Modeling & Simulation of Discrete Event Systems
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Lecture - 06
Statistical Model in Simulation

Welcome to the lecture on statistical models in simulation. So, this is a 6th lecture, and we will start about the statistical models which are used in the simulation purposes. Going to the history out overview of why to use this statistical simulation sorrow when to use, as we know for a model builder in simulation; he sees the world in a probabilistic way rather than deterministic. So, as we have already discussed; that when we talk about these type of systems discrete event simulation systems, we have the probabilistic component.

So, they see it in a probabilistic manner. So, stochastic models are there and they describe the various sense. So, you have different kind of models, difference stochastic models and they will predict the variations. Because things are not deterministic every time the probability is attached for any event to occur. So, that has to be predicted properly. So, that must be following certain kind of pattern. Now these are basically presented well by certain kind of functions.

So, approximate model can be developed by sampling the phenomenon of interest.

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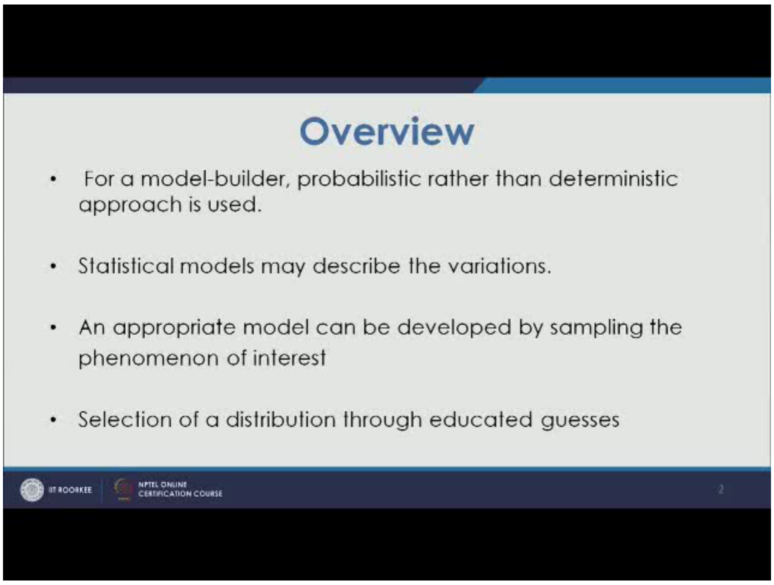

As we discussed; that if suppose we have to generate some numbers, or we have to generate the inter arrival time or service time; you know that will be basically predicted based on what has happened in the past. So, in the past the data are there which are basically recorded which are stored. Based on that you will have it trained that this is likely that this time will be there or this much will be the inter arrival time for that particular customer in between that. So, this way the approximate model can be developed. And many a times guesses are used by the educated persons or the subject matter experts. So, they use the guess to get it.

So, we will have the introduction to different kinds of terminologies; while dealing with these discrete random variables, or continuous random variables, probability distribution functions, density functions, cumulative distribution functions all that.

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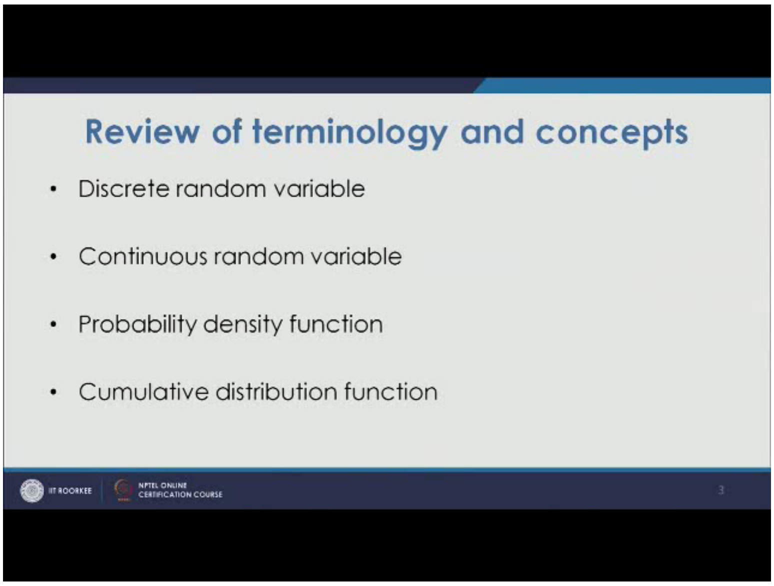

So, let us discuss one by one about them. So, coming to the random variable.

