Modeling & Simulation of Discrete Event Systems
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Lecture – 07
Input Probability Distribution Functions for Discrete Systems

Welcome to the lecture on input probability distribution for discrete systems. So, in the last lecture we discussed about the random variables we had the introduction about the discrete and continuous type of systems. Now, in this lecture we are going to discuss about the typical discrete distribution functions, these distribution functions tell us about the probability values for any discrete value to occur and then we also find the cumulative density functions. So, you have some number of such type of distribution functions which are discrete in nature.

So, let us see what is that.

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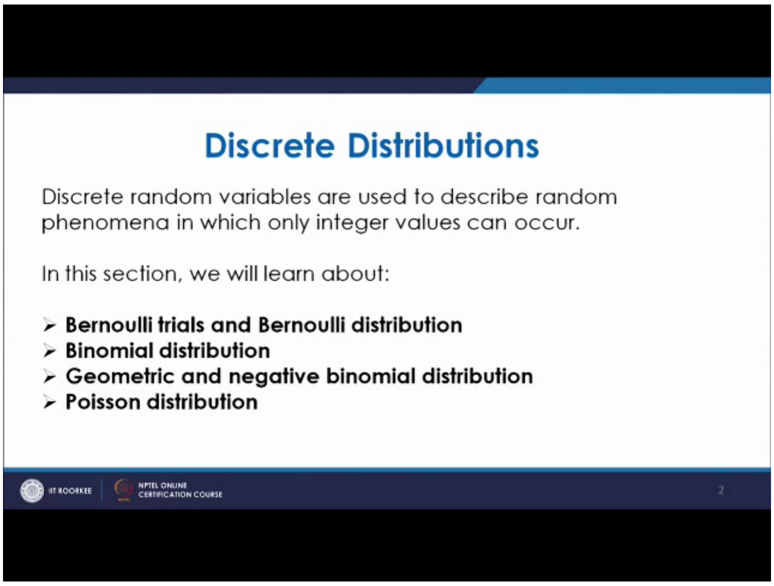

So, as we know discrete random variables they are used to describe the random phenomena in which only integral values occur. So, as we know that in many cases when we talk about the discrete systems you have the probability of having certain discrete values and every value has a certain probability so that is associated. So, they are represented by some typical kind of distribution functions and we will discuss one by one about such kind of distribution functions which will be normally like Bernoulli distribution, binomial distribution, geometric distribution, negative binomial distribution and then Poisson distribution. So, these are the typically used discrete distribution functions in such studies related to modeling and simulation of discrete event systems.

Starting with the Bernoulli distribution.

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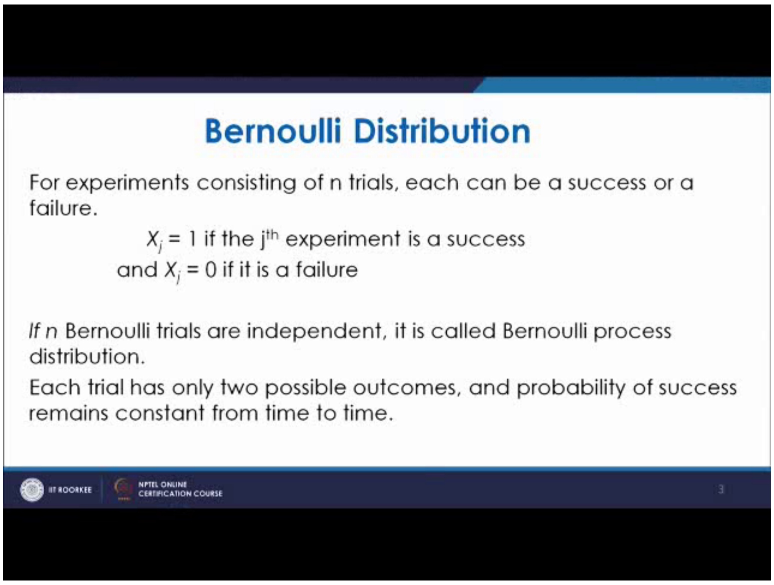

So, you know that when we do the experiment and that consists of n trials and if there is only 2 outcome, either there is success or there is failure. So, that there are they are known as normally Bernoulli trials. So, basically if the n Bernoulli trials are independent than that is known as a Bernoulli's process distribution. So, that is known as a Bernoulli process. So, in that case the distribution which you get that is known as Bernoulli process distribution, Bernoulli distribution. Now, in this cases as we see you have two values x j can be either one if the jth experiment is a success or if it will be a failure it will have the value 0.

