have missed in the beginning of this particular week must have been explained there and now in the next week our focus will be on pert. So, in I thought that when we explain the difference between CPM and pert at that point we must emphasize on this point that CPM is deterministic in nature where as pert is a probabilistic in nature. So, I will come to that again when we start our discussion for the next week and when we will be discussing problems related to pert. But today our idea is to understand that how we can solve simple problems using critical path method. In this week we have already engaged 3 sessions and we have seen that what is project scheduling, we have seen how a Gantt chart can be used for scheduling the various activities of a project, then we have also seen that what are the advantages of network diagrams such as CPM and pert over Gantt charts.

Then we have also seen the role of dummy activities we have seen the Fulkerson's rule that is used for numbering of nodes of network. Then also we have seen if you remember that we have two types of network diagrams, one is activity on arrow, another one is activity on node. And we have seen different types of problems maybe two or 3 different problems we have seen, in one case the activities were there on the arrow in the other case when we have calculated the early time for a particular active activity and the latest time for a particular activity we have taken an example where we have taken activities on nodes. So, all of the learners must keep this thing always in their mind that two different methods are there for representing the overall project in the form of a network and these methods are activity on arrow and activity on node.

But in today's session we will see that we will be depicting the activity on the arrow and we will try to see how to construct a network and what type of calculations can be done based on the network. So, basically our target is that; once we draw the network our main objective is to see; what are the different paths that we have to follow in order to complete the project. The project may be having 10 activities, we have to represent these 10 activities based on the precedence relationship which is already known to us.

So, we have a information regarding the various activities their description, we know we name the activities like a B C D E F H h or we can name the activities like the first node is node 1. So, from 1 to 2 is one activity 1 to 3 is another activity 1 to 4 is another activity. So, we can name activity as 1 2, 1 3, 1 4. So, that is another nomenclature for the activities. So, what is required for making a project network? First is the list of activities must be known to us, the description of the individual activities must also be known to us, the description I mean to say that making a foundation this is being represented by the activity 1 2 maybe erection of walls this may activity may be represented by 2 3. So, that is the way we have the activities and we have a description of the activities.

Then we have the precedence relationship among the activities, that few activities may definitely have some predecessor which has to be completed before this particular activity may start. For example, erection of walls necessarily will be done once your foundations are ready or the roof can always be laid once your pillars and the supporting arrangement are in place. So, roofing is dependent on some other activities and that those activities are the predecessors for the activity that is making of a roof. So, there will be a

precedence relationship among the activities. Finally, we required the time required for each activity. So, if we have this information again I am summarizing the list of activities the description of activities the predecessor relationship or precedence relationship among the activities and the time required for individual activity. If we have this information available with us we can construct the network and then do the calculations based on this network.

So, first job is to construct the network once we have constructed the network then we need to do the calculation. What we can do? We can see what are the different paths, starting from the starting node to the end node already I have highlighted when we are representing a project in the form of a network it must always have one start node and one end node. Sometimes I see when the learner are learning to draw a network some of the nodes with they will leave in between only they are not connected to the end node. So, it is always advisable that you start the network from one node and you finish the network at another node. If there is a node which has no activity coming out of it you can just draw a dummy activity to the end node to just represent the relationship that the end node represents the completion of the project. So, once we have the activities the precedence relationship that time we can construct the network. So, our first job is over that is construction of the network.

Second stage is that we have to write down all the paths, starting from node one we can write the either the node numbers as the path or we can write the activities as the path for example, there are 10 activities, so a B D and F maybe on the critical path. So, we can say a B D F is one path in the network or 1 2 5 7 numberings are nodes, 1 2 5 7 represents the activities on the critical path. So, we can represent the critical path in anyway.