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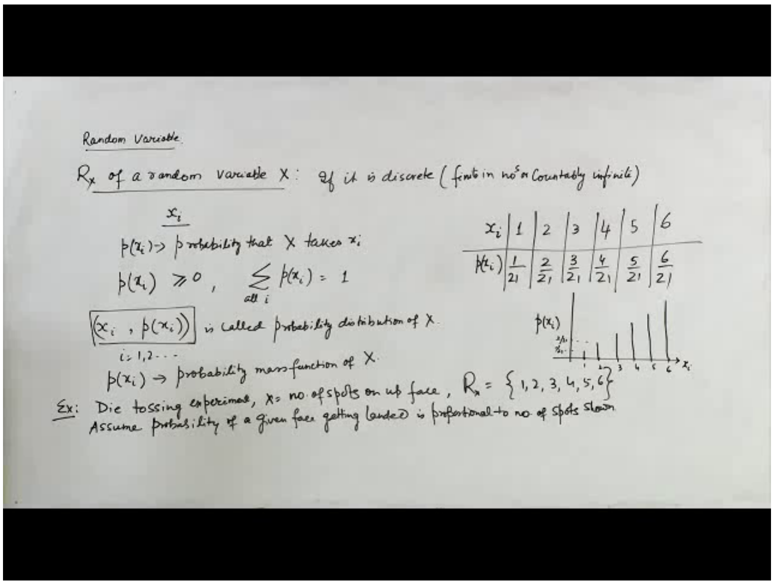

As we know you have this is a variable, which has randomness attached. And these random variables are of 2 types; when we studied is modeling and simulation you have a discrete random variable, and we have a continuous random variable. Discrete random variable means, it possesses many values, it will possess the values which are discrete, which are finite or countably infinite; which you can have a discrete identity.

So, suppose any random variable X, capital X is there it may have many values. So, it will have a range of the range or space. So, you have the R X. So, if X is the random variable R X will be the range of space for the random variable. It will, so how many? How much value it can attain? Suppose you are tossing a die, then there are 6 probable values. Either it will be 1 2 3 4 5 or 6. So, this is the range is from 1 to 6. So, that that is a random variable. When we toss a coin what appears on the top phase, that is basically; so, that is the outcome it is a random variable. And the space is so range of space that will be from 1 to 6.

So, either we can get or 1 or 2 or 3 or 4 or 5 or 6. So, that will be this is a kind of discrete random variable. Now they are defined like this. So, R X of a random variable x if it is discrete; that is if it is countable finite in numbers or countably infinite. So, in this case R X will be going from. So, one value to other and normally the value which is attains that is normally x i.

So, x i is basically the values. So, i will be wearing from one to whatever value it takes. And p x i; so, p x i is the probability that X takes x i. Now as we know we toss a coin if you look at the probability normal case having any face coming to the top is 1 by 6. So, 1 by 6 is the probability value and x i is either 1 or 2 or 3 or 4 or 5 or 6. So now, what we have certain properties about it, what we get in this case is that p x i will be greater than or equal to 0. In the case of discrete random variables, the probability of the individual outcomes if we take the so if we take the value of them, it has to be either 0 or it will be more than 0. And summation of p x i for all i it has to be 1.

This is the property of the discrete random variable. Because you have the range X will have a range that is R X. So, it will be denoted by R X. Then x i and p x i; basically, this is known as for i equal to 1 2 and all, this is called probability distribution of X. So, you will have on the X axis, you have the outcome, which is there x i. And on the y axis you have the probability of that particular outcome. And in that case, that is; so, x i p x i it is known as the probability distribution of X. And p x i is called the probability mass function.

So, X is the discrete random variable, and you have the possible values that is x i. Now let us take an example. An example is like; if in a die tossing experiment, X is number of spots on up face. So, we supposed do a die tossing experiment, and if X is defined as the number of spots on the up face, in that case as we know the R X will be it vary from 1 2 3 4 5 and 6. So, this is a discrete random variable.

Now, if the condition is given like this, that assume probability of a given face getting landed is proportional to number of spots shown. So, if we see that the probability of any given face which is getting landed is proportional to the number of spots shown. In that case you will have x i and p x i. So, you will have x i and you will have p x i. As we know x i you have 6 results 1 2 3 4 5 and 6. So, in that case the p x i values, if we say if we take that they are equally likely in that case it will be all 1 by 6, but then here the thing condition is that the probability is proportional to the number of spots which is being shown.