So, having there will be a some probability attached. So, either it will be 1 or it will be 0 and each trial has only 2 possible outcomes and probability of success remains constant from time to time. So, what we see in Bernoulli distribution we can see that in Bernoulli distribution P j x j it is nothing, but p x j.

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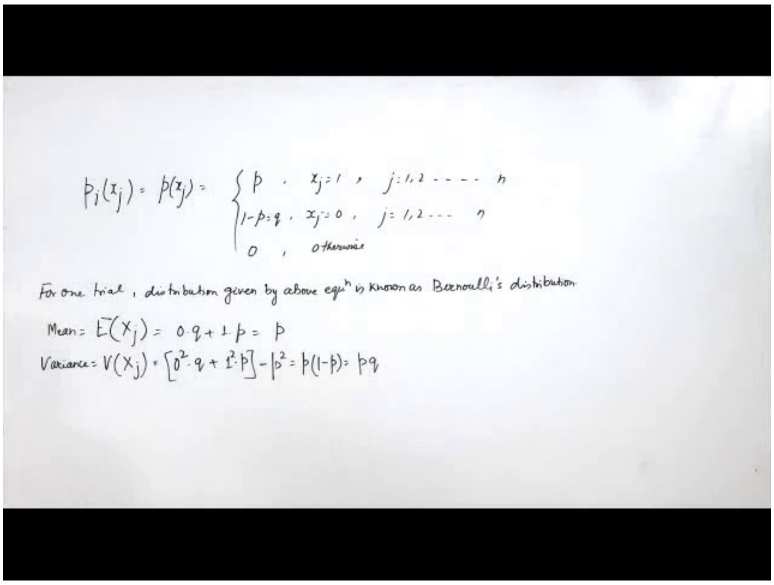

So, if the events are independent in that case you will have 2 probability values you will have either value probability p when x j is 1. So, it will be p if x j is 1. So, j will be from 1 to n. So, if you have n trails then either it will be 1 or 0. So, as we discussed x j maybe either 1 or 0 one will be success and 0 will be failure. So, if the probability of having the success is p then in that case having the only outcome other than p its probability will be 1 minus p that is q.

So, that is also known as q so for that. So, when x j is 0 this is your probability of having 0 at the outcome. So, here again j is from 1 to n and the probability of having any value other than this is 0. So, this is about the Bernoulli distribution, now for one trial the distribution given by above equation is known as Bernoulli distribution. So, in this case we can find the mean and variance of the process mean will be e x j that is mean and mean will be nothing, but you have the probability p for the value 1 and probability q for the value 0. So, p into 1 plus q into 0 so it will be 0 into q plus 1 into p it will be p.

So, mean has the value p and similarly variance, variance will be is it is denoted by v x j and this will be 0 square into q plus 1 square into p minus mean square. So, it will be p minus p square that is p into 1 minus p or it is something like p into q. So, this is the mean and variance calculated in the case of Bernoulli distribution, now we will move to the next kind of distribution.

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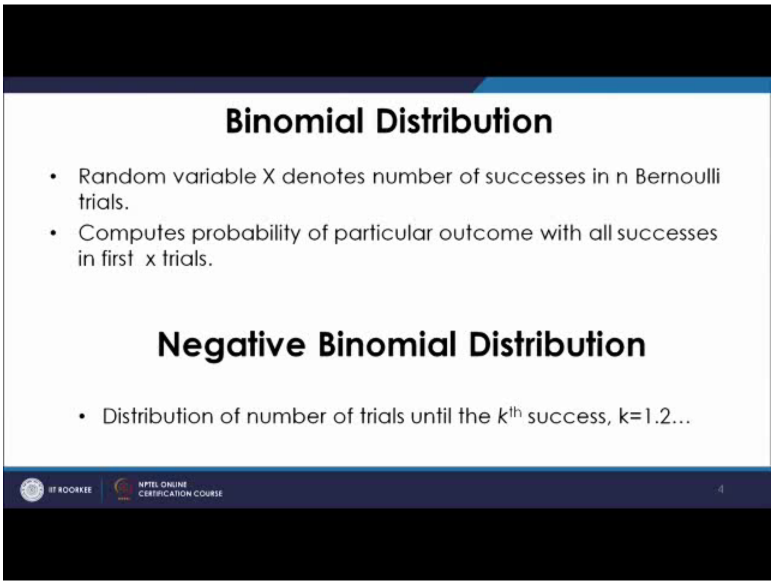

