Modeling & Simulation of Discrete Event Systems
Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 36
Problem Solving & Case Studies on Simulation of Manufacturing System

Welcome to the lecture on Problem Solving and Case Studies on Simulation of Manufacturing System. So, in this lecture we will discuss about the problem solving and also some aspects, what we need to discuss in case of simulation of manufacturing system.

Now what we have seen that you have different type of distribution functions to represent certain kind of events. Now the distribution it is mean in variance basically affects the output performance measures. Suppose you have a single machine which is used for serving for doing the operations by using the tools. So, and suppose the inter arrival time of the jobs which come on that machine, it has the distribution that the exponential distribution with mean of 1 minute and similarly the service time is also exponentially distributed and we mean with 0.9 minute.

Now, in that case the utilization percentage of utilization that for the machine that becomes 0.99. So, basically the inter arrival time is with mean of 1 and the service time is with 0.99 as mean. So, in that case the utilization proportion of time it is utilized utilization factor is basically lambda by mu. So, in that case lambda will be 1 by 1 and mu will be 0.99. So, that way you will get it as 0.99.

Now, in that case what we see if we do the simulation for a very long time. So, the purpose of doing this study will be that how the performance measures may change. Now in that case what is seen is that if you basically solve it. So, while solving you have to take the numbers this inter arrival time will come, but that will come in random. So, it may range anything from either from 0 to any values. So, depending upon the distribution function, it will take any value it has certain probability.

So, now again that will be taken against the numbers. So, if you basically correlated with the numbers or stochastic simulation you do, in that case against that numbers it will comes it do not know what will come it is not deterministic. So, in that case if you go.

So, if you go for the long run then what you see is the average delay in queue may be quite high value. It may it will be quite it has been reported in the examples that may go quite high whereas, if you see the deterministic value if you look at the deterministic value then what will happen there is no queue.

So, inter arrival time is mean time is 1 minute and service time is 0.99 minutes so in fact, ideally there is no queue. So, what it says that because these are all probabilistic we do not know that what time the job will come. So, that can be seen by only simulation. So, in the simulation you can simulate it quite easily, that you go into the excel sheet you have the inter arrival times.

So, that will be basically pertaining to certain randomization you know inputs. So, for that you can have the random numbers and then you can assign them. So, you will have the inter arrival numbers, inter arrival times and this basically is quite stochastic. So, quite probabilistic and it will range from anything. So, that is why the it is seen in (Refer Time: 04:40) it has been reported that it may go as high as more than 50, 60 or even 90 minutes which is not imaginable.

If you look at the deterministic approach like it is coming after 1 minute and every 0.99 minute if it is getting served. So, in that case there is no queue whereas, that queue may go as high as 90 or so. So basically the prediction of these performance measures depends on the mean and variance of the distribution function what we use. So, that is why this is important when we analyze such systems. So, what we will see that in the case of manufacturing system, basically you have many types of times like up time you have time in the down time you have waiting time repair time.

Now, all these are basically the times either you are having those data or you do not have those data. So, basically in those times you will have to fit them you have to assume certain kind of distribution function, which is reported that it is normally fits it normally fits. So, many a times the data is given many a times you have some data and it is seen that you basically fit many number of times and then you see suppose some data fits with certain type of distribution and it is tested many times. So, in that case may be that the mean value for which it fits that may vary every time little bit. So, we can take the average value in the end. So, this is how these are the basically approaches which is normally used in case of simulation of manufacturing systems.

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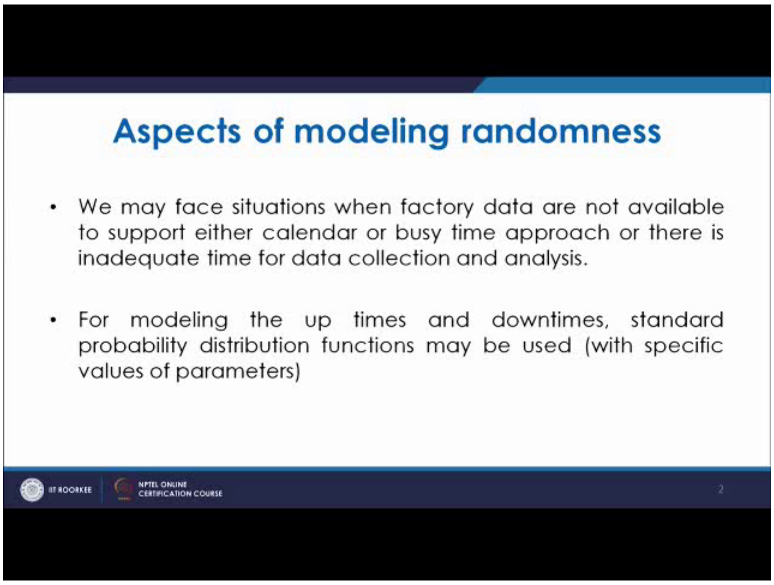

