Modeling & Simulation of Discrete Event Systems

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Lecture - 05
Simulation Examples

Welcome to the lecture on Simulation Examples. So, in the last lectures, we had the introduction about the different kinds of problems involving this simulation of discrete systems. And we discussed about the concept of model system and simulation, we also saw the applications and advantages and disadvantages of these simulations.

So, we will discuss that how we come across certain examples where this concept of simulation is used. And earlier we have also discussed about the applications. So, what we have so far discussed that we have the application of simulation in normally these areas.

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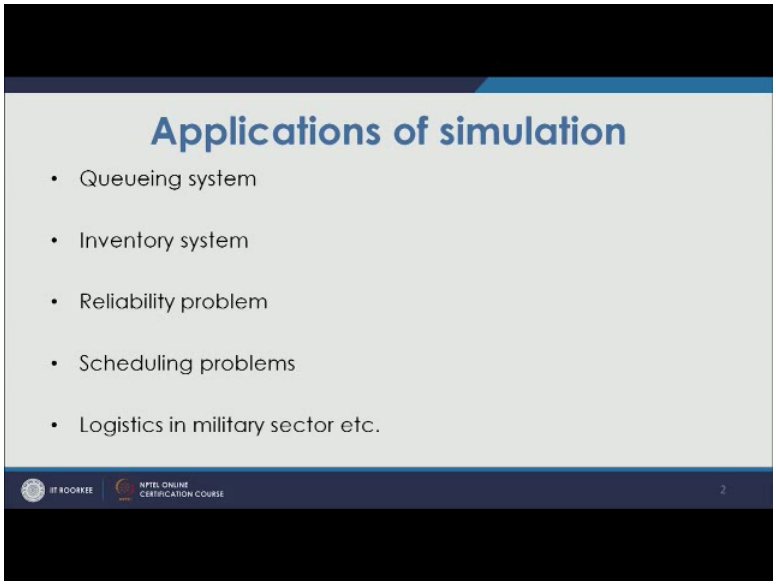

Like queuing system, inventory system, reliability problems, scheduling problems, logistics in military sectors etc. So, what we have understood with the concept of discrete event system simulation is that normally the process is stochastic there is randomness, we do not know that what is to be you know when the things will happen, when the event will occur. So, there are random components and that is why you have the generation of random numbers many a times in these cases.

About them basically we will discuss in more detail in our later chapters or later lectures. But we will see that how these random numbers are used for solving such problems in the cases of queueing, like in queueing what we have earlier studied also or earlier discussed that as we take any example of practical example of queueing problems the problem is that we have to predict first that when somebody is going to come, then how much time he is going to be in the queue, then how much time he is going to take for getting the service and then when he is going out.

So, these are the components which are basically random components about then the prediction is done and since there is probabilistic component involved. So, we use these random numbers there are many kinds of density functions or distribution functions that we will start in our next lecture.

So, there once we get that then we will have a chart and we can predict based on the past observations that it is likely that the customer is going to come at a particular time when he comes then the server is going to be either idle or it will be engaged. And then it will also take some amount of time that is again probabilistic to be served. So, there is service time. So, based on that the queueing performance measures can be estimated like average time spent in the queue average idle time of the server all that.

Similarly, inventory system as we know in inventory system you have basically to order the inventories. So, how much to order what will be the demand on a particular day these are the probabilistic components and these things are basically coming from the past data's. So, depending upon the past data we are generating it and in generating that. In fact, the random numbers are very much used. So, we will discuss about one of the problem like in queueing system, and then also we will discuss in brief about other systems like inventory again. So, let us go to one problem of queueing.