So, what we have to do? We have to list down all the possible paths of reaching the end node starting from the starting node. So, suppose our starting node is 1 and our last node is 10. So, we have to look in the network and see; what are the various paths that we can follow in order to reach from the start node to the end node. Then we can list down all these paths and we can calculate the time required for each path. So, the longest path will represent the critical path for the network and it will represent the minimum possible duration required to complete the project. So, these two statements must be clear earlier in may be session 2 I have a little slip of tongue where we were not I was not able to

explain it properly. But today again I am emphasising that the critical path will be the longest path in the network requiring the maximum amount of time, but it will represent the min minimum duration that is required to complete all the activities. So, if we go, if the project is not completed in this duration the project deadline will get extended. So, we have to do the calculations in order to find the longest path in the project network.

So, that is what we are going to do today. We are doing going to do the calculations today and trying to find out that how we can mathematically calculate the critical path and finally, we will try to understand what are the uses or what is the application of finding out the critical path. So, already in the previous session if you remember we have seen calculations related to the forward pass and the backward pass. Today we will try to see how individually at various nodes how we can calculate because in the last session I was myself not very satisfied or that very convinced that I have been able to explain the forward and the backward pass in the best possible manner. Today we have tried to maybe find tune tweak the calculations in such a way that it is easier for the learners to understand.

So, let us start quickly we will take two problems today.

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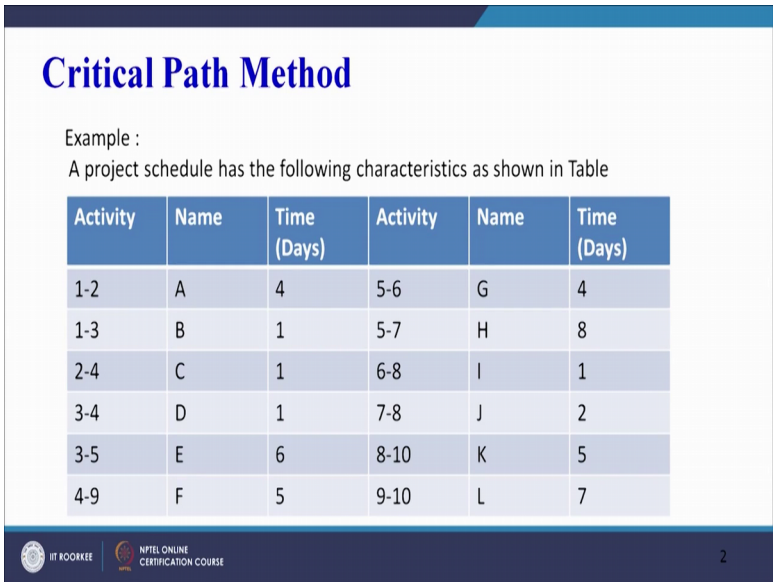

This is a first problem a project schedule has the following characteristics as shown in table. So, as I have already told you there are different ways of representing an activity and here you see we have taken both ways we have taken both 1 2 and A. So, you can

represent an activity by the alphabet A or you can represented by the start node and the end node. So, 1 2 is our activity a which is the start node and the end node for activity A the time required for this activity in days or number of days is 4. Similarly we see that there are may be 12 activities here 1 2, 1 3, 2 4, 3 4, 3 5, 4 9. So, A to L we have 12 different activities here.

So, precedence relationship is not to be given in this because already is the node numbers are known. So, if we construct the network based on the node numbers no precedence relationship is required automatically the presidents relationship will be established once you construct a network based on the node numbers given in this network.

