Operations Management
Dr. Inderdeep Singh
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 33
Critical Path Method

Friends welcome to session 33 in our course on operations management, currently we are discussing the fundamental aspects of project scheduling and we are into third session in this week for dedicated for project scheduling and if you remember in the very first session we have tried to compare the Gantt charts and the network diagrams, and the second session we have tried to see that how we can draw on network, how a project can be represented in the form of a network.

We have seen that we can have a combination of nodes and arrows and these nodes and arrows can defect the various activities which are required to be completed in order to ensure the completion of the project. There are basically 2 different methods for representing the network, one is the activity on arrow as we have seen in our previous session activity is represented on a arrow.

So, we have one node there is another node there is a start node and end node and in between we have a activity.

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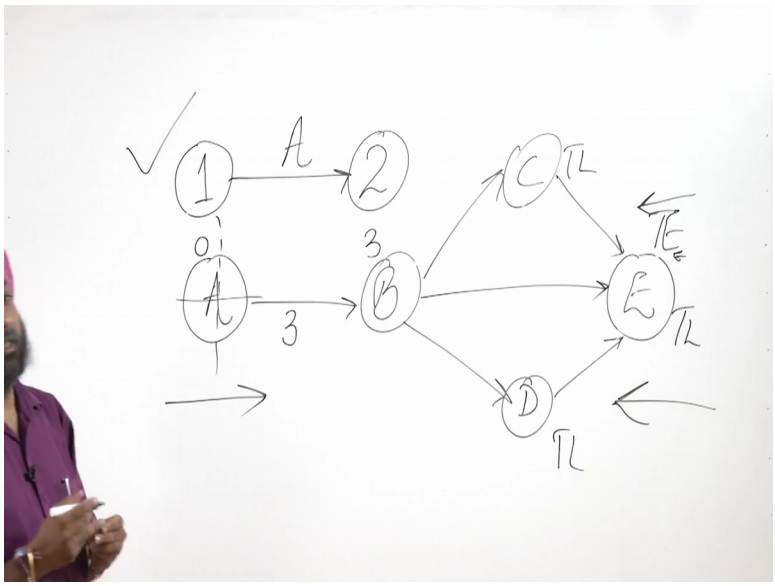

So, let us quickly see that how we can represent the networks. This is our node one or we can called as the start node and then there is an arrow and there is another end node for this activity, which we usually maybe defect by number 2 and activity is represented like this. So, activity a starts from node 1 and it ends that node 2 this type of networks are called activity on arrow type of networks, there other types of networks in which we will represent the activity on the node itself.

So, here activity A is at this node and activity B is here and this type of networks can be called as the activity on node or A O N type of network and this type of network can be called as A O A activity on arrow type of network. The calculations will remain same for both types of networks and in the previous session we have basically seen this type of network, where the activities are represented on the arrows and we have learnt of fulkerson rule for numbering this nodes in the previous session.

So, today our focus will be to see the examples based on A and B that is activity on node and we will try to seen our subsequent sessions because this method we are trying to learn in another 2 sessions also there are 5 sessions of half an hour each dedicated to critical path method today is the third session.

So, we will further see some calculations based on both types of networks. So, that we are able to understand the working of both types of network that is, you can use a network in which activities are represented on the arrow and you can also use a network in which activities are represented on the node. So, today my focus will be on activity on node type of networks and we will try to do the calculations for early start late start early finish late finish for each node that is for this node.

Now, suppose this is my start node and this is the second node in my network and this activity takes maybe supposed 3 days' time. So, the earliest start for this activity a can be calculated as day 0 and then it may take 3 days' time to complete that activity A and then this activity B can only start. So, we will divide this total node into various sections and we will do the calculations for early start, late start, early finish, late finish for this node also early start late start early finish late finish for this node and we will carry forward these calculation for all the nodes in the network and try to find out that what is the critical path.

So, this is the objective of today's session to find out that what is the critical path based on the calculation so quickly we will rush through the presentation and try to learn that how we can calculate the critical path and what is the meaning of the critical path. once we know how to calculate then I will try to explain the decisions that we need to take based on the critical path method.

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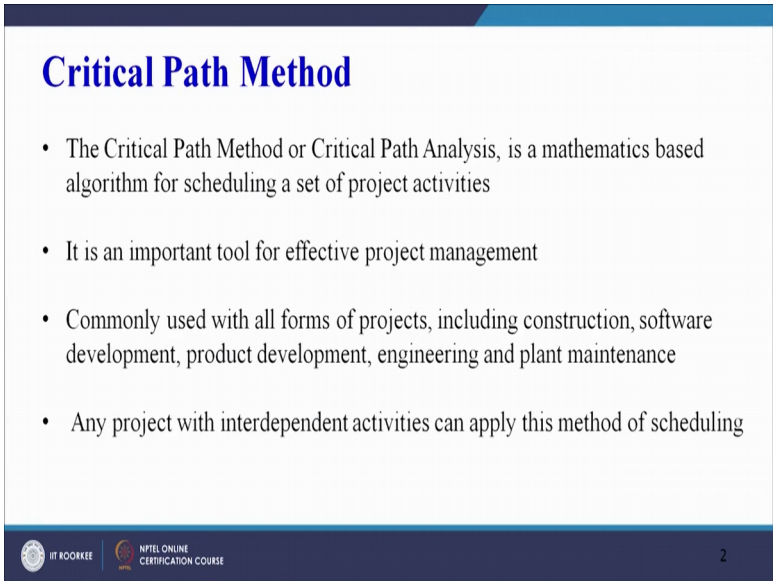

