Modeling & Simulation of Discrete Event Systems
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Lecture – 30
Problem Solving on Output Analysis of Single and Alternative Systems

Welcome to the lecture on problem solving on output analysis of single system and also alternative system configurations. So, in the last week, in the last four lectures, we studied about the output analysis of single system, also we studied about the output analysis of alternative system configurations, in that we had to compare one system with another and we also did the output analysis of comparison basically for one standard system with many of the systems. And then we had to achieve ultimately the objective was to achieve certain confidence level I mean we had to make the interval to have overall certain confidence level.

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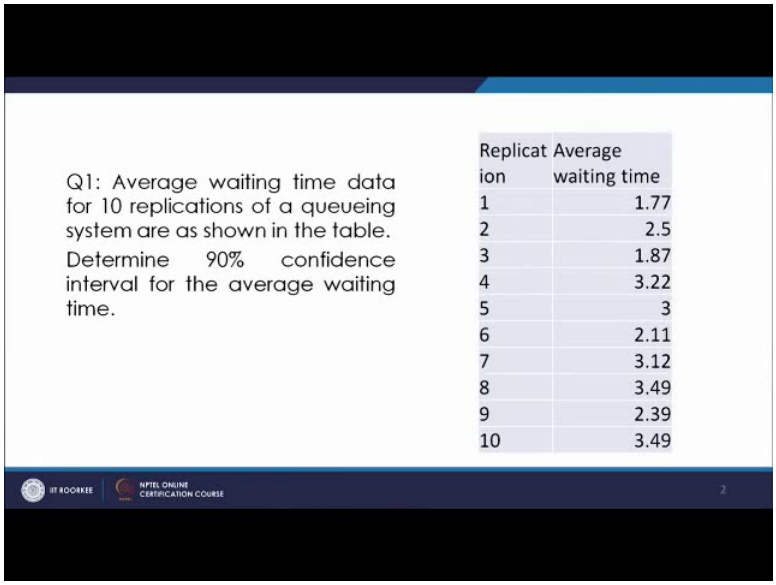

So, we will deal with the questions one by one. So, the first question is about the finding the confidence interval and this is the output analysis for a single system. So, here the problem tells that average waiting time data for 10 replications of a queuing system is shown here. So, you have the 10 replications and this is the average waiting time data. So, what we have to see we have to find is determine 90 percent confidence interval for the average waiting time.

So, as we know that in this case we have to find the mean waiting time first sample mean. So, sample mean will be found by adding this waiting times together and then dividing it by n that is 10 and then so that will be X bar n and then we are getting plus minus the half length. So, half length will be t n minus 1 and 1 minus alpha by 2, so that is basically in this case since it is 90 percent overall confidence level, so alpha is 0.1. So, in that case 1 minus alpha by 2 will be 0.95. So, ultimately we have to search from the table the value of t nu, nu there will be n minus 1. So, 9 and then you have the this alpha 1 minus alpha by 2 that is 0.95 and then we are getting the value for the intervals, so that will be sample mean plus minus this half length.

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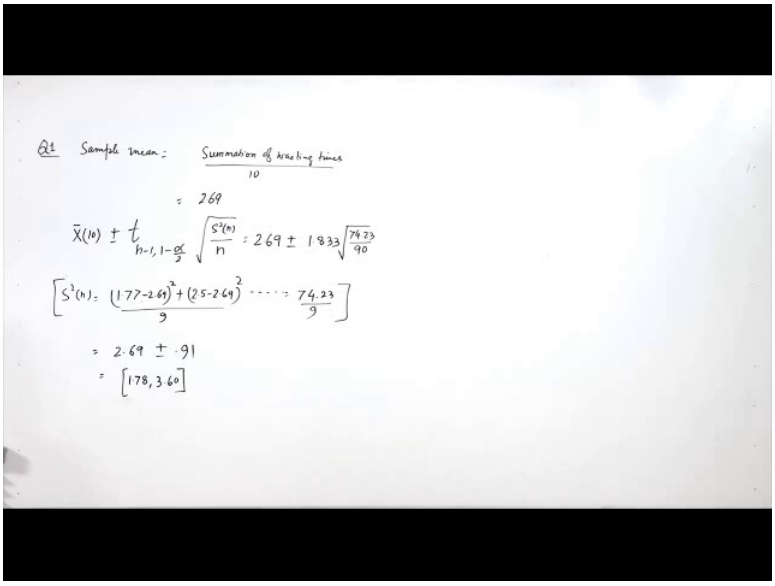