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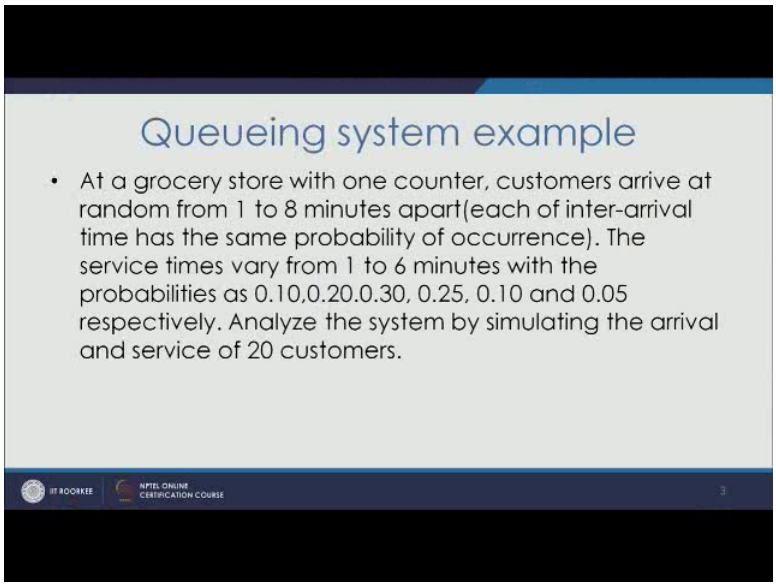

Now, let us see there is a queueing system example like at a grocery store with one counter customers arrive at random from one to 8 minutes apart. So, what we see is that we have a grocery store and customers are coming randomly from one to 8 minutes apart. So, that is given inter arrival time one customer is coming. So, and after that the next customer will come in between them the time of apart maybe 1 minute to 3, 4, 5, 6, 7 or 8 minutes. So, there are 8 values.

Now, they that is each of inter arrival time has the same probability of occurrence it means the probability of having 1 minute is 12 and half percent. Similarly probability of having 2 minute is also 12 and half percent. So, they have the equal probability the service times vary from 1 to 6 minutes with the probabilities as 0.10, 0.20, 0.30, 0.25 0.10 and 0.05 for 1 to 6 minutes respectively. So, for one minute the probability is 0.10; 10 percent; probability is there that the first person who will get service will have 10 percent probability of 1 minute.

Similarly, 20 percent probability is there that he will be served in 2 minutes 30 percent probability is there that he will be served in 3 minutes 25 percent probability is there that he will be served in 4 minutes 10 percent probability is there that he will be served in 5 minutes and 5 percent probability is there that he will be served in suppose 6 minutes. So, these are the random components the probabilistic components given to you and

based on that you have to simulate. So, you have to analyze the system by stimulating the arrival and service of 20 customers.

So, how they do how the simulation is carried out for that we will have an idea that how the simulation will proceed. So, what we see is now in this case as we have understood that you have you know arrival time is given inter arrival time is given and you have the service time given and for that you have to assign the random digits. Now in this case basically this random digits are given this random digits can be produced. So, you have softwares by which you can produce the random numbers you have the methods by which you can produce the random numbers. This we will discuss in our later chapters, but here for example, we have been given that this is how you are going to use the random numbers for the inter arrival time.

Now, how to you know interpret that. So, what we have seen we have known that this inter arrival time varies from 1 to 8.

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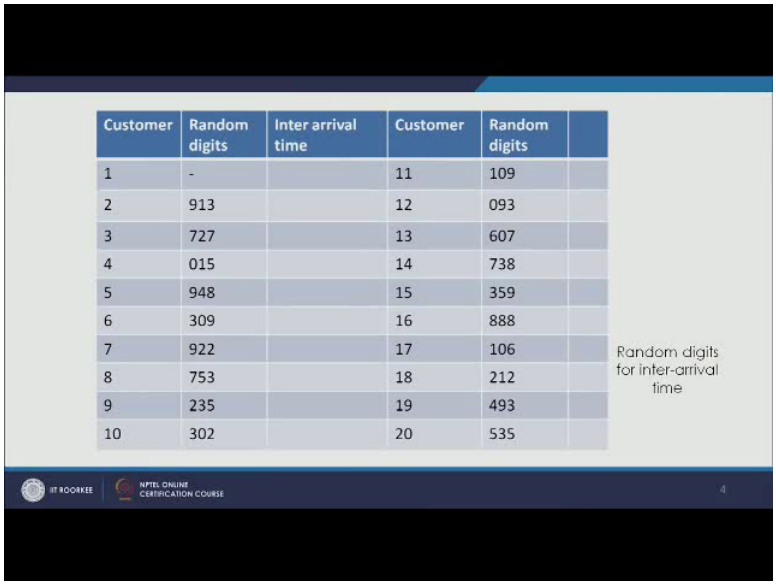

So, you have 1, 2, 3, 4, 5, 6, 7 and 8.