So, as your as you all can see on your screen the critical path method of the critical path analysis is a mathematics-based algorithm for scheduling a set of project activity. So, we have to calculate based on the precedence relationships that how these activities must be schedule scheduled in order to complete the project in the minimum possible duration. So, it is an important tool for effective project management because we are able to find out the minimum time required for the completion of the project in addition to the completion of all the activities that comprise of the project, as we have seen that each and every project will have defined identifiable activities which have to be completed in order to ensure the completion of the project.

So, commonly used with all forms of projects including construction, software development, product development, engineering and plant maintenance. So, C P M method as I have already earlier also highlighted is not only related to mechanical engineering and manufacturing engineering it is related to civil engineering also it is related to product development also it is related to software industry also, it is related to

plant maintenance also. So, the application areas of project management are huge and critical path method is one of the important tools that is used for project management. So, any project with interdependent activities can apply the method of C P M for scheduling the various activities.

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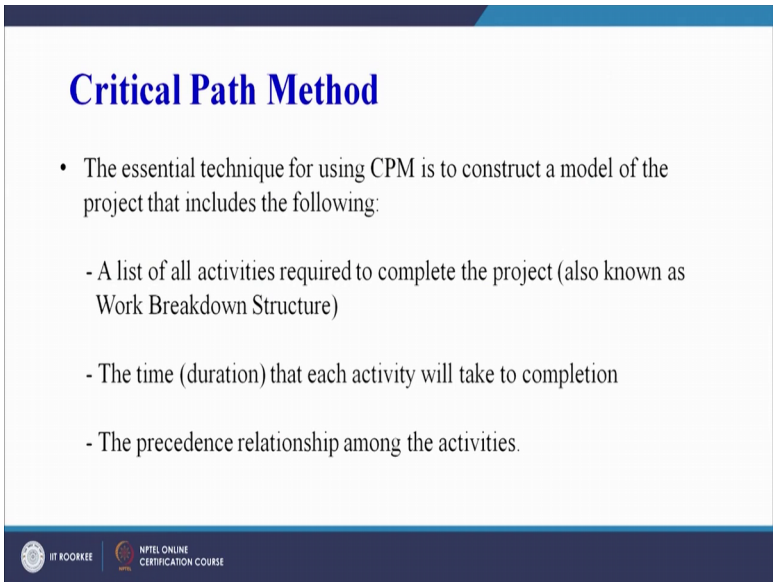

Now, in critical path method the essential technique for using C P M is to construct a model of the project then that includes the following. So, basically, we will use the, what the nodes and arrows that I have drawn on the whiteboard. So, these type of nodes and arrow combinations will be used to represent the various activities that comprise of the project. So, we will see that we need to have a list of all activities required to complete the project this is also known as the work breakdown structure W B S, the time that it activity will take for completion and also the precedence relationship among the various activities.

Now, some of you may be wondering that, why this light is important? This light is important because when we have to construct a network for the project we need to have this information. First of all, we need to have a work breakdown structure of the complete project that these are the number of activities that have to be completed necessarily for ensuring the completion of the project. So, the total project may be divided into 8 divided into 8 to 10 to 15 to 20 activities, which each activity which will

require some time some resources some manpower for it is completion and the combination of these activities will result in the completion of the project.

So, first we have to break down the complete project into individual activities that is point number 1, point number 2 is we must know the resources required for the completion of the activity the resource can be time it can be manpower it can be caused or money involved for the completion of the activity, that is second data that we must know and third as we all understand that we must have the information regarding the precedence relationship that is the predecessor and successor for each and every activity that is included in the project.

So, we must have that information also. So, we require the list of activities the time required for each activity as well as the relationship among the various activities in order to draw the project network, and then once we have drawn the project network we will do the calculations to find out that what is the critical path for this project and what are the activities that are lying on the critical path.