So, the thing what we need to know that we may face situations, when factory data are not available to support either calendar or busy time approach or there is inadequate time for data collection and analysis. So, these are also the situations which may we may come across. So, for that you have basically for modeling the uptimes and down times, you have the standard probability distribution functions which can be used with specific values of parameters that we will see that how if you have some data how we will see that how it fits in to it.

Before that let us see one example of another example where a machine is there which is expected to come and it is said that it will be down for 10 percent of the time. So, about the machine it is written. So, one machine is there, a machine which is expected to be down for 10 percent of the times.

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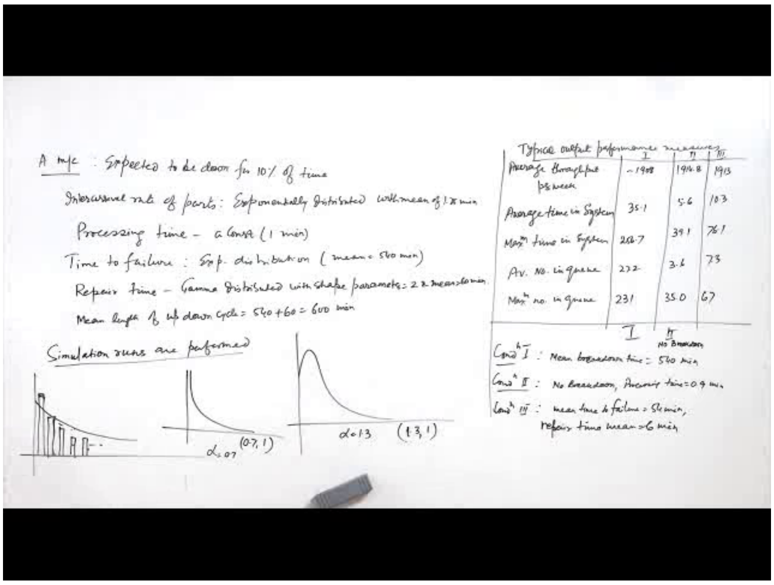

So, expected to be down for 10 percent of time. Now as we see that we may have the data in different forms. So, means for 90 percent of the time it is up and then 10 percent time it is down.

Now, other data may be given like the inter arrival rate of parts. So, that is basically given as suppose exponential it is exponentially distributed and that is exponential distributed with mean of 1.25 minutes; so exponentially distributed with mean of 1.25 minutes. So, then it means you have a distribution function and it will talk about the times and the associated probabilities. So, you can draw these exponential distributions in computer.

So, there are many softwares now available, but even you can do your own. So, if you go to excel, you just write exponential and then you can draw it. So, they will ask you for values against which it has to be drawn, x values and then for the lambda value. So, if you put those values then it will draw against the x values. So, you will have the exponential this. So, that inter arrival rate of parts is suppose given as exponential distribution with mean of 1.25 minutes, similarly there may be given as processing time and suppose processing time is taken as a constant and that is 1 minute. So, processing time it may be a variable, it may be following certain distribution function, but suppose it is given as a constant of 1 minute.

Now, time to failure; so time to failure it is said to be the having exponential distribution and here the mean is mean time is 540 minutes. So, mean 540 minutes means for 540 minutes the machine will be up. So, basically it is using the calendar time approach, you have to go it is not given that in that how much time it was idle or how much time it was busy. So, it is given that simply your time to failure. So, that is using an exponential distribution and that mean is 540 minutes. Similarly the repair time and repair time may be given. So, repair time is said to be gamma distributed.

So, gamma distributed as we know in the gamma distribution you have 2 parameters shape parameter and scale parameter. So, in the gamma distribution basically the shape changes with the change in say shape parameter values. So, it is said that it is gamma distribution with shape parameter as two. So, that is alpha and beta we call it sometimes beta and theta we call it. So, shape parameter as 2 and mean is 60 minute. So, what we see is certainly 90 percent of time it is up and the 10 percent of time that is 60 minutes. So, that is you know ultimately it is down. So, your whole cycle of up and down process is normally you have 600 minutes and then you have. So, that is why your mean length of up down cycle becomes 540 plus 60. So, that will be 600 minutes.

