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## Lecture - 14 Quantitative Methods-1

Friends, welcome to session 14 in our course on operations management. We are currently in the third week of our discussion, and in this course we have weekly discussion on specific topics. In week one, we discuss the basics and fundamentals of operations management. In week 2 our complete focus was on product design and development, and in week 3 our focus is on forecasting. We want to figure out we want to find out that how much quantity we should produce, in order to be successful and competitive in the market. And in that direction we are trying to find out the numbers actually.

We are trying to find out the demand that is being created in the market. We are trying to figure out the sales expected in the market. And how we can find out if you remember, just to have a brief overview of what we have covered till today in our discussion on forecasting if you remember, in first session we discuss the need requirement of forecasting. We have seen how forecasting can help us or how forecasted values can help us in making various decisions; such as the manpower planning the materials planning, the planning of the various activities, the time schedule of various activities, all that depends upon the forecast that what actually we are going to produce in a specific period of future time, or in a specific period of time in future. We have seen the basics of forecasting, then how to do forecasting. in the second session we discuss the forecasting system. in which we which we discussed or we in which we have seen that what are the various elements of the forecasting system.

We have seen that there are inputs. There is a forecasting method and there is a output. Then we have seen this system operates under certain constraints. Constraints can be in terms of time constraints can be in terms of data that is available. Constraints can be in terms of money that is available to carry this excise constraint, can be in terms of skill that is required to make a forecast. So, there are constraints under which we this overall system or under which we have to operate to use this inputs, using a method to make a

forecast, and then apart from the constraints there are certain decisions that we have to take and what are these decision we have to take a decision that which method is applicable for a specific set up problem; which data is relevant for making a forecast. So, 2 things the method and the data we have to select. So, that we have covered in the forecasting system.

Then the next stage we have covered the qualitative methods of forecasting, in which we have seen the estimate or the survey technique, and we have seen the Delphi method of forecasting. And we have seen that what are the advantages of qualitative method. if you remember we have seen that they require less time for making a developing a forecast, the money required is less the skill set required is less. So, basically all the most of the constraints are taken care of by the qualitative methods of forecasting. Specially the survey technique and the Delphi method. But still these methods are applicable if you remember we have seen they are applicable where we do not have any previous data available with us the historical data is absent.

Or we are coming up with the completely new product, and there is no comparative data no historical data available with us. We are doing a long term strategic planning, we are not relying on the existing data, we want to forecast 10 years from here on. So, there also our qualitative methods are good. But suppose our product is not new it is an old product, and we have may be last 15 20 years data available with us to be more specific last 5 years data available with us data related to the demand of the product. The actual sales of the product in the last 5 years. So, that data available with us.

So, once such information is available with us, we can definitely make use of this information for developing a forecast, and how this information we are going to use that we will see today. So, we have seen 3 things still now the need requirement and the importance of forecasting we have seen the forecasting system, we have seen the qualitative methods of forecasting that is survey technique and Delphi method. And today we are going to cover the simplest of the quantitative methods of forecasting. They these are simple methods do not require much skill and are definitely applicable, when we have the previous year sales data available with us. So, let us now start the discussion related to the quantitative methods of forecasting.

So, again coming back to the classification, we have qualitative methods of forecasting and we have quantitative methods of forecasting.

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The qualitative methods we have covered in the last class today our session will focus on quantitative methods. And in quantitative methods we have average, moving average, exponentials smoothing, and we can also have casual method like linear like linear regression and multiple regression. So, we will try to cover all these methods in our sessions that are left in the week on forecasting. As I have already told we are focusing, on various topics on weekly basis.

First week was basics of operation management. Week 2 was product design and development. Week 3 we are focusing on forecasting. Today is the 4th session, and fifth session we will wind up our discussion on forecasting. So, let us now see the simplest methods of making forecast using the previous year's data, that is available to most of the companies. So, this is something or this method is something which is applied where we have a same product, it is it may not be a new product. And we have the previous data available with us. Now the data for previous years may vary in a different pattern.

So, let us see what are the variations the top most this is a trend.