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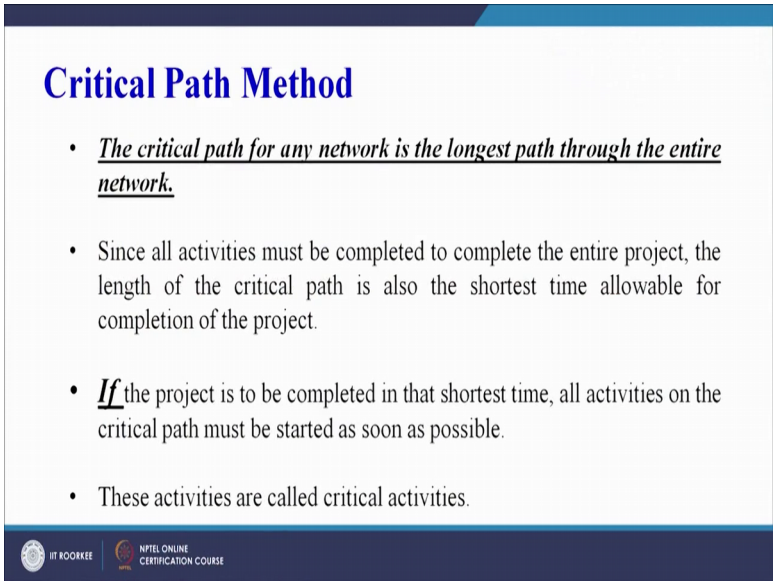

So, what is the critical path, the critical path of any network is the longest path through the entire network. Now starting from the start node to the end node critical path represents the longest path or the maximum time taken for the completion of the project. So, sorry not the maximum it will represent the overall time duration for completion of

the project. So, since all activities must be completed to and complete the entire project the length of the critical path is the shortest time allowable for completion of the project

So, as in the beginning of today's session I have told it will give us the shortest time required for completion of the project I think in between I have slipped ah I may have called the longest time, but we will see that it is the shortest time allowable we will try to understand this with the help of an example also. So, we will see all activities have to be completed and they have to be completed in such a way that time required for completion of all the activities is the minimum time required. So, it is the, it will be the longest path in the network, but with minimum possible time. So, if the project is to be completed in that shortest time all activities on the critical path must be started as soon as possible.

So, we will see that the time required for each activity is counted for and if each and every activity has to be completed we have to ensure that all activities are completed by that time, it may not happen that by the time that we have calculated there are 12 activities 8 are completed 4 are not completed we cannot say that the project has been completed.

So, we have to ensure that within that time frame that we have calculated using summing up of the time of the activities that lie on the critical path, we must ensure that all activities are completed and therefore, when we draw a network there is one start node and one end node. So, one end node will ensure that all other activities between the start and the end node have been completed in that time that we have calculated so, that the overall project can be said to be completed. So, these activities that fall on the critical path are called as the critical activities.

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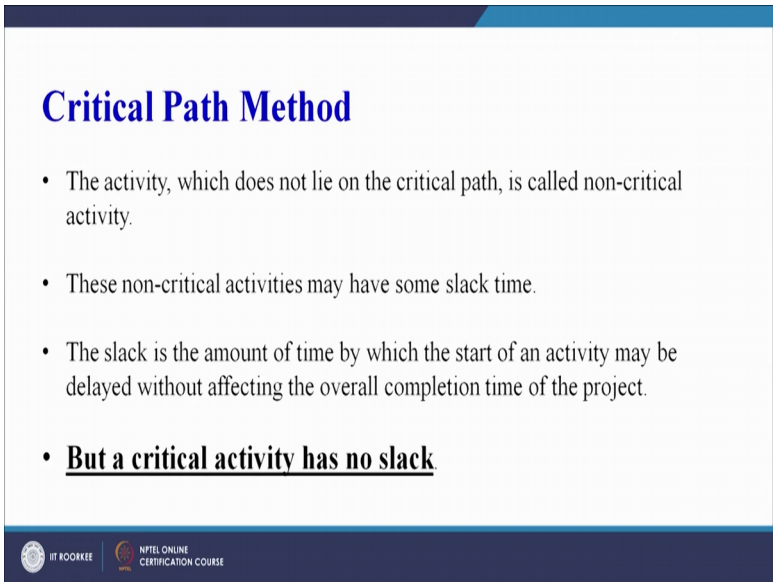

Critical path method again the summary the activity does not lie on the critical path is called a non-critical activity which is just maybe opposite to the critical activities. So, these non-critical activeties may have some slack time, now new word has come that is slack time now slack time basically is the duration in which the activity can be extended further without compromising with the overall completion time of the project.

So, slack is the amount of time by which the activity or the start of an activity may be delayed without affecting the overall completion of the project, but a critical activity has no slack. You cannot change the duration of any critical activity if we delay any critical activity the overall project duration will get delayed.

So, this is very, very important to understand the basic we can say philosophy behind critical path method. The critical path will represent the longest path that is one thing and it will be the shortest time required for completion of the project. So, these 2 words we have to always keep in mind the path that we are calculating will be the longest, but it will be the minimum time required for completion of the we can say project. So, these 2 things must be absolutely clear and they will become more clear when we will take an example.