Next is the binomial distribution, now what happens in case of binomial distribution. So, binomial distribution as we see random variable x denotes number of successes in n Bernoulli trials. So, you will have n trails, which is taken and you want you know successes. So, you want successes in first x trails though in the first x trails you want all the successes.

So, it computes the probability of a particular outcome with all successes in first x trials. So, that is known as Bernoulli that is binomial distribution, what happens in binomial distribution? So, let us see what happens in the case of binomial distribution.

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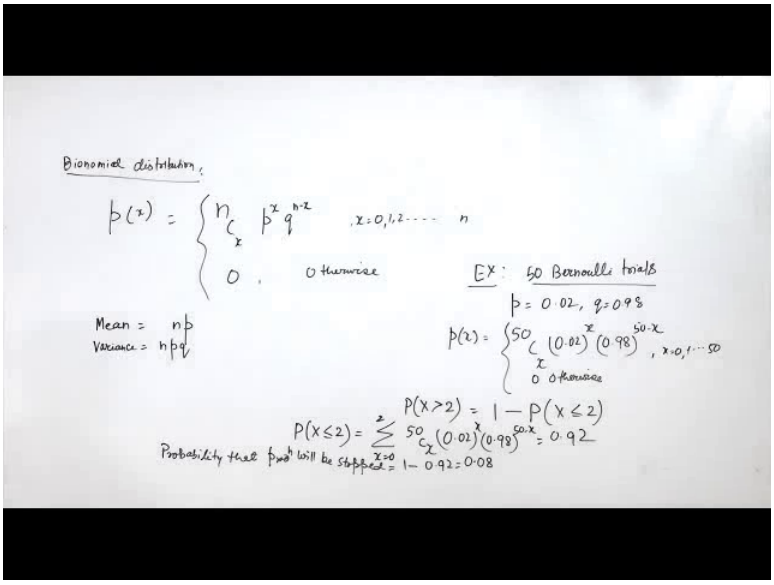

So, in the binomial distribution, here p x will be n C x, p x q n minus x as we have discussed where x is 0, 1, 1 to n. So, what we discussed is you have p as the probability of success, q as the probability of failure and if you have x number of successes you want in that case the probability will be n C x, p raise to the power x and q raise to the power n minus x and it will be 0 otherwise.

So, it means if you have x successes in succession the in that case you will have n minus x failures, if you have n trials and if you have x successes in succession in that case you have remaining n minus x as failures and that is why probability will be calculated as n c x p raised to the power x success raise to the power x and this is for failure raised to the power n minus x the mean in this case mean as you know you have n successes. So, it will be n into p and variance again will be n into p q. So, this is typically about the binomial distribution.

Then next we will move now in this case what we see let us before that we will discuss about the negative binomial distribution. Now, in the case of negative binomial distribution what is there in this case is distribution of number of trials is till you get the kth success. So, that is known as negative binomial distribution.

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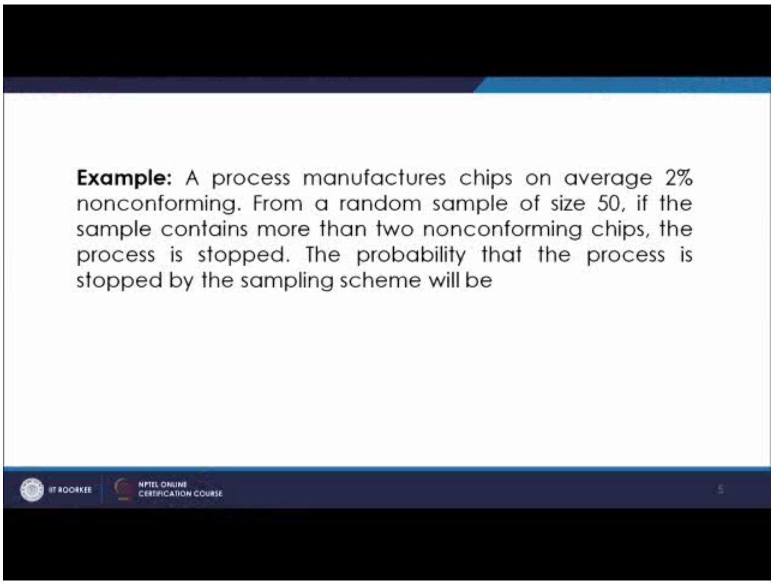