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We see a trend component this is an increasing trend component. So, on x axis we have time, and on y axis we have the demand data. So, we see a continuously increasing trend in the demand over the last few years. Similarly, contrary to that we see a trend which is decreasing in nature. So, with time the demand is decreasing over the last 5 to 10 years. Similarly, we can see that there are seasonal variations. This type of plots we have seen in our session one or 2, I do not exactly remember where we have seen the variation of demand data with time. So, there also we have seen the seasonal variation. Then there is a business cycle variation; in which we see that there is a increasing demand and then it is decreasing. So, this is a business cycle may be many time we correlated with the economic boom or the economic depression. So, may be that cycle may continue for 5 to 10 years time.

So, for 5 years we have may be economic boom. So, the demand will be high, but may be after 5 year there can be depression. So, the demand comes down. So, basically this is the variation of the demand in the previous years. Now based on this we have to make a decision that what we what can be or what will be the forecast for the next year. on your screen if we see the first graph to take an example the demand is continuously increasing. So, we can expect that the demand will be higher in the next year. So, for last 5 years if we have a increasing trend we can expect that the next year or 6 or 7th year forecast must be higher than the demand of the fifth year. So, we are already focusing on the last 5 years data and trying to forecast for the 6th and 7th year. Since the trend is increasing or there is an increasing trend in the demand data we forecast that yes next year the demand will further increase.

So, we make use of this data in a judicious manner for making a forecast. Similarly, if there is a seasonal variation we can take into account the seasonal variation, and include it in our time series model, and calculate the seasonal in disease, and make a forecast on season to season basis, usually we do it on quarter to quarter basis. So, the whole year is divide in to 4 quarter. And the forecast is made on quarterly basis in case of seasonal variations. Similarly there can be random or erratic behaviour of the data. And whenever there is random or erratic behaviour of the data, we can use a simple average method for making a forecast. So, we can average out the readings and use that averaged value as the forecast.

So, this is the variation of the data. Why this slide is put here, because now we are going to see the methods that make use of this data. So, first of all we have to see, that what is the variation in the data. Suppose we see a trend we should select our method accordingly. There is no trend there is random variation of data. So, we will say or normal average or simple average can give us the forecast for the next year.

So, looking at the data we have to take a decision that which particular method we have to apply. So, everything is inter related, if we have seen in forecasting system that there is a input there are a combination of forecasting methods; and which produce a forecast. And what decision we have to take we have to take a decision that which method we are going to use. And here also looking at the distribution looking at the variation in the data over the period of time we try to select the best possible method for making a forecast. And we will try to see some of the simple methods of making a forecast for the next year.

Now, again this is just the classifications based on qualitative and quantitative.

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And in quantitative models which are we see as more formal because in case of qualitative we say survey techniques which can be slightly subjective also, but here the things will be more objective more formal in nature. So, we usually use time series models or casual methods of forecasting for example, regression we can develop a curve fitting equation and make use of that curve fitting equation for making a forecast.

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Now, let us see a simple moving average method. Simple average is clear to everybody. So, here we can see it is simple average only. So, here we have taking last may be end data points, and we are adding all this data points and dividing it by n. So, this we can see is a simple average method. So, moving I will explain may be slightly later, but here we can just conclude that is the simple average of the data only, if you see that we are adding from D t plus D t minus 1 plus D t plus 1 minus n.

So, basically we are focusing on the last 5 years or 7 years data. Adding all the demand data for the last 7 years, and dividing it by the number of years for which we have added to the data dividing it by 7 for last 7 years divided by 7. So, we get a average value which is simple average method. So, where forecast f t we can say or f t plus 1 is average of n previous observations or actual or D t. So, D t is a demand data for t period. So, we add all these data points and divided by the numbers of years and we get the simple average method.

Now, why simple moving average is a required, the previous 1 it is return simple moving average, but it is actually simple average and here we have simple moving average. Now why a simple moving average is important; now let us see data that we have with us.

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Again on x axis, we have time; which can be in terms of years, and on y axis we have suppose the demand data that is available with us. I am plotting may be last 5 years data, and this is the trend on your screen. So now, if you see, we use a simple average method of making a forecast, what will we the average it will be the some were around this point.