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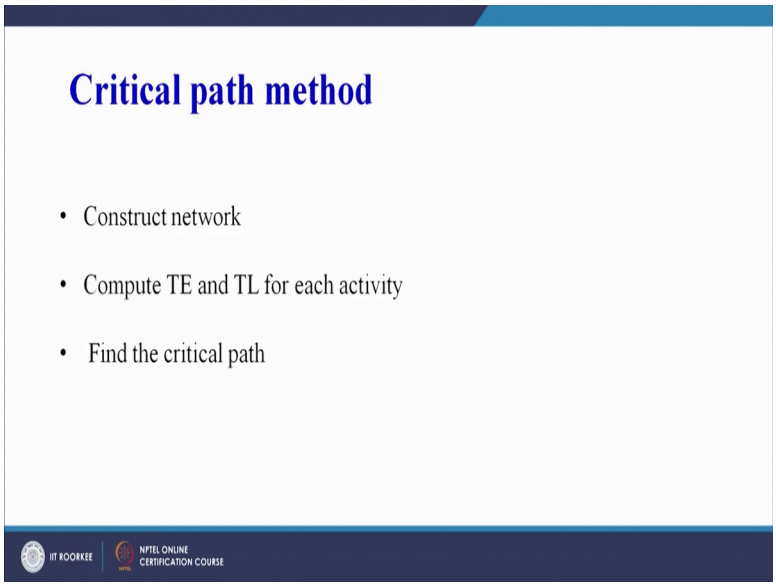

So, we need to construct a network then we have to calculate TE and TL which we have seen in the previous class that is the early time and the latest time for each activity and finally, we have to calculate the critical path. The critical path will represent that path where the activities for which we have calculated TE and TL the slack is 0, which means that these activities cannot be delayed beyond their normal time otherwise that overall project duration will get delayed. So, that is the purpose of finding out the critical path that the activities that are lying on the critical path are very very critical which cannot be delayed or may be postponed otherwise the overall project duration will get delayed. So, we have 3 tasks here task number one is to construct the network based on the information provided to us.

Now, instead of giving the node numbers the precedence relationship can also be given and then accordingly we have to construct a network. So, fortunately in this example we have taken that directly the node numbers are known to us. So, based on those node numbers we can construct a network, so easier to construct a network when this type of information is available.

Then we have to calculate TE and TL which we have already seen in the last session how to calculate by using of forward pass and backward pass and then we have to find the critical path for which the slack will be 0 or the activities for which the slack will be 0. So, first stage is the solution is the network.

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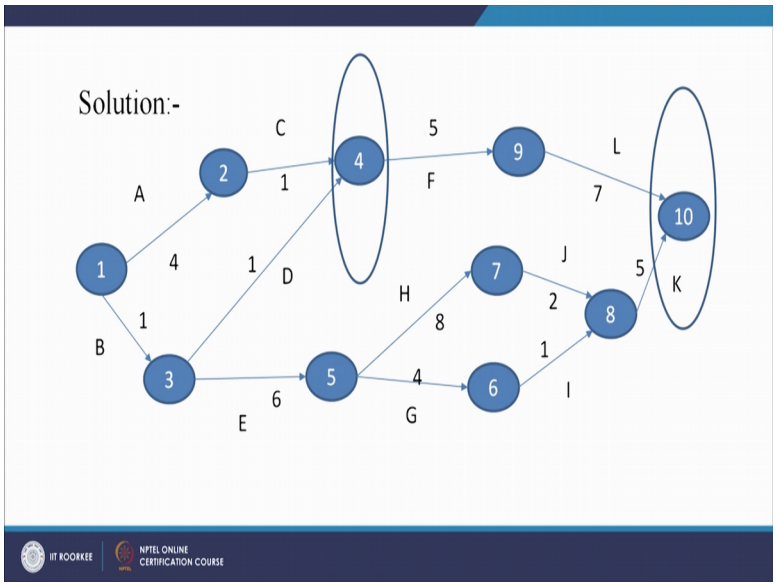

So, this is the network here you can see we have activity A B C D E F H I J K and L. So, one important thing that you can note here is that we have a starting node as 1 and only one end node that is 10 and all the node numbers have been depicted properly. So, activity is on arrow here. So, this is 1 2 is representing one activity A which will require 4 days to complete. Similarly activity C will require one day activity, B will require one day activity, D we will require one day. So, B C D all 3 activities will require one day let us check whether our network is correct or not.