Now, let us see we can see a example problem where it is shown that a process manufactures chips on average 2 percent nonconforming. So, a process is there which is normally making the chips and on an average 2 percent is nonconforming.

Now, from a random sample of 50 if the sample contains more than 2 nonconforming chips the process is stopped. So, you have to find the probability that the process is stopped by the sampling scheme. So, such problems you can have the solution, now in that case what we see is if you get more than 2 chips which is nonconforming then it will be rejected that is what the question is. So, what we can see is here you have 50 Bernoulli trials.

So, if you look at this example in that you have 50 Bernoulli trails; now in this case as we know p is point 0 to. So, q will be 0.98 the thing is that if you take p as the 0.02 value q will be 0.98.

Now, in that case p x will be 50 c x 0.02 raise to the power x and 0.98 raise to the power n minus x that is 50 minus x. So, it will be 50 minus x for the for x value from 0 to 50 and it is 0 otherwise so this is the pdf. Now, in this case what we see is we will have the probability that you will have more than 2 times. So, p x more than 2 that will be 1 minus p x less than equal to 2, p x less than equal to 2 means either you have the value as 0, 1 or 2. So, p x less than equal to 2 means it will be x varying from 0 to 2 you have 50 c x 0.02 raise to the power x and 0.98 raise to the power 50 minus x. So, if to compute this value it is coming as 0.92. So, what we see that probability that production will be stopped will be 1 minus 0.92 that is 0.08. So, 8 percent probability is there that the production will be stopped. So, such problems can be solved in this manner.

Now, you have the geometric distribution.

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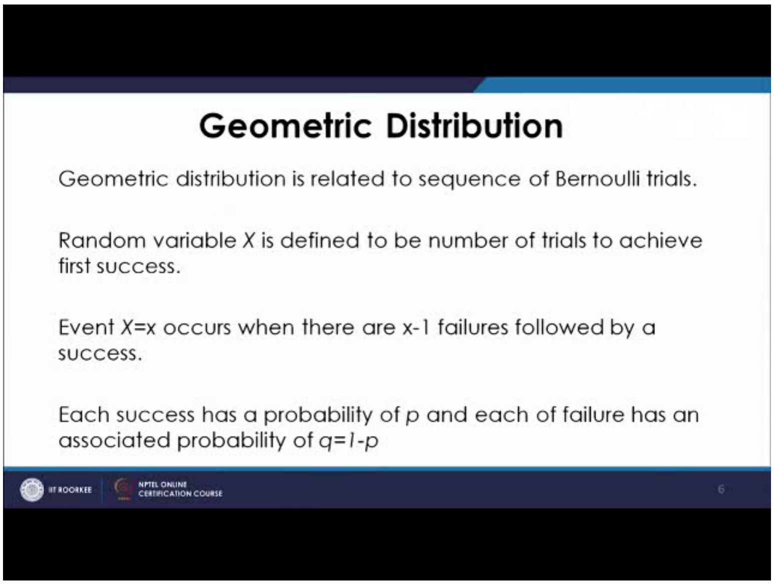

The geometric distribution is related to sequence of Bernoulli trials. So, in this case the random variable x is defined to be number of trials to achieve first success. So, this is for such cases where you go for number of trials till you get the first success. So, x equal to x occurs when there are x minus 1 failures. So, you go to x number of trials in that case up to x minus 1 cases you have failures and then the xth event is a success. So, such distribution are presented in terms of geometric distribution. So, probability of p is it will be for success and failure has the again probability of 1 minus p.