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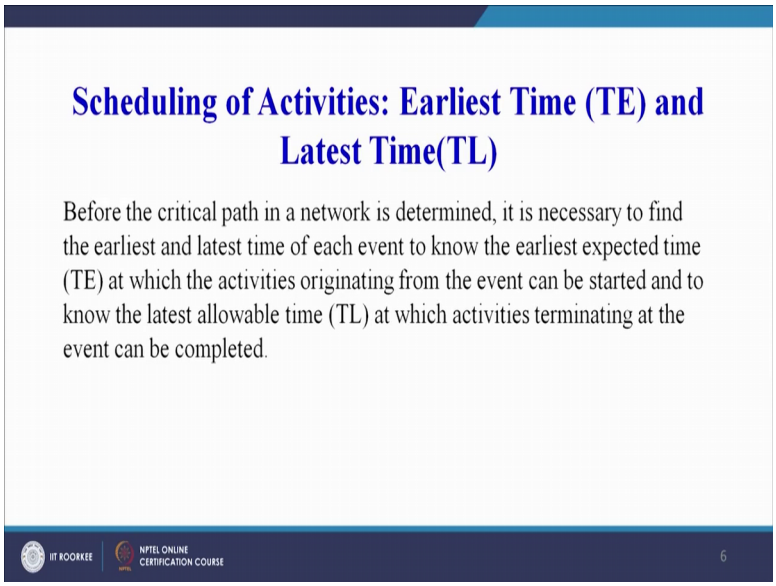

Now, scheduling of activities we can I will read what is given on the slide and we will try to understand this with the help of an example. So, for every event as we have seen here activity A and activity B represented on the node so, they represent an event. So, before a critical path in a network is determined, first of all we have to construct the network. So, once the network is ready it is necessary to find the earliest and latest time of each event to know the earliest expected time at which the activities originating from the event can be started and to know the alley latest allowable time by which the activities terminating at the event can be completed. So, I think little confusing here we can try to understand this with this network.

So, this is A so, we must be able to find out that when activity a can be started at the earliest and we need to calculate that when activity a can be started at the latest. Similarly, for this we can do the calculation early start, late start, early finish, late finish, similarly for this activity B which is at node second node here we can do the calculation what is the latest start for activity B what is the earliest start for activity B.

So, if A B is dependent on A only suppose so and a will take 3 days' time. So, we can say the earliest start for activity B can be 3 days because A has to be completed before B can start and A is the only predecessor for activity B and A will take 3 days' time for completion. So, earliest start for B will become 3 and latest start is calculated when we do our calculations from the end node.

Now, suppose this is my end node and there are 2 other activities here. So, A B C D and E now suppose this is the complete network for calculating when B can start at the latest we will start our calculation, that is the backward direction and earliest when can b start we will do the calculations in the forward pass directions.

So, we will start moving from this direction and try to find out the earliest maybe time when B can start earliest time when C can start earliest time when E can start then from this side we can do the reverse calculation and find out the latest times involved for each and every activity. So, we will try to understand this with the help of an example. So, we can calculate for each node the earliest expected time and the latest allowable time, and the activity which will be on the critical path there will be no slack available which means the earliest start and late start will be the difference between the early start and late start will be 0.

So, this is forward pass calculation.

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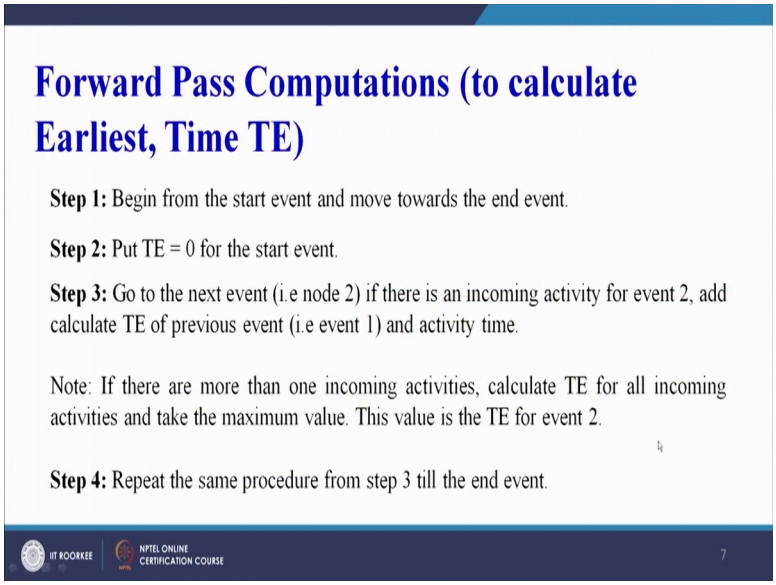

So, begin from the start event and move towards the end events. So, from here we have therefore, deliberately I have drawn this diagram so, that we can understand this so begin from the start event. So, we start from A and we go till E. Put TE is equal to 0 for the start event. So, for A the earliest start time to calculate the earliest time TE we will put it at 0 go to the next event that is node 2 that is where we have presented B.

If there is an incoming activity for event B so, we can see there is one incoming activity for event B, add the add calculate TE of previous event that is event one and the activity time. So, we have TE is 0 here and activity time is 3 here. So, 3 plus 0 we will add and TE for B will become 3. So, we get the TE for B note if there are more than so there can be an event where more than 2 3 activities are coming together.