Now, you can go for doing the simulation of this problem. So, in that problem basically you will have the time for the ups that is how much time it was up and that first of all the time of arrival. So, your time of arrival is basically exponentially distributed with the inter arrival time as 1.25 as minute as the mean. So, it will vary from anything from 0 to some value where the probability becomes 0. So, we have to have those values and then in that you will have the processing time. So, it will be coming and then it will be coming as the processing time. So, as we know that if the inter arrival time is less lesser values in that case the larger queue will be generated and if it is more in that case the processing will go on and it will reduce the queues.

So, if we look that in that case suppose there is no breakdown in that case. So, there may be such ideal cases, but then what we will see here, we have to how to model such cases. So, what we it has been seen that normally if you do the 5. So, simulation runs are done. So, if the simulation runs are performed. So, what has been basically the value? So, you have some basically output performance measures, now what are these typical output performance measures. So, the typical output performance measures for which we are interested will be basically average throughput per week. So, these are the typical output performance measures. Now what we do here, here basically 160 hours of simulation that is 20 days of 8 hour a day suppose you are running the simulation and then you are basically finding these output performance measures.

So, suppose in a week you have 5 working days, you have 8 hours of running. So, you will simulate for that 8 hours then it will go for 5 days and then you can go for suppose. So, in 1 week you have 18 to 5 that is 40 hours similarly if you go for 4 weeks. So, if you go for 20 days like 4 weeks. So, in that case you will get you will have these you need to know about these output performance measures.

Now, what are these also next will be average time in system then you will have maximum time in system, further you have average number in queue and then maximum number in queue. Now these are the typical output performance measures for which we are interested in and we need to know that what are these performance measures what are the values of these performance measures per week suppose average throughput how much of the product is getting you know serviced as it is going out like average time in the system maximum time in the system and average number of in queue.

So, using this values like if you take the mean down time as this time to failure as 540 minutes. So, what we seen is that you have different conditions. So, if you take the different conditions like condition 1 and so, in condition 1 you can have condition 1 here. So, if you do the simulation the it has be reported in the law and (Refer Time: 16:43) exercise if you do here there you will find it or you can do on the arena basically you can work on arena or you can have your own code and you can solve it.

What you see is the value is coming as something close to 1908 that is average throughput per week, similarly average time in system is 35.1 and then it is 256.7 and further 27.2 and then 231. So, this what this condition 1 is? Condition 1 is using that mean breakdown time. So, mean breakdown time is taken as 540 minutes and we are taking these values like inter arrival rate is exponentially distributed with mean of 1.25 minutes, processing time constant for 1 minute. So, if you for this 160 hours of simulation 5 independence simulations were done using different random numbers certainly and then you average it out and then you find the values. So, you are getting 1 9 0 8 this is normally the average throughput per week.

Now, then average time in the system is 35.1, similarly maximum time in the system is 256.7, average time in the queue is 27.2, maximum time number in the queue is 231 what is the meaning of this? So, what we see that in the first case you are your values are 1908 and so, this is average throughput per week, i you do the second simulation where.

So, in that you have the 5 independent simulations. So, if we do another simulation where we do not take any breakdown. So, in this case you do not take any breakdown: so no breakdown. So, if that being the case the system performances system performance measures are given as basically this value are reported as 1914.8 and this value is 5.6 now then this is 39.1 this is 3.6 and this is 35. Now what we see the difference? The basic difference what we see now in this case the processing time has been taken as 0.9. So, here in the condition 2 you do not have you have not taken no breakdown. So, you assume that there is no breakdown and also you assumed that processing time is 0.9 minute.

So, if there is no breakdown and processing time becomes 0.9 minute, in that case it is seen that it is going similarly somewhat and now what is that meaning. In fact, in that case if there is no breakdown. So, what we seen that in 5 days if you look at we have the 5 days week and so, what we see is inter arrival time that is 1.25. So, in an hour you get basically 48. So, basically you have 48 arrivals in a day, no in hour. So, in the 8 hours you have number of arrivals. So, you have number of arrivals in 5 days.