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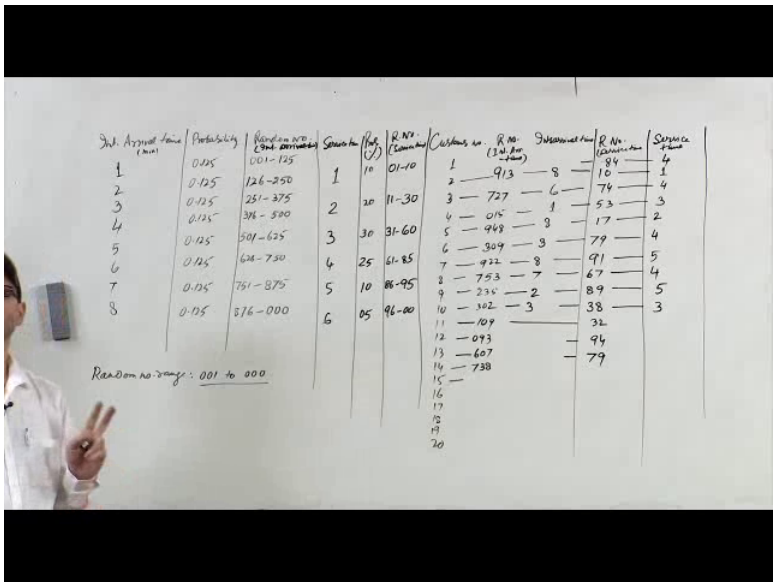

So, these are the 8 values of inter arrival time. Now what we see is that its probability of each component each time is probability is 1 by 8. So, probability is 0.125 for each one. Now this is the probability for each of the component the practice is that you have to assign the random numbers in such a way that if there is random number is chosen you know they have equal probability that it will be among any of these. So, the practice is that you have a set of random numbers you have the numbers in a certain domain or in a certain range and in that you assign this numbers. So, suppose you what you do for them is its better since it is by 8. So, its better we can take the random numbers up to 8 or in a multiple of 8 so, but then normally the better practice is to take the random numbers from one to hundred or one to thousand or so.

So, being multiple of 8. So, 1000 is the multiple of 8. So, we are taking the random number from 1 to 1000 and in that case if, and then we are basically distributing them. So, if we are taking the random number range from 0 0 1 to 0 0 0. So, basically before that you have a 9 9 9. So, all together you have 1000 numbers. So, in this case what is seen is now in this case you have. So, if you take random number; so random number 0 0 1 to 0. So, to 1 2 5 this will be for 1.

So, this number will be. So, if the random number which comes while doing the simulation if the first number is in this range 1 to 125, then the service the inter arrival time will be taken as one minute. So, this is in the minute. So, similarly it will go further

126 to 250, then 251 to 375, then 376 to 500, then 501 to 625, then 626 to 750, 751 to 875 and 876 to many a times you start here from 0 0 0 and you end to 9 9 9. So, either that convention is followed or this convention is followed any of the convention maybe followed.

So, here we start from 0 0 1. So, we will end with 1000. So, going to 4 digit, we do not go to 4 digit; we take this digit. So, this digit may be taken as the equivalent to 1000; so, that if that value comes it will be corresponding to 8 minutes. So, what we see is in this case if we see the first customer since first customer is coming at 0 time. So, there is no random digit assigned coming to second customer. So, customer number if you look at customer number 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20.

So, you have these customer numbers and as you know that your random number is given for inter arrival time is 9 1 3 first for first there is nothing because first is coming at 0 time. So, here that is determined. So, second the random number is 9 1 3. So, inter arrival time now inter arrival time if you look at the 9 1 3 the first random number which is there in the table according to that 9 1 3 comes here. So, it means the inter arrival time is 8 for second it means the first customer comes at 0 time the second customer will come 8 minutes after the arrival of the first customer.