So, maybe if we can draw another line here because here we see each event has only one entry entering activity. So, suppose we draw another activity here like this maybe. So, here we can see event E is having 3 incoming arrows. So, which one we will select there will be 3 this time will be a activity for time for C will be added for D will be added and for B will be added. So, which time we will select here TE.

So, we will take the maximum time that is coming from the 3 node which you have a very maximum time that time will be taken here. So, for calculation of TE at activity TE at activity E when we will calculate we will take the maximum of the 3 arrows similarly for calculating the late time that is given in the previous TL for calculating the TL here we will take TL value which will be equal to TE and from here we will start our reverse calculation for calculating the TL for this event similarly TL for this how we will calculate we will subtract the time for each activity and calculate the TL value.

So, this we will try to understand with the help of an diagram. So, here you can see I will read it for you on your screen begin from the end event and move towards the start event. So, for latest time we will start from the right-hand side that is end event.

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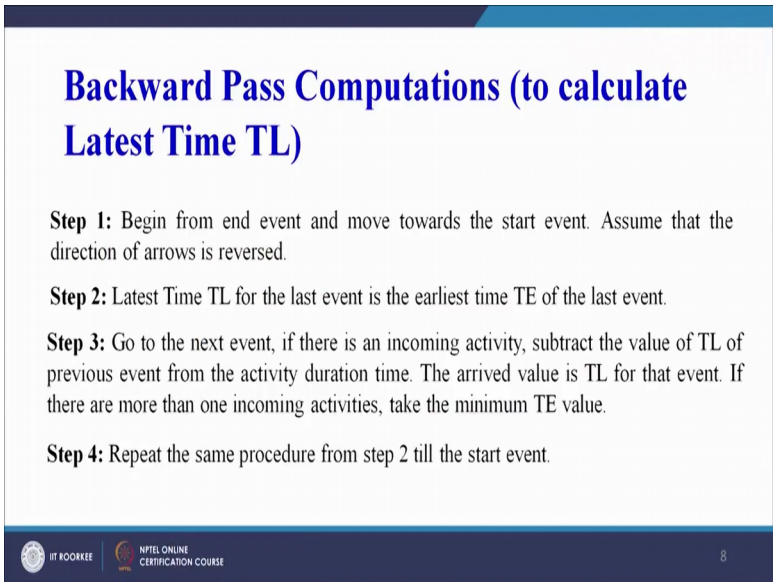

And move towards the left-hand side. So, assume that the direction of arrow is reverse now. So, for late calculations we perform the backward pass latest time TL for the last event is the earliest time TE for the last event which I have already explained and then go to the next event, if there is an incoming activity subtract the value of TL of the previous event from the activity duration. The arrived value of TL is the value of TL for that event. If there are more than one incoming activities then we can take the minimum of the TE values. So, while we performing the forward pass if there are more than 3 or 4 incoming events or arrows into one node.

We were taking the maximum value for TE here if there are 2 or more incoming arrows into the node we will say that we have to take the minimum value while doing the backward pass calculations. So, we will try to understand this with the help of an example. So, we will try to calculate the first is calculation of in general the earliest expected time for each event and the latest expected time for each even then we do it for activity wise and here we can see we will calculate the nomenclature is given at t I j is the duration of the activity.

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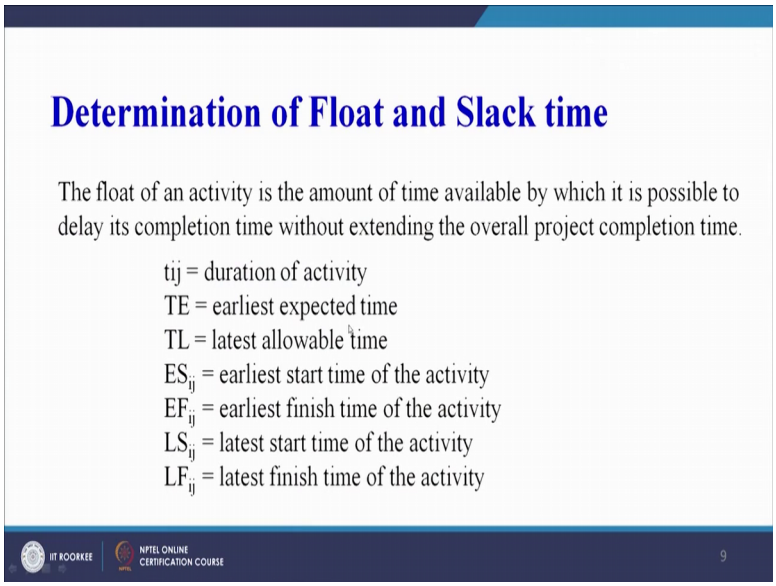

TE is the earliest expected time that we have calculated while moving from left to right or from the start node to the end node. TL is the latest allowable time that we have calculated by moving from right to the left then, ES is earliest start time for the activity, EF is earliest finish time for the activity, LS is a latest start time for the activity and LF is a latest finish time for the activity. So, for each activity for example, this node is representing activity A, for activity A we will cal we can calculate all these 4 values early start time late start time early finish time and the late finish time and from these values we will calculate the float and slack time.

