Modeling & Simulation of Discrete Event Systems
Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 39
Monte Carlo Simulation of Queueing Problems

Welcome to the lecture on Monte Carlo Simulation of Queueing Problems. So, we have already seen that we use this random numbers to solve different kind of problems. And the different kind of problems may be either in queuing, or replacement inventory control, reliability. So, we will see that how it can be used for solving a problem in the case of queuing. So, let us take an example of a queuing problem, which is at the doctors clinic a doctor is there; a dentist is there and it is Monte Carlo simulation will be carried out to see the average waiting time of the customers.

So, basically the simulation will start at 8 am in the morning and the arrival of the customer is fixed. So, they are coming at the scheduled arrival time although arrival time may also be the random.

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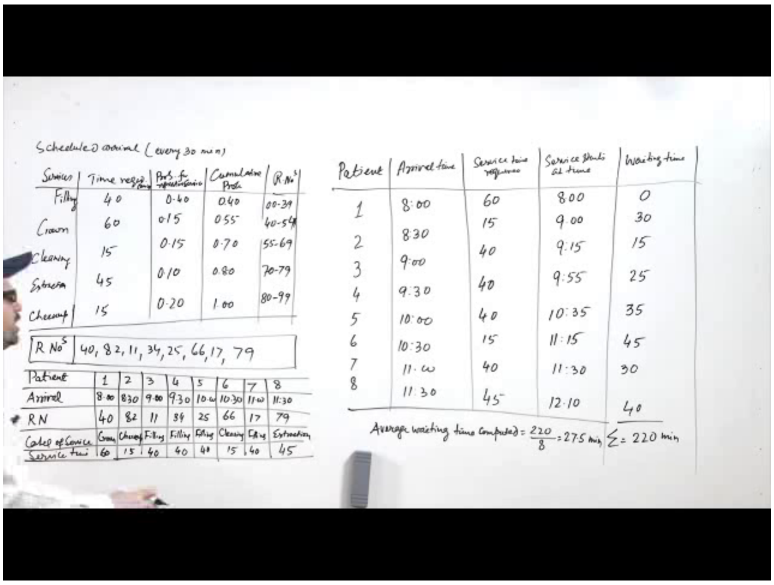

But let us see in this case. We assume that the arrival is occurring at a scheduled time. So, at a scheduled arrival case is there. Every 30 minutes one customer is coming. So, this is the case of queuing. Now in that case: although they are coming after every 30 minutes, but they are coming for five different treatments to be carried out upon them. So, they have 5 different kinds of service.

So, these services are like you have filling. So, the services which the doctors provide is filling, then you have the crown, and you have cleaning, extraction and checkup. So, see this is schedule arrival. It may be unscheduled arrival random arrival. And that will be again depending upon see if you have some probability distribution, then based on that we can have the arrival time also. But in this case, they are coming. So, first customer coming at 8; second is coming at 8:30. Third who is coming at 9? And then 4th is coming at 9:30. Now, they are coming for these different services, and they are coming for these services which take certain time, and the time required for the service is different. So, time required for the service is for filling it is 40 minutes, and then for crown it is 60 minutes it is 15 minutes, that is cleaning extraction is 45 minutes, and checkup is 15 minutes. So, this is in minutes.

Now, it was see it has been seen by previous record, it was seen that the customers who come for filling has a probability of probability for respective service. So, this is the probability that they are coming for filling. For filling the service time is 40 that is fixed, but probability that they are coming for filling is 40 percent. So, it is 0.40. Similarly, probability for coming that they are coming for the crown, placement that is 15 percent. So, that is 0.15, probability for cleaning they are coming is again 15 percent. Probability of extraction they are coming for that is around 10 percent.

Similarly, probability for checkup is they are coming, it is around 20 percent. So, let us this data is there, and you are told that you find the average waiting time. So now, the thing is that we do not know for what purpose he is coming. So, he will be coming and that guess will be done by again by referring to the random numbers which are generated, and here there are stream of random numbers. You have some random numbers which are assigned and based on that we will do. But anyway, that random number in the program can be called from the respective of our routine.