So, here you can see as per the information available activity B C and D all are taking one day each. E and F takes 6 and 5 days, on your screen you can see E is taking 6 days and F is taking 5 days. So, 6 and 5 for E and F again you can check E 6 and F 5. So,

similarly we have constructed this network representing all the activities. Now, you see second stage is to see what are the number of paths available here. So, one path if we start from here number one and the direction of arrows we follow 1 to 2, 2 to 4, 4 to 9, 9 to 10, this is one path. Then 1 3 4 9 and 10 second path, 1 3 5 7 8 10 another path, 1 3 5 8 sorry 6 8 and 10 another path, so we can in our notebook write down all these paths and then we can calculate what is the time required for each path. And which time we have to select? We have to select the shortest of the smaller number or the larger number.

So, I think all of you must be able to answer the longest path we have to take we will take the largest number. So, in this case we can calculate for 4 paths and we can see what is the longest duration required. Apart from that we also have to see what are the activities lying on the critical path and what is the time that is available with us in order to focus on the non critical activities in which we have a little bit of flexibility available.

As I have already told you we cannot reschedule the critical activities because the overall project duration will get delayed. But we have the flexibility of modifying that time or for rescheduling our non critical activities. Now, we have to find out that; what is the time available with us for non critical activities that we can reschedule them so that the overall project duration that we have calculated in the form of the critical path remains the same. So, now, we will see that how we can calculate the time available for individual activity which can be rescheduled.

And we will see that activities that are lying on the critical path we have no flexibility available, we have to complete them on time otherwise the overall project duration will get delayed. So, we will calculate the early start and late start for each activity similarly early finish and late finish for each activity and we will see that there can be a difference between the early start and the late start of a activity. If there is a difference; that means, that it can start earliest maybe after day 4, but it is possible that even if we start it on day 6 still the project duration will remain whatever we have calculated as the critical path.

So, that is the flexibility we can calculate by calculating the early start, early finish, late start, late finish for individual activity. And if we have the data we can make use of that data for our scheduling operations for scheduling of the operations required for the completion of the project. Let us quickly see which we have this information already I have shared with you in the previous session where we have seen the forward pass and

the backward pass. So, now, let us see for these two nodes that is activity 4 sorry activity F when can it start at the earliest and when can it start at the low latest. So, let us see for first 4 activity 4.

Let us do the forward pass we are starting from node 1 that is day 0. This is our day 0, activity A and activity B are starting on day 0. Now, how much time activity A will take? Already information is available with you, it will take 4 days, is it correct. Activity A will take 4 days, activity B will take one day only. So, C activity at the earliest it can start after activity A has been completed. So, C can start at the earliest on day 4 after day 4, after 4 days it can start. Now activity C is taking only one day. So, it will start after 4 days and it will take one day to complete. So, this activity F is dependent on the completion of C and D, F is dependent on the completion of activity C and activity D. So, when can its start at the earliest we have to calculate that F can start at the earliest on which day.

Now activity a will take 4 days C will take one day if we follow this path 1 to 4, F can start after 5 days. But if we follow the path B and D, B will take one day, D can start on second day it will also take one day. So, as per this path F can start on third day. So, F can start either on third day or after two days or F can start after 5 days after the completion of A and C. And as I have told you that in the forward pass we will take the maximum value if there are two paths coming together at one particular node as is the case with 4. A and C are also maybe leading to the node 4, and B and D are also leading to the node 4 from fair activity F is starting.

So, F has a predecessor C and D which must be completed in order to calculate or in order to start the activity F. So, both ways if we see 4 plus 1, 5 activity F can start on the 6th day or it can start on the third day. So, we will take the maximum value. So, activity F will start on the 6th day, why because a will take 4 days C will take one day. So, 5 day will go for the completion of A and C and on the 6th day we can start F and that is the maximum value of the two paths that are meeting at node 4.