So, if you compute these values, you will get like this. So, this is question number one for the output analysis of single system. Now, in that case, if we find the mean of so sample mean sample mean will be summation of the all these waiting times. So, it will be summation of all waiting time, and then we will divide it by 10. So, once we do that if we add all of them and we are basically dividing it by 10, then we get it like 2.69. So, that you can do the exercise if you add all of them, then in that case you are going to get this number basically we are getting going to get only 2.69 we should have because we have only two significant digits.

So, in that case that then what will be the interval confidence interval, so that we have to find. So, we know that that will be given by X bar 10 plus minus t n minus 1 and 1 minus

alpha by 2. So, after this sample mean, we have to find the sample variance and sample variance will also be computed and sample variance will be computed. For that we are getting, so here it will be s square n by n. So, we are getting this sample variance as again we have to get it like mean is there like 2.69. So, this s square n we will be getting like it will be the first entry is suppose 1.77. So, it will be 1.77 square. So, it is not square it will be 1.77 minus mean. So, it will be minus 2.69 square plus the second value is 2.5 minus 2.69 square. So, like that it will go and that will be basically divided by n. So, in that case you are getting the summation, summation you are getting as 74.23. So, this summation is coming out to be 74.23, and it will be divided by n minus 1. So, it will be divided by 9. So, it will be divided by 9 and this value will come 74.23 by 90. So, what we get is X bar 10, we have got as 2.69 then plus minus now we have to see this value n is 10. So, we have to look at the table.

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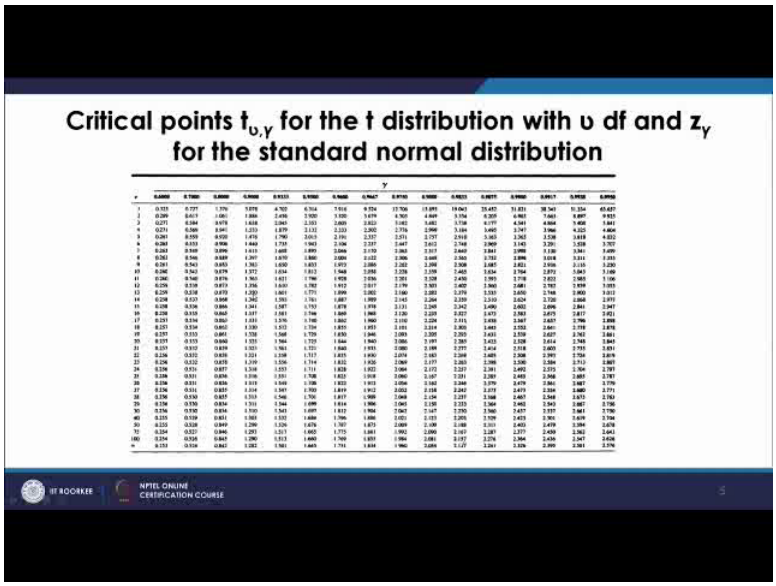

So, this table we will get here. Now, in this table we are going to see for this 9, we are going to get for 0.95. So, this 0.95, if you look at this value comes out to be 1.833, so this will be 1.833. So, this will be 1.833 and then under root and this will be 74.23 divided by 90. So, we got that. Now, if we do this computation, we are going to get, it will be 2.69 plus minus if you compute this it will be something close to 0.91. So, this is the confidence interval. So, we have the interval that is it will be minus first of all. So, once we do the minus, it will be 1.78. And if you do the plus, it will be again 3.60. So, this is the confidence interval you get for getting I mean with 95 percent of confidence with 90

percent of confidence basically you get this confidence interval. So, with 90 percent of confidence, you can say that the value will lie between this and this, so that is what the way of solving such questions are.