Now, if you look at this; so, to assign the random numbers again we need the cumulative probability. So, the cumulative probability will be; in this case the probability will be here 0.40. Then it will be 0.55. Then it will be 0.70. It will be 0.80, and then it will be 1. So, this is the cumulative probability. So, as we have done in the previous examples, we will have to assign the random numbers from 0 0 to 99, in that and the if you assign the random numbers which are to be assigned. So, that will be from 0 0 to 39. So, there are 40 random numbers including 0 0.

So, there are 40 random numbers which are assigned, and if any random number comes in between them then we will take the service of filling. Similarly, you have again you will go from 0.40 to 0.54. So, that not 0.5. So, it is 4 0 to 5 4. So, this is the number. So, it will be 40 to 54. Similarly, it will be 55 to 69. Then 70 to 79, and 80 to 99. So, these are the random numbers which are assigned. So, that you can go and predict; that when the customer who is coming he is coming for what kind of service. In that case you can say that if he is coming for cleaning that time will be taken as the 15 minutes of time.

Now, what we see is; that we can find that how these customers are coming. Now for that the random numbers which are to be used are basically. So, random numbers which are given to be used are basically 40, 82, 11, 34, 25, 66, 17 and 79. Suppose this is the random number which is to be used for knowing what kind of service will be required by to the I mean patient by the doctor. So, what we will see we will have the patient here, and then we will have his arrival. And then we will have the random number. Further we will have category of service and service time. So, this table can be made. Now if you make this table the patient number you have one to so, 1 2 3 4 5 6 7 and 8.

Now, this patient arrival is fixed. So, this patient is coming at 8 he is coming at 8:30. He will be coming at 9. He will be forth one come at 9:30. Fifth one will come at 10, and then he will be coming at; 6th one at 10:30, 7th at 11 and 8th at 11:30. Then the random numbers assigned are shown. So, that can be taken from the program, and if it is given here will take that random numbers: 40, 82, 11, 34, 25, 66, 17 and 79. So, these are the random numbers which are being put in.

Now, based on the random numbers we have to see, what kind of service will be assigned to him. So, if it is 40, the 40 is coming here. So, it will he will be going for crown, then 82, 82 again in this range. So, he will be go for checkup. Further you will have the so, for crown you have the time of 60 minutes he will be having a service requirement of 60 minutes of time for checkup. He will have requirement of service time has 15 minutes. Then you have then further 11, 11 will be in the first. So, it he will be go for filling. So, filling will take about 40 minutes. 34 again is in the filling.

So, again he will take going for filling and it will take 40 minutes, 25 is further in filling. So, he will go for filling he will it take another 40 minutes, 66 is in this cleaning. In cleaning it will take 15 minutes. Then in 17 again he is coming for filling. So, for filling again he is taken the 40 minutes, and for last one that is 79 is in this range. So, it will be extraction. So, extraction will take again extraction is taking about 45 minutes. So, this way what we see that the patient is coming and he is taking that much of time.

Now, we have to see that how we have to find that when the customer comes and when we leaves, so that we can find the final time, when he leaves and average waiting time. So, for that again we will make the table. So, it will be patient. Then you have arrival time of the patient. Now service time required for the patient, and the service starts at what time? And then further the waiting time. So, you have 8 we will go for 1 2 3 4 5 6 7 and 8. Now in that we know that arrival time is fixed. So, the first customer is coming at 8, he is going at second is coming at 8:30. He is coming at 9 then one is that 9:30; 10 10:30, 11 and 11:30.

So now service time requirement we have already calculated; that what is the service time requirement as per the random number which has come into the stream the random number, as we discussed the random number can be anything anytime we calculate in the in the extent the random number it will be different. And the random numbers are they are uniformly distributed random numbers, from there the sample is taken, but we can have the generation of random numbers as per certain particular distribution also. So, the service time is 60, 15, 40, 40, 40 and then further 40 15 40 45 40 15 40 and 45. So, this time is already there for a particular job or which the patient is undergoing. So, this time is fixed.

Now, the service starts at what time first customer when he is coming there is no queues. So, it will be just joining the queue. So, his service will start at 8. So, he has the waiting time of 0. Then the second customers service will start at 8 plus 60 minutes. So, it will it will start at 9. So, for him the waiting time will be 30 minutes. The third customer so now, the customer has come to get the service at 9. And he needs to get time of 15 minutes for the service to complete.