Similarly, I may not explain that much in detail similarly we can calculate for 10 also. So, if we see what are the paths leading to 10. So, first we have to calculate when can L start at the earliest. So, L can start at the earliest the F will take 5 days 4 plus 1, 5 plus , 10. So, L can start at the latest on 11th day or maybe 4 plus 5 plus 5, yes it can start on

11th day and similarly we can do the calculation and see that when can activity K start. So, we have to take the maximum value of L and K and that we have tried to include in the next slide.

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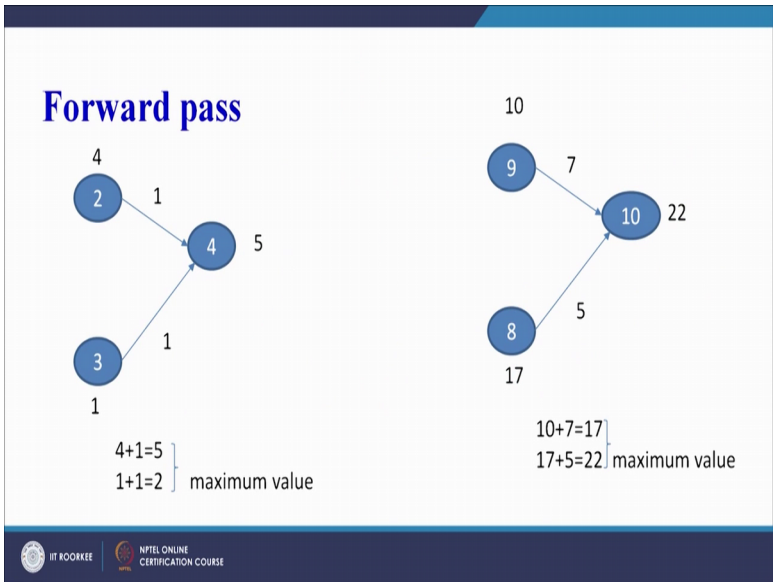

So, here you can see in forward pass we have seen at here 4 you have to do the calculation when can it start. So, here 4 plus 1 5, it can start after 5 days and from this side 1 plus 1, 2. So, maximum of 5 and 2 we have to take. So, it can start after 5 days. Activity F is coming from F is starting at node 4 as we have seen here activity F is starting on node 4, so 4 plus 1, 5 and 1 plus 1 2. So, 5 is maximum here. So, it can only start after 5 days.

Similarly, we see for node 10 how much time it will take. So, we can see 9 how the value 10 is coming here 4 plus 1, 5 plus 5 10, so 10. So, it can start after 10 days L will start after 10 days. So, therefore, the value 10 is coming and similarly calculating the different paths there are two paths here at node 8 we can do the calculation, when can activity K start or after how many days activity K can start. So, K can start after 17 days and it will take 5 days. So, 17 plus 5 is 22 10 plus 7 is 17. So, we have to take the maximum value, so maximum is 22 so we will take 17 plus 5, 22 as our final value. So, if from left hand side to right hand side when we do the calculation we will be taking the maximum value of time at any node. So, this way we have calculated at the last node that 22 days are required for the completion of this project.

Now, what is the next thing that we have to calculate? We have to calculate the early start and early finish for each activity. So, where here we can see we can make a table like this may be this is a backward pass first and then we will make a final table where we have calculated the early start, early finish, late start, late finish for each and every activity.