Similarly, you have 727. So, 727 comes in this range and that will be 6 minutes, then you have 015, 948, 309, 015, 948, 309 and further you have 9 to 2; 753, 235, 302 like that; it will move so forth; it is 015, 015 will come in this range and time will be 1 minute 948 will be again in this range. So, time will be 8 minutes 309 will be in this range, it will be 3 minutes 922 will be again 8 minutes, 753 will be again here 7 minutes 235 will be in this range, it will be 2 minutes 302 will be in this. So, 3 minutes.

Like that. So, then for 11 to 20; 109, 093, 3 then you have 607, 738. So, like that if the time we have we can solve that, but we can have the idea that ideas will be clear from here similarly. So, this way you have these inter arrival times defined with you now let us go to the next the service time. So, this random number is for inter arrival time then another set of number is random number for service time this number is also given to you now this random number for service time is given now for service time if you look at. So, this in this was for the inter arrival time.

So, this random number was for inter arrival time now for service time for service time we have been given the probability for service time as we see we have been given the probability of 10 percent, 20, 30, 25, 10 and 05. So, this random number can be assigned from 1 to 100. So, we can start from 0 1 to 0 0 like that. So, in this case, if suppose if you go for; so, for customer if we take the service time service time we have 6 values 1, 2, 3, 4, 5 and 6 the probability values are given as 10, 20, 30 and then 25, 10 and 05 percent.

So, the random number for service time will be again assigned based on that. So, for this you can have the random number range from 1 to 100. So, we can have a start from 0 1; it will go up to 0 0. So, for this it will be 0, 1, 2, 1, 0; similarly it will be; so, what we do in normally is we get the cumulative probability, so based on that we can get it anyway so it will be 30. So it will be from 11 to 30 because this is 20 percent. So, after this you have to have 20 numbers. So, this will be 11 to 30 then further it is 30. So, it will be 30 1 to 60.

Then you have 61 to 85, then 86 to 95 and 96 to 0 0. So, this way you can have the numbers which is assigned random number assigned for the service time. So, what we see here is you have different random digits for service time and these digits are like 84, 10, 74, 53, 84. So, here for first also it will come for first it is 84; 84, then you have 10, 74, 53, 10, 74, 53, further you have 17, 79, 91 and then you have 67, 89, 38. So, let us see how we do in this range; now in this range as we know; the service time which will be approximated or which will be predicted by using these random numbers will be again based on this table.

So, if you have service time of 84 it means 84 is coming in this range and the service time which is expected of the first customer is 4 minutes, similarly the second number is 10 this 10 is coming here. So, it will be one minute 74 is coming here. So, we can take it as 4 minutes, similarly 53 you will have here 3 minutes 17 is 2 minutes 79, you have here 4 minutes, 91 is here 5 minutes 67 is here, 4 minutes 89; 89 will be coming here as 5 minutes and 90, 38; 38 will be here 3 minutes.

So, this will go further you have another numbers written like 30 2 ninety 4 79 like that. So, you can have the prediction of these random numbers. So, what we know by this time; we are knowing that when the customers are coming and how much time they are going to take for getting the service. So, in how much time the server will be serving

them that is the service time. So, once you have these 2 components known, then you can find other measures of performances like how much the customer has waited in the queue how much the customer has waited in the system for how much time the server was idle.

Now, let us see how this table is made. So, that is known as a simulation table this stimulation table is like this where you have first you have the customer. So, customer in that tables you will have 1 to 20. So, if you try to see we can have certain idea that how we will proceed in this case. So, let us see you have the customer.