So, whether it is a correct forecast. We see if it consider the last 3 data points only. What will be the average the average will be somewhere here.

So, considering the last 5 years data this is my average, and considering the last 3 years data this is my average. So, from this graph we say that for examples let us see this is the actual demand for this year, this is a demand or forecast actual demand we should not say actual demand actual demand for the year may be this we can say 2000s 10 18. So, actual demand is available with us. So, if we see that error actual this was the demand the error for average method for 3 years is this and if he calculate the average based on all the 5 years this is the error.

So, we can very easily see that the error by using the last years data or last 3 years data is less as compare to the le last why considering the last 5 years data. Why we should focus on the trend in the previous slides, if you remember we have seen that there are different data trends. There can be a increasing trend there can be a decreasing trend there can be a random variation there can be a business cycle there can be a seasonal variation. So, as a forecast engineer or a engineer whose going to make a forecast we must focus on the variation of this data try to understand the data so that we are able to make a good forecast.

now here if as normal engineer I use all the 5 data points for making the forecast, my forecast will definitely be lower than the actual demand data. And I will be under forecasting. But if we slightly change our method we only focus on the last 3 years data, I am slightly closer to the actual demand. So, that is basically the meaning of the simple moving average method. That if 5 years data is available with us, we may not like to focus on all the previous 5 years, we will only like to focus on the most recent demand that is available with us or the demand data that is available with us, because that is going to give us more accurate a more reasonable idea of the forecast for the next years.

So, this type of method were we consider may be 1 2 3. Last 3, last 3 years data only we call this as the 3 period moving average method, or 3 period moving average. Again may be when 2018 forecast we have done actual demand data is available we use the 3 period moving average, this was my forecast this was actual demand data. Now may be for next year when I have to do the calculation for 2019. I will delete this data, or this data point and I will focus on this will my become 1, this will become 2, and this will become 3.

The actual demand for 2000. This is actual demand this is the demand data for 2017. This is the demand data for 2016.

So, again I am considering 3 data points only for making a forecast for the year 2019. So, I am considering 3 years data only for doing my average calculations. I am not considering the last 10 years or 15 years and 20 years data. But some of you may be wondering that where simple average can be used. Now in similar graph if my data is varying like this, I have drawn large number of data point. So now, in case 2, I am considering these data points only which are encircled. So, you see this is my data 1 2 3 4 5 6 7 and suppose this is 8. So, I have 8 data points now. There is no trend I can observe. So, if I plot them. So, there is no trend. So, in such 5 cases, if this type of data is available with us we can calculate the average and we can make a forecast. So, when there is a random variation, and there is not much variation. Random suppose the random is varying to a large extent, in one year we have very less sales, next year we have very large sale. So, the variation is large we have to look that which day which method we should follow.

But if the variation is not that much the value is just scattered around a central line. So, we say that yes there is not much variation, let us follow the simple average method only. So, when such type of random variation is observed in the data, we can very easily use a simple average method for making a forecast. And if there is a trend like in the previous data there was a increasing trend in those case is we must follow a moving average methods giving more weightage to the previous year's data; that is a just may be close to the year for which we are forecasting. Suppose we want to make a forecast for 2018 we should go more weightage to 2017, 16, 15 and the exclude all the other data point, when we see that there is a increasing or there is a decreasing trend, and this is called the moving average method.

But we are giving considering last 3 years data only and giving equal weightage to all the 3 years. But in many cases it may so happen that we are seeing that there is a increasing trend. So, I will try to just further do my calculation, that I give more weightage to the last years data, slightly may be less weightage to the previous year data. And even smaller weightage to the previous year data. So, when maximum weightage is given to the last years data, smaller weightage to the further data, even smaller weightege to the further data point, if I am considering 3 data point only. Then we call this method as the

weighted moving average. So, moving average is giving equal weightege to 3 years data, must I also tell you; that it is not 3 years data only. We can have 4 period moving average, we can have 5 period moving average. We can have 2 period moving average. So, that is the decision that lies with the engineer that how or how is going to select his period. So, it can be 3 4 whatever data the observer feels or the engineer feels is good for making a forecast.