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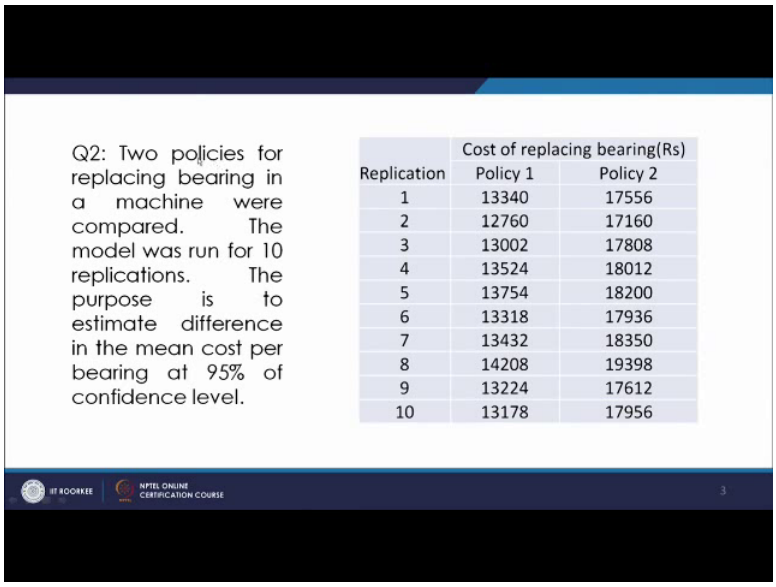

Now, we will solve the second question. The second question is regarding the comparison of one system with other. So, this is the comparison of one system with other one. Now, what we do in that. So, what we do is basically the problem statement is like this that you have two policies for replacing bearings in a machine were compared. We have to be compared this two policies. And the model was run for 10 replications and the purpose is to estimate difference in the mean cost per bearing at 95 percent of confidence level. So, we have earlier seen about such questions. In this what we do is we get the difference of this basically policies, input details and then we are getting the interval for that difference, so that is how it goes. So, we can estimate the difference in the mean and then we can say that difference is going to be something in between this and this. So, we have to have the interval for the difference of the mean I mean values of these two policies. So, in this problem, what we do is we are basically getting the difference of the values here and here and we will make another column for that.

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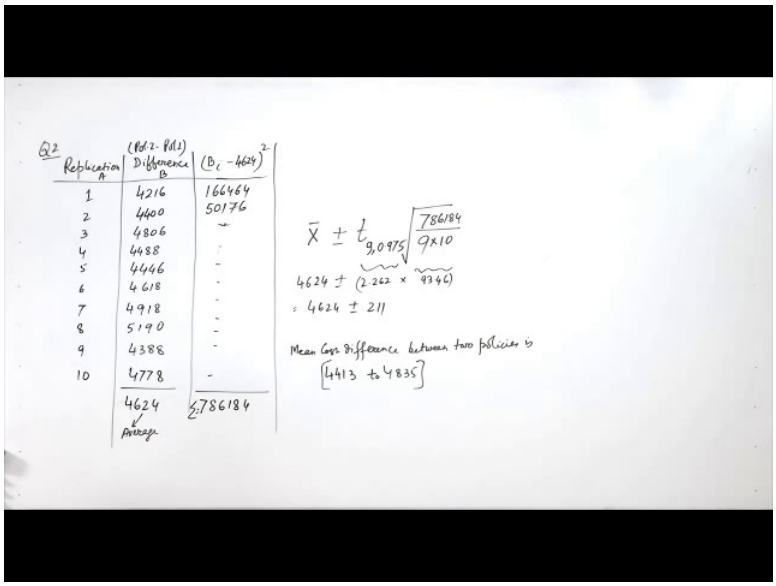

So, for question number two what we discussed is that we have the two policies and for that against the replication, we have to find the difference between these two policies. So, we will have the difference between policy two minus policy one. So, against the replication values in case the replication, so have replication 1 to 10, and then we have the difference. So, we will have the differences. Now, for 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 difference will be from policy two, two minus policy one, we can have even policy 1 minus policy two also. Now, for that again the difference will be if we take the differences, so it will be 17556 minus 13340, so that is coming out to be 4216. So, this way we have to note down all the differences from policy two to policy one. So, it will be 4400, then further you have 4806, further you have 4488, then you have 4446, then 4618, 4918, 5190, 4388 and 4778. So, this is what we get.