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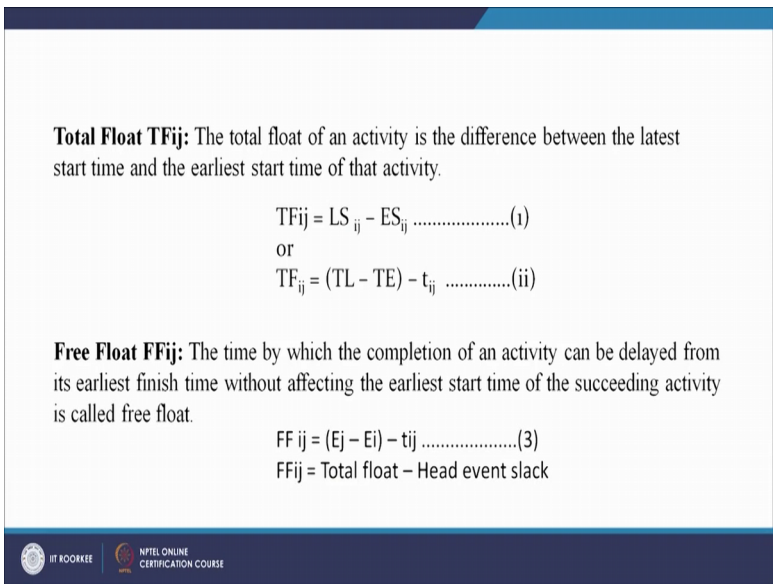

Now, the total float can be calculated the formula is given on the screen latest start minus early start or TL minus TE minus the time duration for that activity both ways we can calculate the value and we will try to calculate it with the help of a example also and free float can also be calculated.

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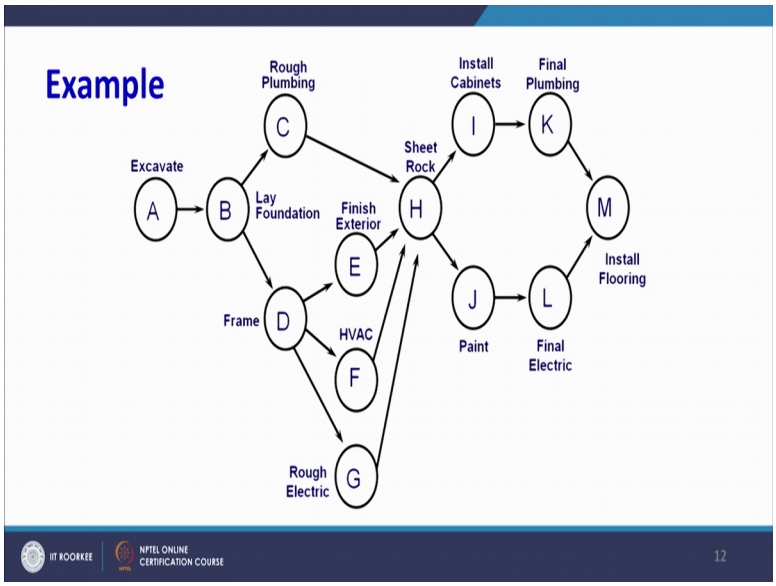

Let us take an example maybe that will be more you can say beneficial. This is one example representing the network as I have drawn a very simple network here, the network on your screen may feel a little complicated. So, this is giving an exce example of construction of a building. So, we have excavate lay foundation, rough plumbing, framing fine in finish the exterior, sheet rock install the cabinets final plumbing in install the flooring. So, you have different H V heating and ventilation air conditioning rough electrical work is done painting is done finally.

So, you have different activities which are required for making a house and in the last session also I think we have taken an example of house building only your construction of a house. So, these are different activities each one of these have got their predecessor relationships and now we want to calculate that how much time will be required to complete this project, what is the shortest possible time required to complete this project? And what is the longest path in the network which will give us the shortest possible time for completion of the project so, that we will try to understand.

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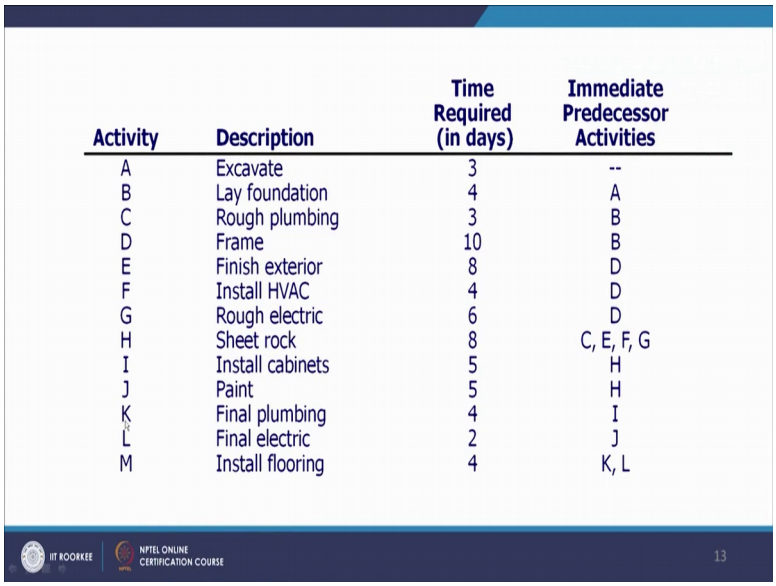