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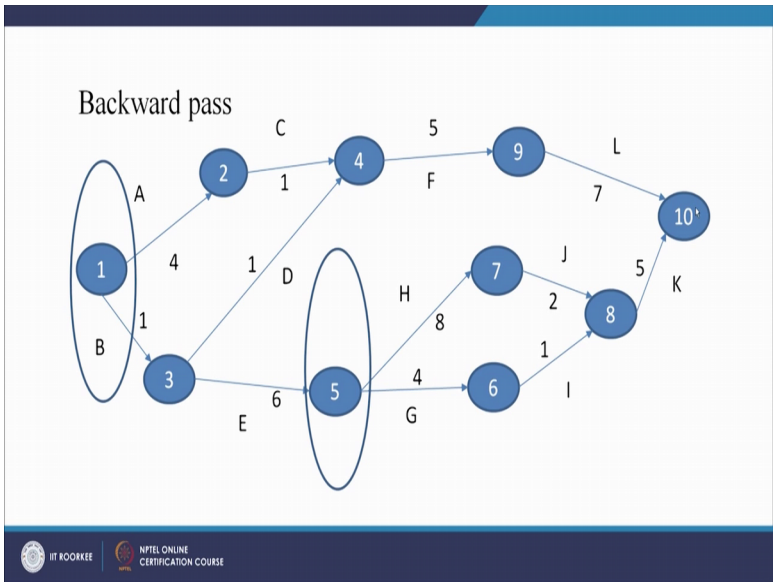

So, let us try to understand first the backward pass because we have calculated into the value of TE for each node, now we can calculate the value of TL for each node starting from the right hand side. So, when we start from right hand side or the last node we that is node number 10. So, here at node number 10 TE and TL will be same. So, this is 22 and 22, so TL becomes 22.

So, similarly when can 8 start at the latest 8 can start at the latest because this has to be finished by 22 days. So, when can K start at the latest at the latest it can start it requires 5 days, so 22 minus 5 it can start at the latest by 17 days, that after 17 days K can start. So, that we are able to meet the target of 22 days.

When can I start at the latest, at the latest it can start 22 minus 5, 17 minus 1 16, so activity I must start after 16 days in order to complete the project in 22 days. Similarly coming we have to do the calculation for each of the activity, when can activity L start at the latest at the latest L can start 22 minus 7 that is 15. So, after 15 days L must start otherwise the project will get delayed. So, this way we will do the backward pass calculation starting from the end node moving towards the start node. And let us take two

examples here at 5 we have to take a decision because their two paths are closing down. So, at 5 we have to take a decision similarly at one also two different paths are coming. So, we have to take a decision. So, let us take an example of 5.

I will start my calculation from here 22 minus 5 17. So, 17 K must start after 17 days. Similarly when J must start 17 minus 2 16, so J must start after 15 days. Similarly 22 minus 5, 17, I must start after 16 days 17 minus 1, 16. So, here we have 16 and here we have 22 minus 5 minus 2, 15. So, 15 minus 8, 7, H must start after 7 days, 15 minus 8. Similarly 16 minus 4, 12, H must start after 12 days. And now we have two different paths entering into 5. So, we have to take the minimum value of the 2. So, we can see here for 5, again I am doing the calculation 22 minus 5, 17 minus 2, 15 minus 8 minus 15 minus 8 that is 7 and if we follow this path 22 minus 5, 17 minus 1, 16 minus 4, 12. So, if we come from this path from this path we have it must start H must start after 12 days and if we come from this path H we have to see that when can H start at the latest that we have already calculated after 7 days.

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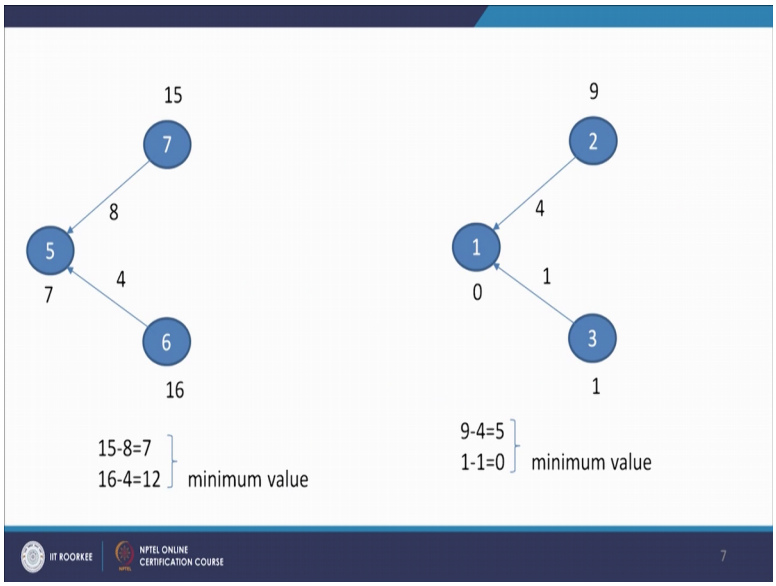