So, what we have to do we have to find the confidence interval for this mean difference. So, mean of this is to be obtained and its mean is so we have to sum it and then we have to take its mean. So, sum is coming out to be something like 4624. So, this is the mean basically this is 4624 is the basically the mean value. So, we will write 4624 as the average. So, sample mean that is difference of the two policy values that is coming out to be 4624.

Now, we have to find the component in the second half of this plus minus sign that is in the left part, we have the sample mean; in the right part we have to configured the half

length, so we have to get the half length. So, for half length again we have to find this square values. And if we get the square values, so for this the square values can be obtained suppose this is a, and this is b. So, it will be b I minus and then it will be 4624 and then its square. So, you will have this column again coming. So, if you look at 4624 minus 4216. So, it will be 408. So, 408 will have the square of 416 6464. So, like that you will have the values. So, similarly the next one will be 224 for that it will be 50176. So, like that you will have all the values you have or you can fill all the values and ultimately you will sum them. So, this summation will come out and then that summation is coming out to be something like 7886184. So, this summation will come 786184, this is the summation of the values.

Now, this is to be divided with n into n minus 1 that is 90. So, basically what we do is now you find the confidence interval. So, confidence interval will be X prime plus minus t n minus 1 and 1 minus alpha by 2. Now, here the confidence interval is 95 percent, confidence level is 95 percent. So, again 1 minus alpha is 0.95. So, alpha is 0.05. So, 1 minus alpha 2 alpha by 2 will be 1 minus 0.025, so it will be 97.5 percent. So, it will be n minus 1 first, n minus 1 is 9, and then it will be 0.975. And then S square n by n, S square n will this by 9, so it will be 786184 by 9, and then further divided by n that is 10, so 90. So, this way you get that parameter.

Now, again we have to see from here the value of this t nu alpha. So, we have to see that value. Now, this value is coming out to be if you go to against 9 and go to 0.975, so it will be 2.262 here, it is coming as 2.262. So, it will be 2.262. And this will be 93.46. So, this value is coming as 2.262 and this will be 93.46. So, X bar anyway you know as 4624. So, this 4624 will be plus minus this multiplied by this, so that comes out to be 211, so 4624 plus minus 211.

Now, what we get out of it. So, we can say the with 95 percent confidence that the difference of the mean will be from 462 So, it will lie in that interval. So, mean difference between these two policies will be between. So, basically we can conclude from here that the mean cost difference between the two policies, policies is so 4624 minus 211, so it will be 4413. And then two it will vary from 4413 to 4835. So, it will vary from this range to this range, this value to this value, so that is how you can get the outcome when you compare the two systems.

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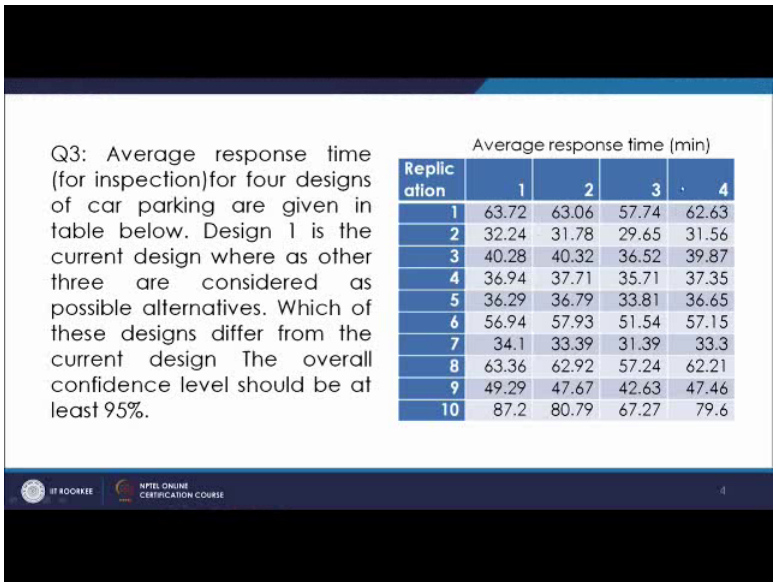

Now, we will deal with the third question. This third question tells that this actually there has been four responses response times. So, this is basically the inspection time which is given for a parking station in the car parking. So, the four values are given, for four you know designs the values are given. Now, for that the first system is basically taken as the standard one. So, it is written that average response time for inspection of four designs of car parking are given in the table. Now design one is the current design, whereas other three are considered as possible alternatives. So, you have one design as the current design - the standard design, and the three designs are to be compared against that. Now, which of three these designs differ from the current design and the overall confidence level should be at least 95 percent.