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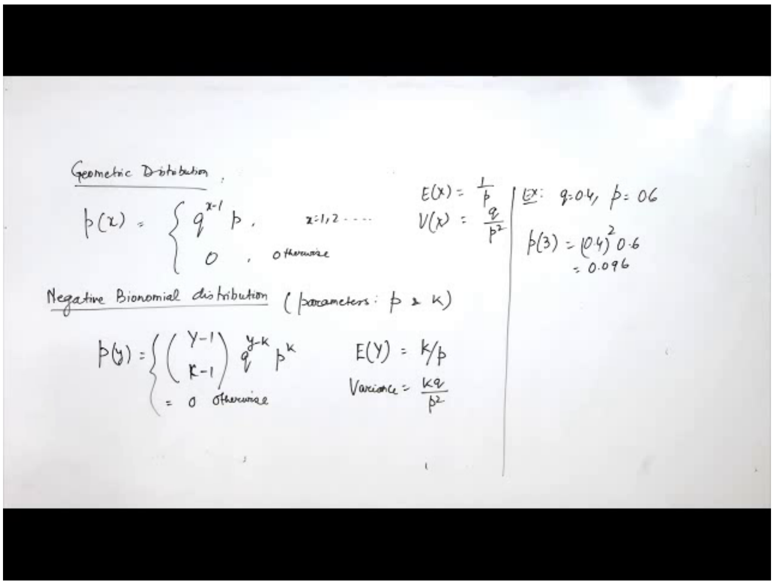

So, in these cases for geometric distribution, for geometric distribution Tthe p x it is defined as q raise to the power x minus 1 into p x equal to 1, 2 like; that means, we are going for x number of trials success is the 1. So, that is why on p you have the exponent 1 and then on q because you have x minus 1 times the failures. So, it is q raise to the power x minus 1 and it is 0 otherwise ok.

So, this is this is a kind of distribution which is known as the geometric distribution in in for such cases the mean is 1 by p and variance is q upon p square. So, this is the trait of the geometric distribution, we had discussed about the negative binomial distribution. So, as we see the negative binomial distribution in that we are going for number of trials until you get the kth success. So, such means you have any negative sense unless you get the success you go on going for the trials. So, such case is represented by the negative binomial distribution. So, you have the, you have to get certain number of successes and for that you go for trials so for the negative binomial distribution.

So, since it is a meaning having until that is why it is negative binomial and in this case you have 2 parameters p and k, p and k. So, as we know p is the probability value for the success and q is the probability value for the failure and k is a parameter because we want k successes up to kth success we will go for having the trials and for that p y will be y minus 1 and k minus 1. So, it will be c and in that you have because you have y minus k and success is k you are getting the kth success. So, that is why on this you have the power k and on the failure probability you have the value y minus k. So, this is the probability distribution function for this negative binomial distribution and it will be 0 otherwise. So, this is the representation for negative binomial distribution for such distribution the mean is defined as k upon p and variance is k q upon p square. So, this is known as so this is the example for negative binomial distribution.

Now, if we look at this kind of problem which we had seen, this has already been solved now there is another example like.

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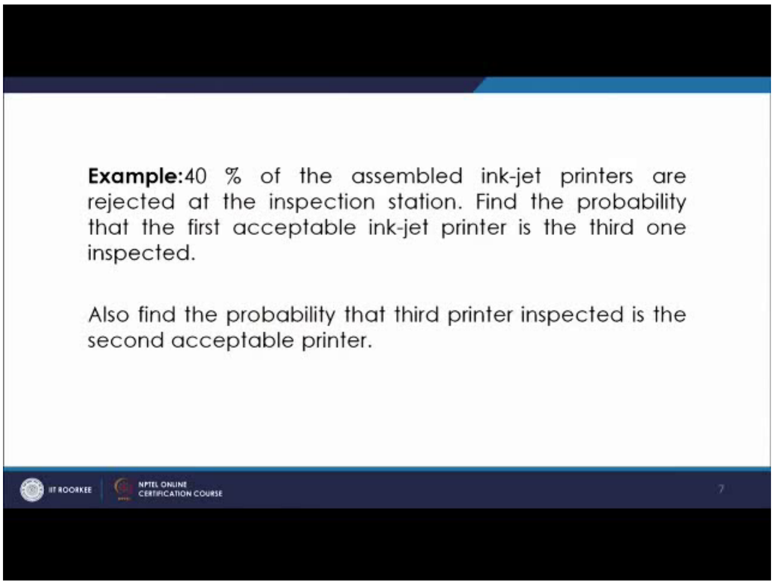

Forty percent of assembled inkjet printers are rejected at the inspection station find the probability that first acceptable inkjet printer is the third inspected. So, for such cases what happens 40 percent is rejected, it means 60 percent is accepted. So, if you try to solve such examples.