So, it will be 1 by some quantity it will be 2 by some certain quantity. So, in that case you have to sum them. So, it will be 1 by 21, 2 by 21, 3 by 21, 4 by 21, 5 by 21 and 6 by 21. So, what we see is; in this condition this is how the probability of appearing any face varies. And you can have a graph. What we see is on this you will have x i and this will be p x i. So, p x i will vary from 1 by 21 to 2 by 21 to. So, so you will have outcome 1 2 3 4 5 and 6. So, what we can see is; this will be 1 by 2 3 4 5 and 6. This kind of distribution function we see this.

So, this shows a distribution function, and this is showing the probability values. So, this will be 1 by 21, this will be 2 by 21, like that. If we take the summation of p x i, if we sum them it will be 1. If we see the p x i values, they are all either greater than or equal to 0. So, these are the conditions which are to be followed. So, this is a case of discrete random variable. Now we will see the next kind of random variable that is continuous random variable.

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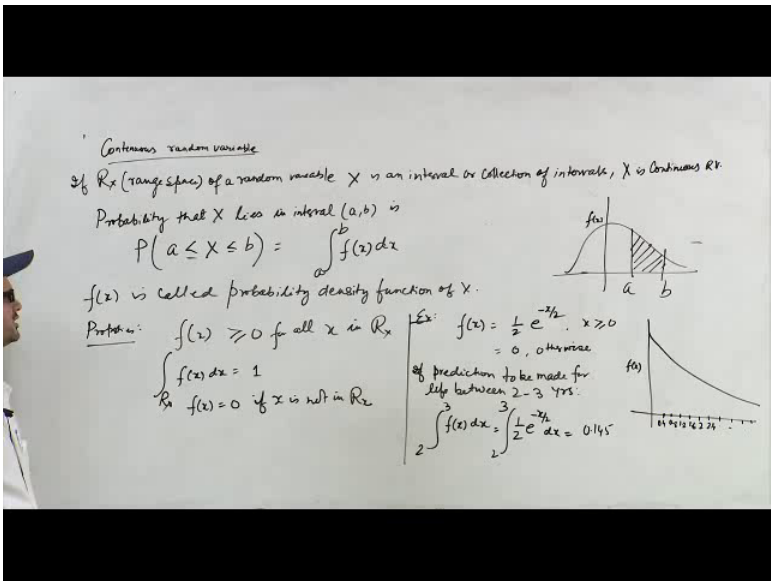

When the random variable does not have the discrete values, it cannot be defined on a discrete point. Any outcome is not as a discrete value in that case it is a continuous value. So, here you have to express this in terms of the intervals. When we get certain values or the probabilities, it has to be in certain interval or certain number of intervals.

So, if R X; that is range space of a random variable X is an interval or collection of intervals, X is continuous random variable. So, what we see is; the range space of a random variable. Either it is an interval or a collection of intervals; in that case it is known as the continuous random variables. You does not have that freedom to define at a particular point, in this cases at view define at a particular point the probability of that will be normally 0. Now in this case probability. So, probability will be always expressed for a certain interval between one quantity and the other.

So, probability that X lies in interval a b is; so, probability will be, X will be in between a and b. And it will be defined as f x dx a to b. So, in this case, normally this is nothing but if you have a f x dx. So, if you have a and b which is nothing but this area. So, if you want to find the probability of this kind of random variable. It will basically the area under this curve is taken as unity and in that case the value of this as a fraction it will be the probability values. So, this is this kind of random variable is known as continuous random variable. And in this f x is called probability density function of X. So, this f x is known as the probability density function of X. Now it is properties are f x will be greater than equal to 0 for all x in range space. If it is in the range space, then for that for all x the f x value will be more than or equal to 0.

Then summation or integral into for the range space f x dx will be 1. So, if we have this range in this graph; if we take the area whole it will be taken as the one. And f x will be 0 if X is not in R x. If X does not lie in the range in that case f x value will be 0. So, in this case we can have the probability values calculation like in many cases like it exponential distribution or so. Suppose we predict the life of certain instrument to predict the cracks or to predict anything. You may have certain kind of expressions. So, if you want to expect the life in between certain time, that you can calculate using these methods.

Suppose for example, if f x is given as half E minus X by 2 for X greater than equal to 0 and 0 otherwise. So, such maybe the problem, such kind of problem you may come across where this is nothing but this is basically life of certain device which is used to detect the cracks in aircraft components. In that case if you look at you have X more than equal to 0. You have probability of this function value is half E raised minus X by 2. So, if you put X as 0 it will be 0.5. So, if you take the graph of this it will be an exponential graph like this it will go.

So, this will be your f x. And in this case mean is about 2. So, you will have the values here. You take some values you can have these values in in this case. So, it is such; so, this kind of graph this is a case of continuous random variables. Now if you want to predict, if prediction is to be made for life between 2 to 3 years. Now this is basically for prediction of the life of this component and if you want to predict for the life between 2 to 3 years, in that case what we see is; we get the values 2 to 3 f x dx.