So, suppose in my example, we have taken that 3 period moving average; so when I give equal weightege to all the 3 periods, I say it simple moving average method in which no weightage is assign to the previous year, but if I assign weights also that more rate is assign to the 2017 data less weight is assign to 2000 16 data, even lesser weight is assign to 2000 15 data, then that method we call as the weighted moving average method. So, this is the basics about the averaging methods for making the forecast. So, 3 I have tried to explain. The first one was the simple moving average, in which all data are included, they are given equal weightage and we do the calculation as is this example. So, we see there is a random variation in the data. So, view the simple moving average sorry simple average method only. So, we calculate this plus, this plus, this addition of all these data point and dividing it by 1 2 3 4 5 6 7 8.

So, we get a simple average technique for making a forecast. Then we have seen a moving average, when we see there is a increasing trend, we consider the last 3 data points only. And for the next year we consider the previous 3 data point only, that is simple moving average method. That can we 3 period moving average 4 period moving average 5 period moving average. And finally, we have seen that if we assign weights to the previous years, we are using a method that is called the weighted moving average method. By assigning maximum weight to the last years data slightly less weight to the last last year data, and even lesser weight to the last to last to last years data may be 2 years previous data or 3 years previous data. So, these 3 method follow under the averaging methods for making forecast, and let us now quickly see the presentation and try to revise what we have covered on the board.



So, simple moving average may we we will try understand now. Note that the n fast observations are equally weighted. So, in case of move a simple average, all observations are given equal weight issues with the moving average forecast or all past observations are treated equally. observations older than n are not included at all requires that n past observation be retained problem when thousands of items are being forecast. So, simple average has got it is own limitations. Now in simple moving average what we try to do? we try to include the n most recent observations only. As in our example you see we were using only last 3 years observations last 3 years data for making a forecast. So, in simple moving average previous to this we have only covered simple average method or simple moving average were ever the word moving is coming, we will define that our calculation are for 3 period moving average we are considering the last 3 observation.

4 period moving average we are considering the last 4 observations only. please do not get confused I think in the title it was written simple moving average. So, it was simple average only. So, moving you can just eliminate from their. So, in moving average we will con in it will include n most recent observation n is the number. So, if n is 3 will call it n is equal to 3 that is 3 period moving average. if it n is equal to 4, we will call it as 4 period moving average. So, n is a you can say variables. So, they are weight equally.

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So, in our example we have given equal weights to all the 3 periods that us that are previous to the year of forecast, and we ignore the older observation. So, in our example, we had 5 observation. By calculating the simple average we were not getting the actual forecast we are seeing that there is lot of variation it is digressing from, but it expect it, but it, but can be we can estimated. So, we thought let us gives last 3 observations equal weight and try to do the calculation. So, last 3 observation were only considered and the average was calculated for last 3 observations only.

So, as per my slide ignore the old older observation. So, we have ignored, the 2 observations these 2 were ignored and only these 3s were considered. But why we have given equal weight age to the calculation. We have calculated for these 3 values only and we have given equal weightage this plus this plus this, divided by 3 we are getting this average value. So, simple moving average 3 periods considers equal weight given to each of the 3 years. But that also we see digressing or there is a forecast error. So, but how we can overcome this? Equal when equal weightege given there is a problem still, we are far away from the actual demand value. So, this is just an example.

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Let us consider simple average examples only. And then we will go to the weighted moving average. A x y z television supplier found demand of 200, sets in July 225 sets in august and 245 sets in September. So, your demand data is now given to you. Because examples help us understand the theory in a better manner let us take this examples. So, the 3 previous years previous month data is on your screen.

200 sets for July, 225 sets for august and 245 sets in September. So, a supplier found a demand. So, this demand is already known to us. Find the demand forecast the month of October using simple average. What we will do? We will give equal weightege to all the 3 years. 200 plus 225 plus 245, equal weightege means divided by 3. So, that their simple average method of forecasting. So, the simple average we can see, you add the 3 and divided by 3. So, you can calculate the simple average method.