So, here you can see 15 minus 8 and 16 minus 4. So, 16 minus 4 is 12 and 15 minus 8 is 7 and we have to take the minimum value that is in this case 7. So, for 5 during the backward calculation we have to take the minimum of the 2. So, we have to taken the value 7. So, once we do the calculation of early start, early finish, late start, late finish for each node we will get a table like this. So, these are the 12 activities that we have listed

and here the activity name normal time required, earliest time, latest time, total float and we can see that the activities that are lying on the critical path the total float is 0, 0 here, 0 here for activity 3 5, 0 here for activity 5 7, 0 here for 7 8 and 8 10.

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Activity	Activity name	Normal Time	Earliest Time		Latest Time		Total Float
			Start	Finish	Start	Finish	
1-2	A	4	0	4	5	9	5
1-3	B	1	0	1	0	1	0
2-4	C	1	4	5	9	10	5
3-4	D	1	1	2	9	10	8
3-5	E	6	1	7	1	7	0
4-9	F	5	5	10	10	15	5
5-6	G	4	7	11	12	16	5
5-7	H	8	7	15	7	15	0
6-8	I	1	11	12	16	17	5
7-8	J	2	15	17	15	17	0
8-10	K	5	17	22	17	22	0
9-10	L	7	10	17	15	22	5

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And based on this total float we can see this is going to be our critical path that is 1 3 5 7 8 and 10, and it will require 22 days to do the work or to complete the project.

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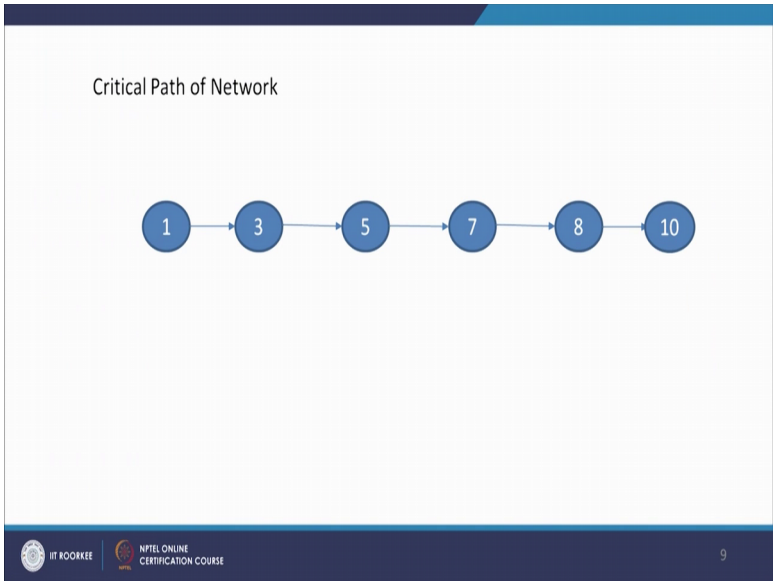

But we can see wherever we have total float some values are there, this is the number of days by which the activities can be rescheduled without affecting the overall project

duration. So, the overall project duration for this project comprising of 22 comprising of 12 activities is 22 days and few activities we can count here 1 2 3 4 5 activities out of these 12 activities are on the critical path, and the remaining 7 activities have some flexibility with them in which they can be rescheduled in order to complete the project.

So, we will try to understand the importance of critical path in our next session. So, with this I conclude the today's session.

Thank you.