So, in at 09:15 he will depart. And once he will depart this person patient thirds service will start. So, he will be coming at 09:15. So, then accordingly this person since he is coming at 9, and he will be joining the service at 09:15, he has to wait for 15 minutes. Then this person will take another 40 minutes. So, at 09:55 he will allow the next customer to go into the service. So, this person has come at 9:30 and joined the queue at 09:55. So, the waiting time is 25 minutes. Then he will take another 40 minutes.

So, he will at 10:35 he will be joining the queue. And since he came at 10 and he join the service at 10:35. So, 35 minutes is the waiting. Then further he will take another 40 minutes. So, 40 minutes will be 11:25, not again 11:25, it will be it is 40 minutes. So, 25 minutes left plus 15. So, 11:15 he will this customer 6 will join the queue, and that way the waiting time be 10:30 to 11:15, so 45 minutes. Then he will take another 15 minutes. So, the next customer will join the queue at 11:30. So, he will be waiting for about 30 minutes 11:30 minus 11. And then at this point he will be taking 40 minutes. So, next customer will join at 12:10, and this person is got to wait for 12:10 minus 11 30 40 minutes. So, what we see; that the different customers have to wait for different number of times.

Now, if we take the total waiting time, total waiting time will be 45 plus 25, 70, 105, 150, 108 plus 20, 220 minutes. So, the average waiting time can be computed. And average waiting time can be computed as 220 divided by 8. So, it will be 27.5 minutes. This is how for when you are given. The sequence of random numbers you can calculate this average waiting time in this fashion. So, what we have seen in the last I mean problem; that you had the random numbers which are specified, but many a times you will have to generate the random numbers as per certain distribution. And then you have to calculate the parameter values.

So, suppose we are dealing with one another questions; where it is said that the inter arrival time and service time they are distributed exponentially, are exponentially distributed with some mean value.

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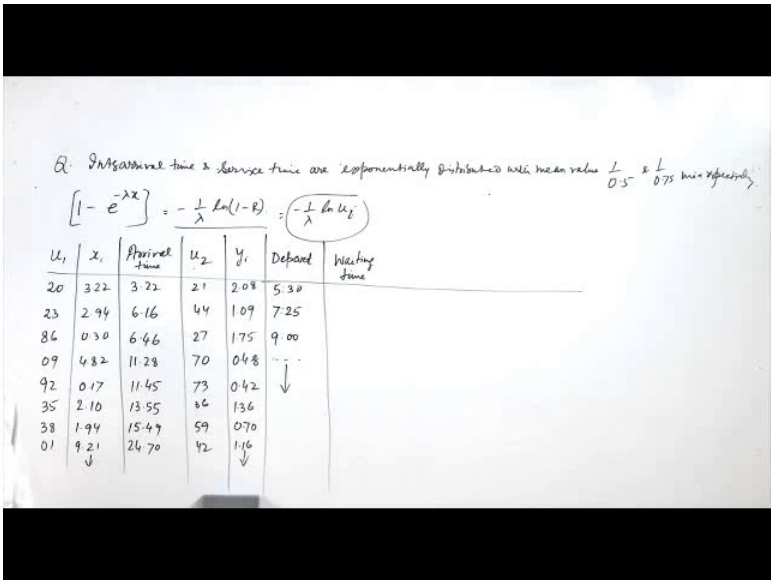

So, as we know that if you have the parameter lambda the mean is 1 by lambda. So, they have the mean values 1 by 0.05 and 1 by 0.75 minute respectively. So, this is given. And you have basically to generate now the random numbers. So, from the random number streams: so using this distribution of exponential distribution you have to generate the random number and accordingly that random number you will have. So, from there you will find directly the inter arrival time itself.

So, as we know that in that case we had seen; that you equate the cumulative distribution function to some uniform distributor number value. And then from there we generate this inter arrival time or service time. And then finally, we are doing the queuing simulation. So, what will happen? In that case you will have. So, what we see in that case we have seen that normally we have 1 minus e raise to the power minus lambda x. That is what it was. So, we were doing the inverse transformation. So, 1 minus random number and then minus of 1 by lambda, and then we do the l n of that particular value. So, what we calculated was that minus of 1 by lambda l n of 1 minus r that is what it was. So, basically if we take this random number as one random number 1 minus r as that then in that case we can have like that minus 1 by lambda l n u 1.