Now, simple moving average method. Now see the x y z ref refrigerator supplier experience following demand for refrigerator during the last 5 month.

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A XYZ re demand for	frigerator supplier h refrigerator during pa	as experienced th st five months.	ne following
	Month	Demand	
	February	20	
	March	30	
	April	40	
	May	60	
	×.	45	

So, 20, 30, 40, 60 and 45. Can you see a trend here? you can see observe the trend 20, then it increase to 30 in march, then 40 in April, 60 in may and finally, there is a decrease of 15 re refrigerator in a month of June. But still there is a increasing trend that we can observe. So, if you use simple average method what will be do? there are 5 years, sorry 5 months data available with us. So, we will add all these data points 20 plus 30 plus 40 plus 60 plus 45 divided by 5. And we will get our forecast for the month of July. But in simple moving average we will try to use the give more weightage to the last 3 or 4 years or 3 4 months data only this problem is related to months. So, we are taking months as our time domain. in that problem we have taken yearly data. So, we were considering year on our x axis or on the times scale.

So, here we can see if we do simple average we will be getting a value some were near let us see if we are solve that.

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Find out the demand forecast for the month of July using 5 period moving average, and 3 period moving average using simple moving average method. So, we will see 5 period moving average, in our case will becomes simple average only 1 2 3 4 5. So, we have 5 month data we are considering giving equal weightege to all 5 month. So, will add up everything and divided by 5.

So, let us see. So, the 5 period moving average considering all 5 months or giving equal weightege to all 5 month. We get the forecast as 39 units. Were as for 3 period moving average if we seek giving n equal to 3, we get 48.33, that is 49 units.

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So, you can see the difference between the 2 forecast. one forecast is giving 39 units or 39 refrigerator another is giving 49. So, basically if we use if we give more weightege to the previous year's data or previous month data it will be more realistic, and it will give us a better forecast as compare to giving equal weightege to all the data that is available to us, why? because when we looked at trend we could see a increasing trend in the data, and from there we could see that we must not give equal weightege to all the 5 data points we should give more weightege to the last 3 data points. So, this I think by now you have understood a difference between simple average and the moving average suppose now we want to forecast for the month of august. So, we will make use of the data points of July, June and May. So, May, June and July data we will consider for making a forecast for august.

Now, we want to make a forecast for September, we will use the actual demand data for august, July and June. So, we will keep on the average will keep on moving, because we are giving more weightage to the previous 3 years, or previous 3 months data only now. Let us see the last example for today. I have explained this with the help of an example there. We have given capital w weightage to the last years data then we have reduce the weightege for last 2 years. So, we will see that with the help of an example let us check this problem the manager of a resto want to make a decision on inventory and overall cost.

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# Weighted Moving Average Method : The manager of a restaurant wants to make decision on inventory and overall cost. He wants to forecast demand for some of the items based on weighted moving average method. For the past three months he experienced a demand for pizzas as follows: Month Demand October 400 November 480 December 550

He wants to forecast demand for some of the items based on weighted moving average method. please note weighted moving average method, for the past 3 months he experience that demand for pizzas as follows. Now if we look at this data that is on your screen, the demand for pizzas in October 400, November 480 and December 550.

Now, there is increase in trend data. So, we wish to give more weightage to the demand of December, and slightly lesser weightage to the previous 2 months. That is the months of November and October. So, let us try to see and solution of this problem. We want to find the demand of January by assuming suitable weights to the demand data. (Refer Slide Time: 35:12)



So, we will give weights to the data of the previous 3 months. The months available or the data available with us for is for the month December, November and October. So, we will looking at the data. We will like to more give weightege to the month of December slightly less weightege to the month of November and slightly less weightege to the data for the month of October. So, we will calculate the weighted moving average by the summation of a multiplication, that is the C i into D i. So, C i we says that is a weight that is a sign to that particular month, and D i is the actual demand for recorded for that particular month.