Probability that the life of this product will be between 2 to 3 years will be f x dx a to b. So, you will have 2 to 3. So, if you will have integral of half E raise to the power minus X by 2 d X and 2 to 3. So, if you compute this value you will get as 0.145. So, here you will have 0.4, 0.8, 1.2, 1.62 2.04 like that. So, this way you can calculate the probability that. So, such problems where the life expectancy will be required to be predicted that any component will last for 2 to 3 years. It is probability about 14.5 percent. So, this is example for the continuous random variables.

Now, further we will define certain other parameters.

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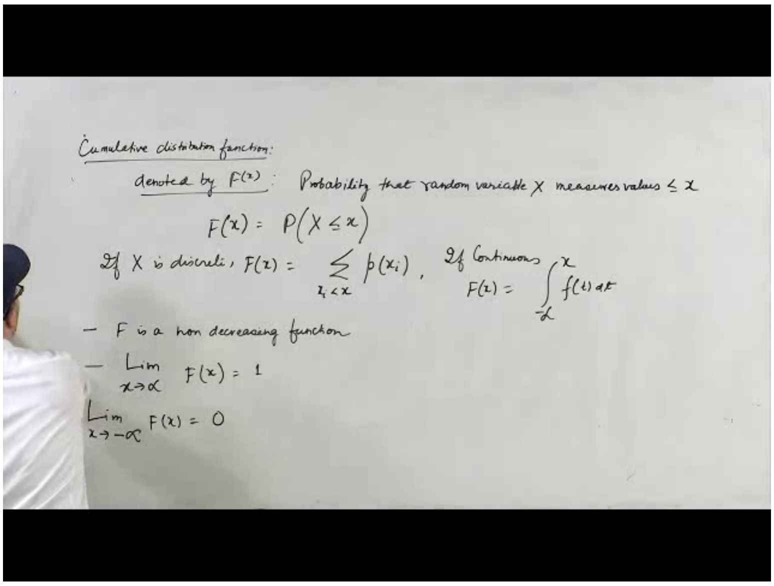

Cumulative distribution function: Cumulative distribution function it will be again a function which talks about the cumulative effect. So, if we take the discrete one and if you want to find the cumulative values at certain point, it will take the summation of all the values before that. So, it will be nothing but the addition of all the probabilities values for all the outcomes which is below that. Similar is the case in case of continuous random variables, where it will take from minus infinity to that value. And then it will have the area under that curve.

So, it is denoted by f x, capital F x and it is probability that random variable X measures values less than equal to X. So, F x will be probability when X is less than equal to x. So, that is what the cumulative distribution function is. If X is discrete, it will be as we discussed it will be summation of all the probability values. And if it is continuous f x will be from minus infinity to x f t d t. So, this way the cumulative distribution function is defined. Now in this case there are certain attributes of this cumulative distribution function, one is that it never decreases.

So, because it is cumulative, it will go on increasing. So, in this case f is a non-decreasing function. Then limit x tending to infinity f x will be 1 because when x is tending towards infinity or towards the forward range of the space. In that case it is taking all the values into account. So, it is value will be coming out to be 0, to be 1. So, it is take that is why it is 1, but if you take the extreme left value then it will be 0. So, similarly limit x tending to minus infinity f x, it will be 0. So, these are the traits of the cumulative distribution function.

Now, there are other parameters other terminologies; while dealing with these statistical models. One of them is expectation.

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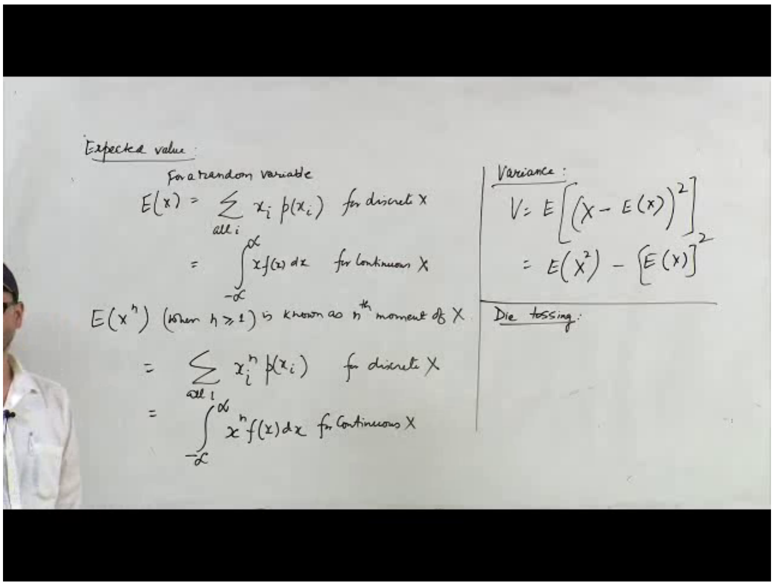