In such in such example where it is written that 40 percent of the printers are rejected means q is 0.4 and p will be 1 minus q. So, it will be 0.6. So, 40 percent of the printers is rejected and that is why 60 percent is inspected.

Now, the first acceptable inkjet printer is a third one inspected. So, you are going for three number of trials and in that case the p 3. So, it will be from here q raise to the power x minus 1. So, x is 3. So, q is 0.3 raise to the power 2 and then multiplied by 0.6 raise to the power 1. So, it will be 0.096 it means approximately 9.6 percent is the only chance that the third printer which is selected is the third one inspected will be the acceptable inkjet printer. So, such problems can be solved using this kind of distribution functions.

Also find the probability that third printer inspected is the second acceptable printer. So, you have to find that the third printer which you have been inspected is a second acceptable we have to use this. So, we are going for 3 experiments and we feel that the second acceptable printer you should get it, for that again you will use this negative binomial distribution.

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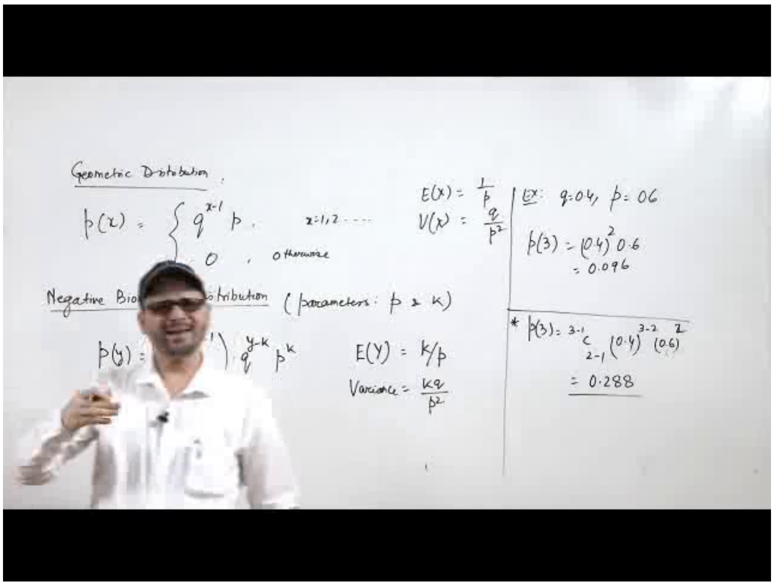

So, for that in such case you have p 3 again. So, it will be 3 minus 1 c 2 minus 1 because you want the kth success here it is this is the k number of successes. So, you have to get the second printer which should be acceptable. So, y will be 3 and k will be 2. So, 2 c 1 and in that case you have 0.4. So, q is there on that you have y minus k. So, that is 3 minus 2 and 0.6 raise to the power 2. So, it will be 0.4 into 0.6 into 0.6. So, and in that so, it will be 0.288. So, when this condition comes that you have to get certain number of successes till that you have to proceed for the trials or in certain number of trials you have to get that many of successes such problems can be solved using these negative binomial distribution.

Next is Poisson distribution.

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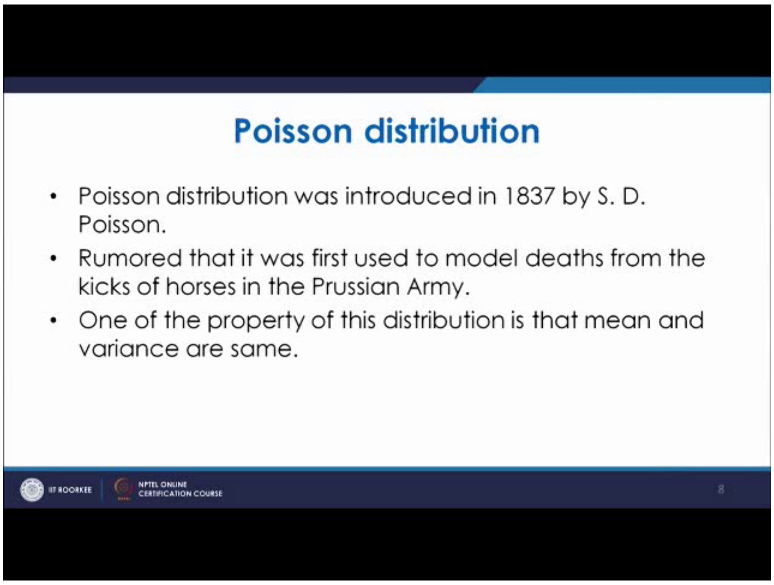