So, C i is the weights for the periods and D i is the demand for the periods. So, let us see C i is 0.25 the weight for the next yea next month is 0.3 and for the latest month is 0.5. So, ideally it this should come equal to 1, but this is C have C 1 may be we can take 0.20, but this is just to explain the idea that how the calculation can be done. Now we will see C 1 is multiplied with D 1 and D 1, you can see is October. So, we are giving less weight, very less weight may be on the 20 percent may be 0.2 only to the October data. So, we can see 0.2. So, 0.2 being multiplied with 400. And the next data is 480, this we are multiplying by 0.3. And the latest data if you see is 550 it is given maximum weightege. So, we are multiplying it by 0.5, and once we add these 3 values we will get our forecast. So, here if we take it as 0.2, we will get this value of 100 as 80 and the total value will be 499 units.

So, the forecast is 499, and for your excise you can do a forecasting using simple average by adding this 3 values, 400 plus 480 plus 550 divided by 3. And certainly you will find a difference in the forecast as we have forecasted using the weighted moving average we have given 0.5 weightege to 550.3 weightege to 480 and 0.2 weightege to 400. So, we have reduced the weightage over a period of time. We have given maximum weightage to the latest data of December we have given slightly less weightege to the data of November, and we can least weightage to the data of October. Similarly we can use this ma method of giving the weights combining it with the moving average also. in our previous case we had 3 data points only we assigned weights to all 3 data point, and did the calculation and make the forecast. Where as in case of if we have large may be last 12 months data available with us. Last 24 months data is available. With us we wish to in exclude some of the data points we will focus only on the recent data points and will get a more accurate forecast.

So, here on your screen you can see the past data on the load on the lathe machine is shown below.

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Weighted Movin The past data on t	g Average M the load on the	ethod : e lathe machin	ne is shown below
	Month	Demand	]
	May	-	
	June	585	
	July	610	
	Aug.	675	
	Sep.	750	
	Oct.	860	-
	Nov.	970	
	NOV.	970	]

So, we have a data starting from June. June, July, august, September, October, November. So, we have 6 months data now available with us. And if you can see 585 610 675 750 860 970. There is an increasing trend. And if we use a simple average method for forecasting, add all these values and divided by 6, we will be under

forecasting we can very easily look at the data, and it is not to going give us the right answer or the right forecast. So, what we need to do? we can focus on the last 3 year, last 3 months, or the last 4 months data. So, if we take the last 3 months data also, there also we see lot of variation. for last 3 months you see 750 860 and 970. So, here what we can do we can assign more weightage to 970 slightly less weightage to 860 and even less weightage to 750. So, we are combining now 2 things. We are using a moving average, and we are assigning weights also to the previous year's data. Or previous months data, and it will give us a fairly accurate forecast based on the information and the data that is available with us this is a solution.

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Compute a weighted three months moving average for December, where the weights are 0.5 for the latest month, 0.3 and 0.2 for the other months respectively. A three month weighted moving average forecast for December = (W Nov. x D Nov.) + (W Oct. x D Oct.) + (W Sep. x D Sep.)= 0.5 x 970 + 0.3 x 860 + 0.2 x 750= 485+258+150= 893 machine hours

What is the problem for which we are solving, we want to compute a weighted, 3 months moving average for December, where the weights are 0.5 for the latest month 0.3 and 0 0.2 for the other months respectively.

So, the weights we are assigning same and we want to compute for the month of December. That is the forecast for the month of December. So, here you can see we have September October November data already available with us. So, 3 periods only we have to consider, and then we assign weights are also given November 0.5 oct October 0.3 and September 0.2. using the same approach we can very easily do the calculation a 3 month weighted moving average forecast for December is given by weightage of November; that is, 0.5 multiplied by the demand for November 970 plus weightage for October,

multiplied by the demand for October, plus the weightage for September multiplied by the demand for September. Add the 3 values we will get our forecast. So, with this we close today session, I think it has been a very busy session. We have try to see the simple average method, were we can apply it is still on the screen; where there is a random variation in the data and the variation is also not that large.

So, if the data points are scattered around the central line, we can make use of simple average method. We have seen moving average method, and we have seen weighted moving average method. We have try to see simple examples which help us to solve problems related to simple average moving average, weighted moving average method. in our next session, we will further try to reinforce our knowledge and information related to quantitative methods of forecasting.

Thank you.