Expected value are mean value, which will also be defined as mean value mu. Now for a random variable, expected value of any random variable is defined as summation all i x i p x i. So, for a discrete random variable you will have the expectation that is summation of x i into p x i. Or you will have integral of x f x dx minus infinity to plus infinity this is for the continuous random variable. So, for all for discrete X and this is for continuous X.

So, this is nothing but it talks about the mean of the values. Now this is also known as the first moment. So, E X n, when n is greater than equal to 1 is known as nth moment of X. So, this is the first moment of X. So, if we got E expected value of X square it is nothing but the second moment of X. And this is calculated as summation for all i x i n p x i for discrete X. So, when we try to find the second moment of X in that case it will be x i raise to power 2 into p x i. And that way if you find the summation that will tell you tell us E X 2. And for a continuous function for continuous X it will be minus infinity to plus infinity x n f x dx.

So, this is for continuous X. So, this 2; these 2 discrete and continuous X. You define this mean or expected value from these formulas. So, from here you calculate the variance. They talk about the central value or the mean value, now many a times you have the data which talk about the variability, or the variance or the spread from the mean. And that is basically expressed in terms of variance. And variance is defined as v equal to E of X minus E X square.

So, this way we find the variance. What we do is; we take the difference from every individual value to the expected value, and then take it is square and the expected value of that. So, that is known as variance. It is also defined as E of X square minus E X of square. So, this way we compute the values of variance. Basically, what happens there maybe positive or negative differences from the central value. So, what is the actually it takes the more value or the modulus value. In fact, it does not take into account any kind of sign.

So, if you take the normal average the plus and minus difference may cancel. So, to nullify that, we take the squares here, and then get the difference, and then we represent it in terms of variance. So, these things are used in case of these a statistical models. So, we may have many kinds of problems, which can be discussed while dealing with such cases. Now let us find try to see how we calculate the mean and variance for the loaded toss die tossing. For die tossing experiment, if we try to find the mean, the mean can be found by so E X.

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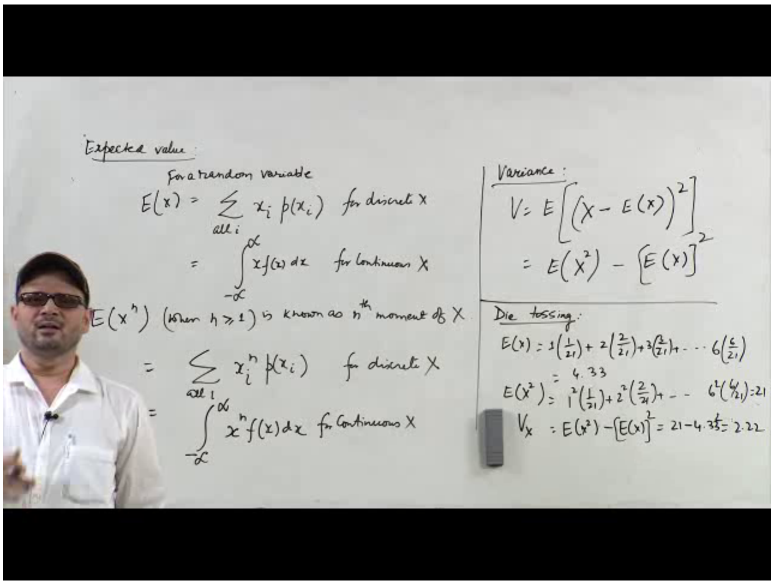

As we know E X will be summation of x i p x i.

So, x i was 1 2 3 4 5 and 6. So, one times 1 by 6, 2 times 2 by 6, 3 times 3 by 6 like that 6 times no it will be 21. So, 21, 21, 21 and if you take that it will be coming as 4.33. Similarly, E X square will come as 1 square 1 by 21, plus 2 square 2 by 21, that way 6 square 6 by 21 and that will be coming as 21.

So, you can get it checked and if you find the V x, variance it will be E of X square minus E X square. So, it will be 21 minus 4.33 square 2.22. So, this way variance can be found. Standard deviation is the another terminology which is used, which is there square root of this variances. You can find the square root of this; you can find the square the standard deviation. So, this way, these are the terminologies which will be used in our frequent discussions and in our subsequent discussions.

Thank you.