So, Poisson distribution is a very popular distribution which is used for the discrete events and it was introduced in 1837 by s d Poisson. So, it was a rumor that normally this distribution came into limelight when it was used to model the death from the kicks of horses in the Prussian army. So, that is a rumor, but normally for such cases where you have the randomness you go to use these type of distribution functions and one of the property of this distribution is that the mean and variance are same. So, we will first discuss about the probability mass function for this Poisson distribution, now in the case of Poisson distribution.

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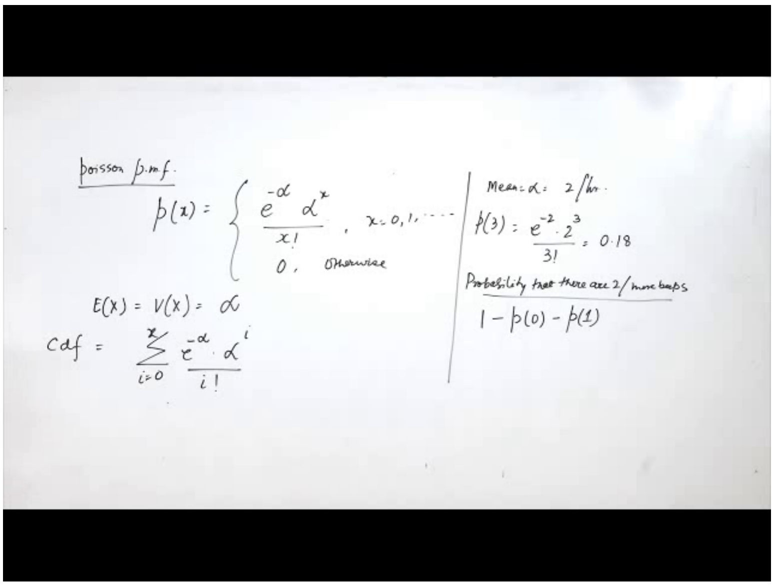

The Poisson probability mass function is defined as p x will be e raise to the power minus alpha and alpha raise to the power x by x factorial for x is 0, 1 or so on and then it is 0 otherwise.

So, value of x is more than equal to 0. So, you get this distribution value probability value as per this expression, now in this case the as we discussed the mean as well as the variance is the same that is alpha, this is the Poisson distribution mass function. Now, in this case the cumulative distribution function once we try to derive, the cumulative distribution function it has got the value i equal to 0 to x e raise to the power minus alpha alpha raise to the power i upon i factorial. So, this is known as the cumulative distribution function for the Poisson distribution means if suppose we try to find the defects which are coming that may be like 0 defect 1 defect or so and if there is a mean rate which is defined this is alpha this is the mean value. So, one mean should be defined and then you can have the probability that how much will be the having the probability if you want to predict that probability having more than mean or less than mean or so. So, this way you can get the probability mass function and the cumulative distribution function.

So, what we see is in this picture.