So, l n u I we can we can write for any particular. And this one random number is known to us. Once it is taken from the uniform distribution uniformly distributed number. So, you can get the u one u 2 like that for. So, once you put these values as or as the different once you can generate that stream of random number which fits for this exponential distribution.

So, you can have these values and you can generate them. So, let us see if you have those random numbers. Like, u take as if you take them as suppose from the table or from the uniform distribution if it comes like 20 23 86 9 and 92 35 then 38 1 like that it goes, suppose it goes the u one goes like that. So, you will have to generate the inter arrival time from there. So, you put in this formula you know the lambda value for this is 0.05. So, 1 by lambda is there.

So, once you do that you will get the inter arrival time. So, that is suppose x i. So, this if you put this value into that expression you will get the value of 3 point suppose 2 2. You may get it as 2.94 further you may get it as 0.30 4.82 then 0.07, then 2.10 further 1.94 and it will go on like that 9.21 and so it so, this will go on. So, this way you can generate the inter arrival time if you do not have that particular distribution. So, using this distribution function you can generate this with the inter arrival time. So, what happens? The once you know the inter arrival time, you can find the arrival time. So, as you see this, this customer comes at suppose 03:22 AM so the first one. So, he will come at 3.22 the next will be from 3.22 to 2.94. So, it will be 6.16, then further 6.46, then you have similarly 11.28, 11.45. Then you have 13 point, you know it is 5 5. Then you have 15.49, like that. Then you have 24.70. So, this way the time will be recorded.

Now, again you have the next parameters that is y i. So, that is your service time. So, for service time this was done using the value of that particular lambda 0.5 if you put 0.75. In that case another value can be found and u 2. For that before calculating this y i you have to have the u 2 values. So, suppose u 2 again you take stream and that value wise 21, 44, 27, 70, 73, 36. Then 59 42 like that it is going. Suppose, so again you are going to put it into that expression, and from there you will get suppose y i. So, that is service time for the customer.

So, it will be 3 point. So, this will be 2.08, 2.08 then we will get 1.09. Further you will have one 0.75 0.48, 0.42, 1.36. Then you have 0.70, 0 point 1.16 and so on. So, this way if you do not have that distribution which is given to a probability is do not given to you earlier; you had the empirical probability given now in this case you are not given. You are simply told to generate these times as per certain distribution function. So, that we have studied that using the inverse transform technique you can generate these numbers directly which are basically exponentially distributed. So, you are getting directly these values.

Now, based upon that you can simulate the particular queuing; so suppose this person comes at 3.22. And he will have a service of 2.08. So, when he will depart so he is going. So, he will going 3.22 and 2.08 it is will be 5:30. So, this way he will be going from here. So, but that is there the next person is coming at 616 the person is going at 5:30, but this person is coming at 616, and he has the service of 109. So, he will be departing at 7:25. So, this person will go out of this queue at 7:25 and at 7:25 this person will be able to go into the queue.

So, he has to wait from 6:46 to 7:25. So, then in that case you have the waiting time, but then from 7 25 onwards has a service time of 175. So, for that he has to wait. So, 7:25 plus 175, it will be 9. Further then from 9, now this customer is coming at 11:28; so at 11:28 it will be coming and service time is 48. So, it will be again it will be going together and he will be. So, further in that case you are going to do the calculation and it will tell you. So, this way you can continue the simulation and get the values of waiting time for the customers. The time can be maintained in respective units, and you can get the time of waiting for the respective customers finally, and you can find the performance measures.

Now, this can be I mean we can take any kind of distribution function. And for that we can generate the random numbers, and you can do the modelling using this random numbers. So, this is how this Monte Carlo simulation is carried out. And it can be done in other areas also it is a powerful tool, which is you wherever this complete uncertainty. And there is very much difficulty in modelling very complex system.

In those cases, this Monte Carlo system is proved to be a very you know, helpful technique very effective technique when you go in the long run, it will tell you the value which are very much accurate to the actual phenomena.

Thank you very much.