What is the information required? We have to have the activities. So, there named as A to M and the description of each activities given here time required for each activity is given in days and the immediate predecessor activities are also given. Predecessor activity as we have told in the previous session also are the activities which are proceeding a particular activity for example, h has to be preceded by C E F and G that is sheetrock can only take place once C E F and G are completed. That is rough plumbing is done finish exterior is done rough electrical is done and installation of H V A C is done.

So, when C E F and G are complete then only H can start. This is another way a very good way of representing the early start time, early finish time, late start time and late finish time and ti is the duration required to perform activity i and t i is a duration and i is the name of the activity.

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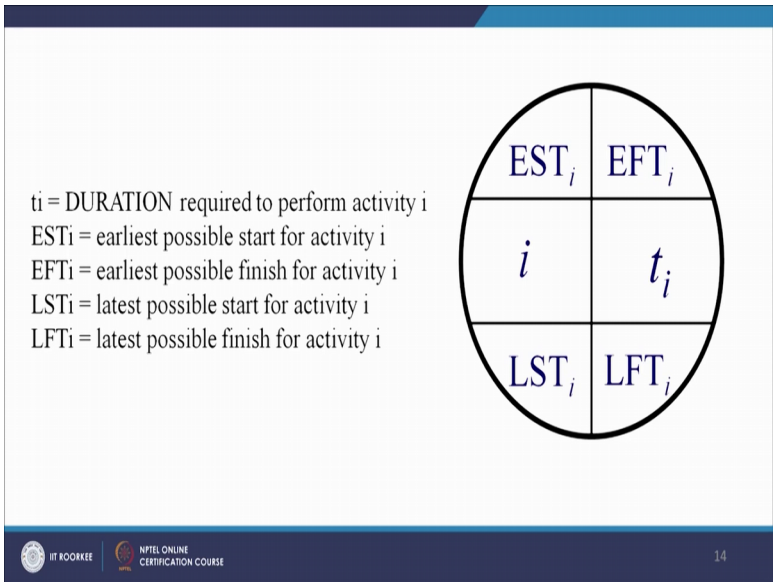

So, in our case i can be a in the very first node and then for a we can calculate the early start time the early finish time the late start time and the late finish time.

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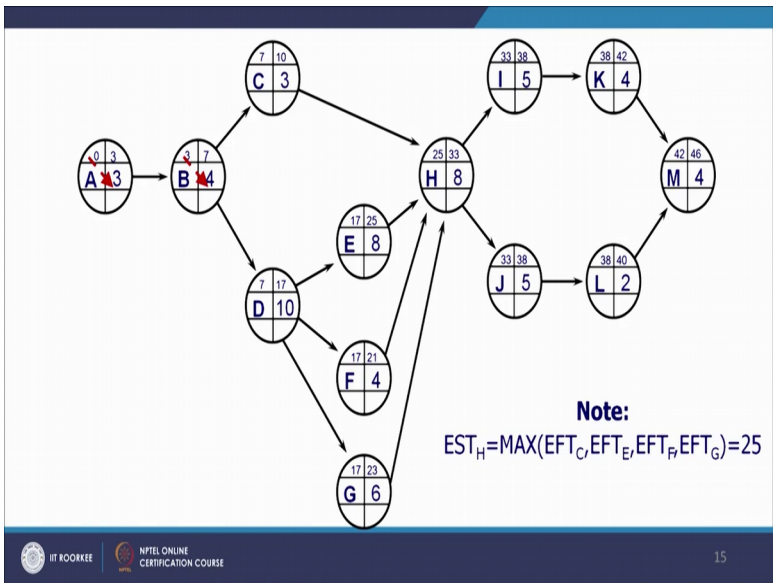

So, quickly we can try to see that how we are representing first we are doing the forward pass. So, on your network when we are doing the forward pass we have to move in the from the start node to the end node. Now start node here is A and the end node here is M. So, we start from A so the first point here is the earliest start. So, earliest start is 0 because we have to start our project on day 0 and it requires 3 days so this is a time

duration. So, early finish can be 3 days it cannot take more less than 3 days it cannot take more than 3 days. It can take more than 3 days, but then the project may be delayed if there is no slack in the activity. So, we get the early finish as 0 plus 3 as 3.

Now, for activity B we see what is the earliest start that is 3 how much time it requires 4 days. So, 3 plus 4 earliest finish time will be 7 as given on your screen. Similarly, for C what is the earliest start earliest start can be because A and B have to be completed before C can take place. So, 3 days plus 4 days 7 days have to be spent before C can start. So, the earliest start for C is 7 and it will require 3 days. So, earliest finish for C becomes 10. So, here for all these 3 activities only one arrow is entering into these activities, but if we take an example of H there are 4 activities entering into H.