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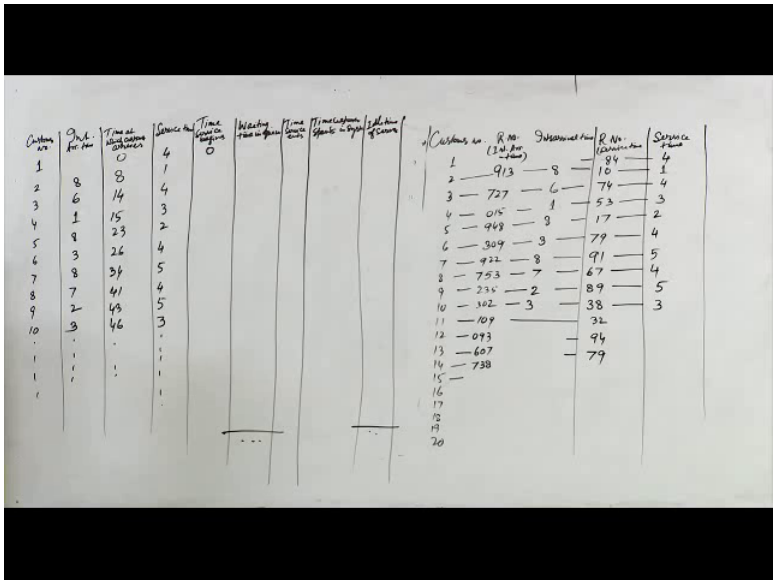

So, customer number will move from 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 and then it will move inter arrival time, already it is predicted. So, the first one is inter arrival time first one is coming anyway as 0 for the second one; it is 8 6 1. So, it is going like that 8 6 1, then 8 3 8 7 2 and 3.

So, we are just limiting our studies to 10 persons then in that case the; if suppose it is at time the time at which the customer comes. So, what we see is time at which customer arrives. So, the arrival time of the customer if it is 0 it will be 8, because there is 8 minutes of inter arrival time between the first and the second customer. So, first customer are comes at 0 time and for him the inter arrival time is 8. So, you have 8 then similarly it will go to 14, 15, 23, 26, 34, 41, 43, 46, like that.

So, whatever num number comes here in that case you can have the arrival time next is service time the service time if you look at the service time again is the probabilistic component which we have predicted from the random number generation. So, here what we see is 4 1 4 3 2. So, for first it is 4 1 4 3 2, then you have 4 5 4 5 3 and then it will move. So, this is the service time now what we see is this customer next table is time service begins, if you look at the time at which the service begins.

So, the customer has come at 0 time and it has taken 4 minutes. So, time service begin anyway he is coming at 0 time and the server is free. So, he will start his service at time 0, then once he has started at time 0 the second customer because he will take 4 minutes it will be taken 4 minutes after 4 minutes that this customer has will depart. So, after 4 minutes, he will this server is ready to further serve.

Then waiting time in queue the next is time service ends further you have time customer spends in system and the last is idle time of server now we have few things two of the parameters we have basically predicted one was inter arrival time and service time. So, inter arrival time we got from here and service time we have got from here now from be here by looking at the problem we can feel all these data. So, depending upon the cases you will have all these data field you can have the waiting time in queue; so, based on that person. So, you can have the average waiting time in the queue. So, once you get for 20 customers you can have the data here and then whatever summation comes based on that the average waiting time in queue can be found.

Similarly, you have the idle time now for how much time when the customer comes when he goes into the queue and when the next customer goes of the service depending upon that the idle time of server can be calculated from this table. So, this can be practiced and you can get the results this results whatever you get here based on that the percentage of the time the server is idle that can be found out. So, what I mean to say that our main job was to predict this and this based on that you can have all this data because when the service will begin certainly when the customer will leave the next customers will join the service.

So, that way service will be coming based on that you can have deviating time in queue when the customer joins if we add the service time then the time service ends there can be found. So, this way all these values can be filled in and once you fill that then after

that all the measures of performances can be found out. So, this is how we solve such problems in the case of queueing coming to other problems like in suppose inventory there will be probabilistic demand. So, again demand will have a probabilistic component it may be said that the demand may vary from 100 to 150.

So, again that maybe based on the random number generations and in the similar way the random number can be generated and demand can be predicted. So, maybe in future we can solve such problems when we discuss about these problems typically and one by one we will try to solve such problems when we discuss about the queueing cases. So, such types of example problems are expected in case of simulation.

Thank you.