So, that number of arrivals becomes 48 into 8 into 5. So, that becomes as 1920. So, that is close to that. So, what we see is that is average throughput per week that basically is your 1914.8 and then processing time is less. So, in normal case what we see that since this being more it means at larger intervals the to mean product is coming for getting the job done, and then it takes very less time to basically serve it. So, you are getting normally close that value. So, it is close to 1920 in a week. So, in a week, and that is why you get this throughput as close to 1920 in both the cases normally what we see is they are comparable.

However you see that this average time in system that differs a lot or maximum time in system that also differs average number in queue and you see maximum number in queue these are differing. Although maximum throughput you do not average throughput you do not see much of the changes, but you see a lot of change in on the other things because in this case here the processing time is less and in this cases there is no breakdown. So, basically these values change these value changes. So, that is how the difference comes.

Similarly, it has further will reported that if you have another condition that is 3 where suppose you have again 5 independent simulation runs, and here the condition is different in this case the mean breakdown time is basically changing from 540 to 54. So, in that case what happens? So, if it is coming from 540 and it is going to 54; so condition 3 is your breakdown time mean time to failure is 54 minutes and similarly you keep the repair time. So, that is service time and if you keep that that time as repair time mean as 6 minutes, now if you do that what we see here again we are the same condition we have 54 minutes of uptime and then you have down time of 6 minutes. So, ratio is 9 is to 1, 10 percent of time it is down.

But then if you do the again the simulation runs from that many hours and you do with 5 independent replications and further you do the average, these values are reported as something very close 1913, then it comes as something like you know 10.3, then it comes as 76, this comes as 7.3 and this is 67. So, what we see that normally you have the although the average throughput is same in this case, but then other things change because here you have the mean time to failure is less. So, these values changing these parameters changing this ultimately the output performance measures are changed. So, what we see that these values basically affect the system.

So, now let us see that when we do not have this data, in that case we must have the idea of the distributions as to what kind of distribution basically will fit for certain type of events like you have as we have seen here, we have machine uptimes, you have machine downtime repair time is there. So, there are many situations many a times you do not have the data. So, you must have that which of the distribution will basically fit better.

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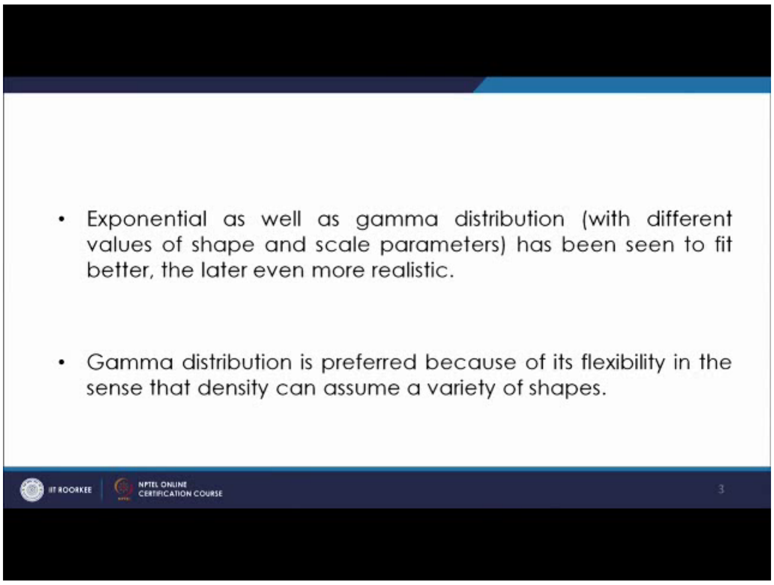

So, exponential as well as gamma distribution with different value of shape and scale parameters, has been seen to fit better even the gamma distribution is set to be even more flexible, because with the with the changing shape parameter of the density the curve takes different shapes. So, gamma distribution basically is seen to be more realistic.

So, what has been seen by practice that if you have the data, if you try to see exponential many a times does not fit very much and especially near that ordinate access basically does not fit much. So, it will meet at that point, but then mostly you will see that maximum values are in that. So, basically the actual values are quite high I mean with high probabilities. So, that way we will see certain cases where we will see that though how they are changing.

Gamma distributions is preferred as we discussed because of its flexibility, what we can see further is certain typical histograms which normally has been seen that your processing times how they are following. So, typically it is seen that the processing time for automotive component follow like this. So, how to see that which distribution it full fits into similarly typical histogram of the time to failure you can see here. So, in this case this is the value of finally, what we see that.