So, we will see the earliest finish time for all the 4 activities. So, which will become the earliest start time for this activity H and if we look at these 4 activities from which are prerequisite for the start of H, the earliest finish for C is 10, the earliest finish for E is 25, the earliest finish for F is 21 and earliest finish for G is 23. So, the maximum value is for activity E that is the earliest finish for activity E is 25 therefore, the earliest start for activity H will become automatically 25 because I have told that when we are moving from left to right or from the start node to the end node we have to take the maximum value only, but on the contrary when we are doing the backward pass that is we are starting from the end node towards the start node we are moving for calculating the late start and late finish.

We have to take the minimum values so let us perform the backward pass now in the backward pass we see here the earliest finish is 46

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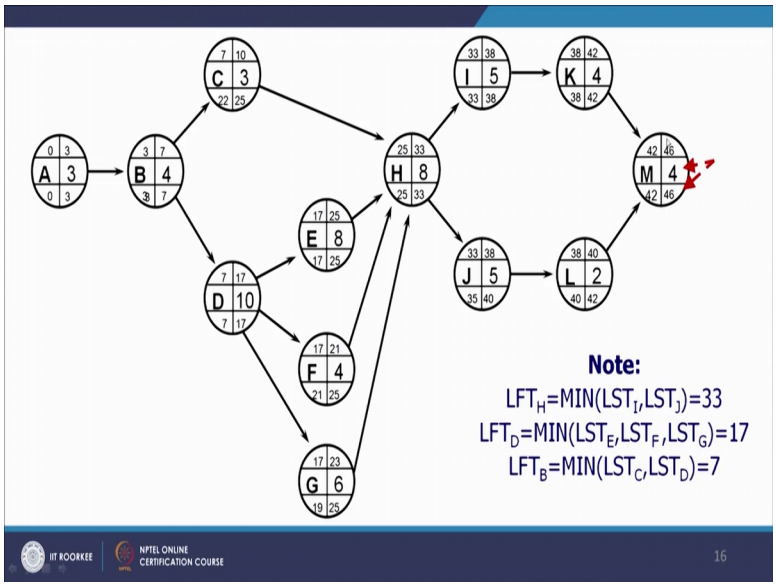

So, we will take it as the TL value that is 46 so 46 minus 4 is to 42. So, here we see only one node is there so there is this node where we have 2 we can say nodes which are entering into this node H. So, when we reverse the direction of arrows we will see that here 2 are entering similarly for B also we have 2 arrows with through which we can to pass through which we can enter that node.

So, for this if we see we have to do find out that what is the latest and we have to take the minimum value for that and here we can see for the late this is 33 here and it is 35 here. So, we will take maximum or minimum while calculating the backward. So, while calculating the back backward pass we have to take the minimum so we will take 33. So, 33 becomes the latest finish for this activity and the latest start will be 33 minus 8 that is 25.

So, while doing the forward pass we have to take the maximum value, while performing the backward pass we have to take the minimum value. So, similarly for B also we can see we have 2 options available here. So, as here we have taken the minimum value between 35 and 33 here also between 7 and 22 we have to take the minimum value. So, the minimum value is 7 out of 7 and 22. So, we can very easily do the forward calculation and the backward calculation also.

Now, how we can use this network so the slack as we have seen will be 0 for the activities that lie on the critical path. So, we can see the difference between early start

and late start or early finish and late finish has to be 0. So, here we can try to look out those activities where the difference between early start and early finish and late start and late finish is equal to 0. So, first activity is A then B, we can see early start early finish late start late finish. So, early start minus early finish there is no gap early late start minus late finish no gaps, A similar is for B then we see here there is a gap here there is no gap 7 minus 70 17 minus 17 0.

So, there is no slack available, then we see other active here there is a gap available slack available 17 minus 17 0, 25 minus 25 0. So, A B D and E are on the critical path, then we C H 25 minus 25 0, 33 minus 33 0. So, H is again on critical path and when we see the other activities where we have no slack, we will find I K and M.

So, where critical path becomes A B D E H I K and M for which the slack is 0 and when you will see the other activities on the required for the completion of this project, you will definitely find some slack. So, here for example, for activity F we can say slack is 25 minus 17 4, 25 minus 21 4. So, we have a slack of 4 days available for F which means that if F does not start on it is earliest start day it can start after 4 days also without affecting the overall project duration, but activity A B D E H I K and M cannot be delayed because if they are delayed the overall project duration will be delayed from 46 days.

So, this is the way we can calculate the critical activities and we can find out the critical path which will be the longest path of the network requiring with that will be the minimum time required for completion of the project. So, we need to find out that it is a longest path, but it is giving us the minimum time required for the completion of the project. We will definitely take some other examples in our next session and try to understand the concept of critical path method.

So, with this I conclude the today's session.

Thank you.