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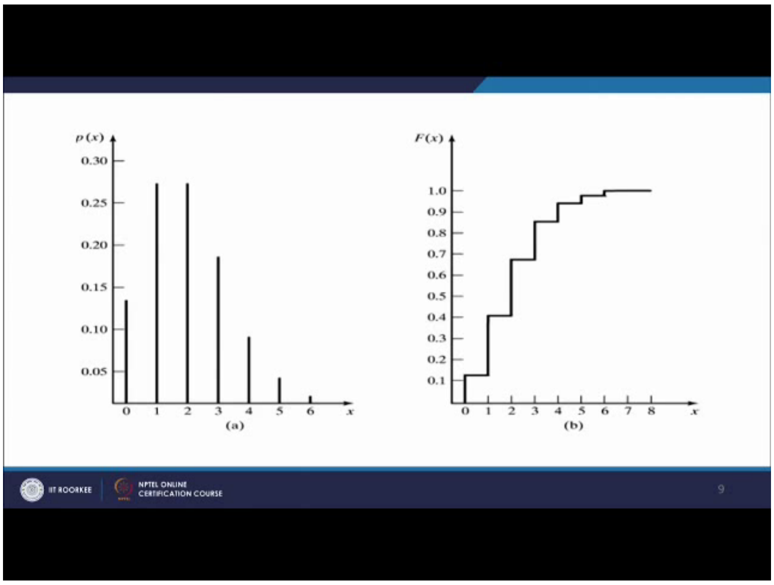

What we see this is the probability mass function we see here that this probability mass function is with alpha equal to 2 when the mean rate is 2 in that case this is the probability mass function. So, this is how the probability value varies for 0, if you put 0 here if you put 0 here and alpha is 2 then we get this value. It is close to 0.14 and if we get the value 1 or 2 you are getting the value which is more than about 0.27 or so like that and once you get the probability values then you can get the cumulative distribution function graph also. So, as we see for 0 this value is shown it will around 0.13 or 4 then it goes for one it is increased and it goes to about 0.4 and so it is increasing slowly. So, this is the cumulative distribution function.

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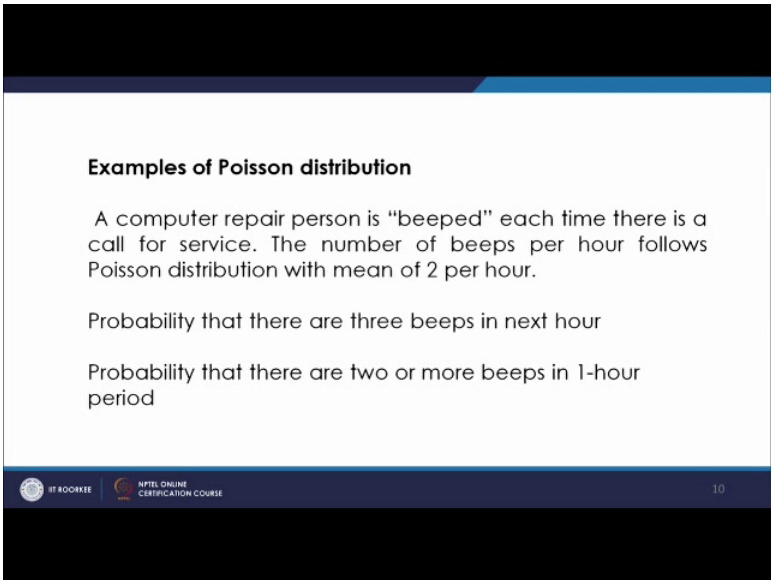

Now, let us take an example of a Poisson distribution problem, now such a problem like a computer repair person is beeped each time there is a call for service, now the number of beeps per hour follow Poisson distribution with mean of 2 per hour. So, the number of beeps per hour which is coming it is showing that you have at an average it is 2 per hour.

So, mean is shown alpha and that is 2 per hour, now the problem is that probability that there are 3 beeps in next hour. So, you have to find the probability that you have 3 beeps means you have to find p 3. So, once you know if it is Poisson distributed if any event is said to be Poisson distributed and if the mean is told in that case you know this mean value and for any for the prediction of any other value you have simply to put the values here. So, p 3 will be e raised to the power minus 2 and then 2 raise to the power 3 because you want to have the probability of value 3 and that will be divided by 3 factorial. So, this value will come out to be 0.18.

So, this way you can find the probability that there are 3 beeps in the next hour, then if you want the probability that there are 2 or more beeps in 1 hour period. So, in that case probability that there are 2 or more beeps 2 or more beeps. So, it means you have to subtract it from 1 the probability of either 0 or 1 beep. So, that will tell you that probability of having 2 or more beeps and you can find it 2 minus cumulative distribution function value at 2 and this if you calculate it was coming as 0.406. So, it will be 0.594. So, this way you can calculate the probability for 2 or more beeps in 1 hour.

Thank you.