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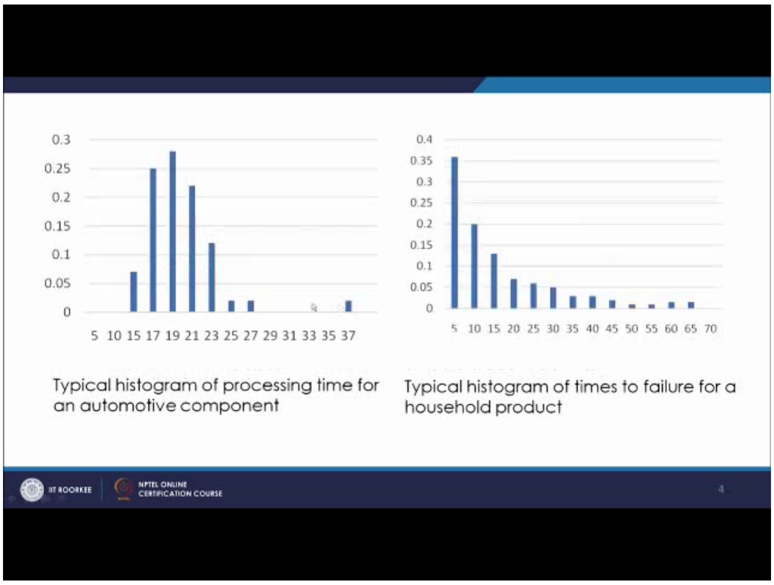

So, we see here that this is the typical histogram for processing time of in automotive component and this is the histogram of time to failure for a household product, it looks like as if it is exponentially distributed if you look at.

But then if you see we exponentially distribute, you will see that this curve goes like this now here it goes like this. And then, but you approximate you will approximate it with exponential whereas, the exponential actual line will come here close to this. So, this will go like this. So, most of time here it is predicting under the value is it is close to more than 0.35 or 0.36 whereas, this graph will come here, then in that case in some cases the land will be up and this values of probability will be down. So, in that case it is not set to be a very good fit. So, in those cases you can go for some other kind of distribution and that is where the gamma distribution is set to be a proper fit.

So, in gamma distribution similarly there is another parameter, which is there like repair time. So, similarly some data has been seen of the past and it was seen that this is normally the time to repair.

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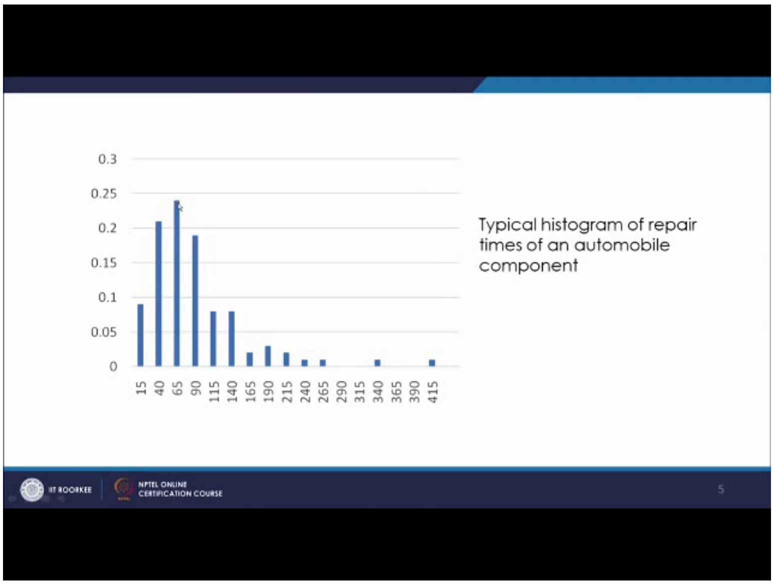

And normally it is in that you have some mean value of repair. And then finally, it goes like this how to see that which type of distribution it fits. So, normally it is seen that this gamma distribution with the different shape parameter is very much fitting to such kind of curves. So, what happens in the case of suppose this earlier one like you have this time to failure.

If you see if you manage it with the basically if you try to draw the potential curve, in exponential curve what happens you have the curves like here what you see is you have the values high and then it is down and further you have this down and all that. So, this is how it is going and so, what we see if you typically draw the exponential curve it goes like that. So, most of the places you will have the values which is here and here also is not able to predict you have a larger values in this case, but if you try to draw the gamma distribution with alpha as 0.7. So, if the alpha is taken as 0.7 in that case, the distribution comes like you have further the gamma distribution will come with. So, this is 0.7 and 1 as the scale parameter. So, in that case curve goes like this which basically fits more with such kind of distribution function.

Similarly, if you go for this typical histogram of repair times, in case of repair times what is seen is that you have. If you take, it does not fit with the exponential you know distribution. So, what we see is here for certain values whenever you have the data you try to fit it. So, many a times it fits with certain kind of gamma distribution with some shape parameter value. So, in that case it was seen that if you draw the gamma distribution with the shape parameter of suppose 1.3 and so, it is 1 point 1.3 and 1 gamma distribution, mean is the product of 1.3 and 1 in that case it was seen that this curve goes like this.

What we see actually this is showing like such kind of distribution. So, what can we said that normally you will have to do this analysis, and then you have to see that how this fits. So, basically the type of distribution what we use that basically tells you that how accurate we are going to model it. You will have to check you must have shown we have studied about a different kinds of distribution functions and we should know that what kind of distribution function is better we can have go forrubblebut then for rubble as we discussed its difficult for mean to be computed. So, we go normally for this gamma distribution in such cases.

Now, in the other way now another thing we need to specify once we have seen that we know these shape parameters, now we need to specify the beta B and D basically for busy time as well as for the down time. So, for that for this gamma functions, we have certain things what we see. Now what we see here is that we have 1 parameter that is efficiency.

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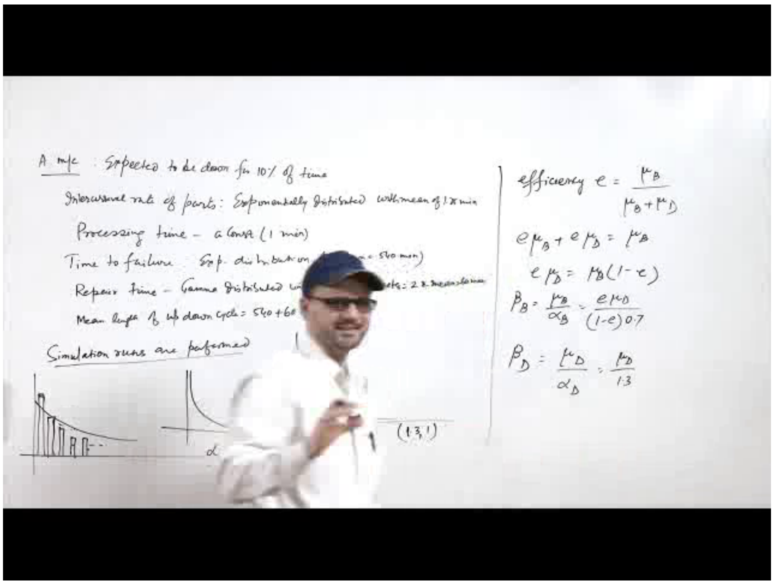

So, we need that efficiency e we define as we can define efficiency as ratio of mu B to mu B plus mu D. So, that is your busy time mean that is for that is the down time mean and similarly you have the busy time mean. So, it is nothing, but the long run proportion of potential processing time during which the machine is actually processing the parts.

So, this is what the efficiency will be defined as now from here what we see is we have seen that we have 2 times 1 is the busy time that is your time to failure and then you have another is time of repair and in one case you have alpha as 0.7 in and in the next case you have alpha as 1.3.

So, this way we have now e is defined as this and from here what we get is we can see that you can get e mu B, plus e mu D. So, that we get it as mu B. So, from here we can get this e mu D as mu B into 1 minus e. So, we can get beta B. So, we wanted to have this beta parameter because alpha B once we know the mean is alpha B into beta B. So, we can get this beta B as mu B by alpha B.

So, once we know mu B and once we know alpha B. So, that will be told as e raise to and then mu D divided by 1 minus e into that is 0.7. So, this is how you calculate this beta B similarly you can have beta downtime also calculation and that will be basically mu D upon alpha D. So, that is alpha D value is 1.3. So, it will be mu D by 1.3. So, these are basically the values of the other scale parameter along for that distribution.

So, this way you can have the main idea about the functions which is to be used the parameter which is to be calculated and once you know the that then you can go for calculating the measures of performance as the simulation progresses.

Thank you very much.