So, what is there in that problem that you have three alternative systems, and you have to compare all these three against the standard one. And ultimately you have to suggest that whether you should go for someone or not, how they behave like that and overall confidence level is you should get as 95 percent. So, in these what we do is the process is that first of all we will get the difference for them that is what for which we have to prepare the mean and then for that particular quantity we have to find the interval with so that overall you have the confidence level of 95 percent.

Now, how to get this overall confidence level of 95 percent, it means for the individual level the confidence level has to be altered. So, as there are three you know intervals now

we have three classes. So, because you have four groups and one is subtracted, anyway from all the three. So, ultimately you have three classes now for that you are dividing. So, individual confidence level for individual entries has to be it will be again 1 minus 0.05 by 3. Now, in this case, it is 1 minus alpha is 0.95. Now, the same thing for individual case will be 1 minus 0.05 by 3. So, in that case it will be 1 minus 0.0167. Now, when we get that, so alpha will be 0.0167 in that case and then 1 minus alpha by 2 will be further 1 minus 0.0167 by 2.

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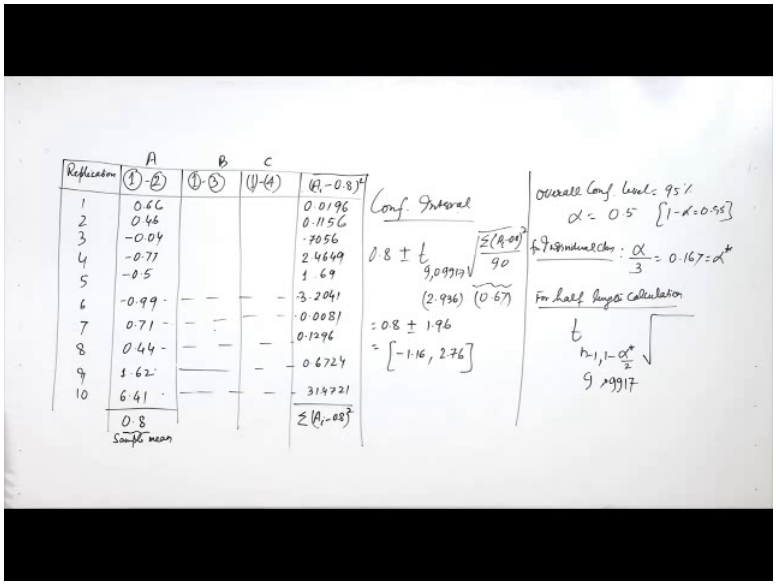

So, what we see here that you have overall confidence level is 95 percent means alpha is 0.5 because 1 minus alpha is 0.95. Now, you have to have the three you know classes. So, in that case, so you will have, so c will be class interval will be c. So, c will be k minus 1 you have k as 4, so it will be 3. Now, in this case, for individual class, so what you have to do is alpha is to be maintained as alpha by 3. So, alpha by 3 will be 0.167. So, this will be basically alpha star suppose and then when we make the half length calculation in that case it will be for half length calculation. So, you have to have this parameters t n minus 1 and 1 minus alpha by 2. So, it will be alpha star by 2 and then we will have other values like s square n upon n or so. So, in that case this will be coming as so this will be 9 and this will be 1 minus again this by 2. So, it will be 1 minus 0.083. So, it will be 99.17 percent. So, 0.9917 because it is this 0.083 will be basically getting subtracted from one. So, we have to see in the table 99.17 percent this value. So, new alpha when we look at we have to see in the table 0.9917.

Now, let us proceed to how to solve. So, first of all we will have the table prepared. So, we have the replication we have 1 to 10, so 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. So, suppose we get 1 minus 2, so one is the standard. So, what we get is 1 minus 2 then we may get 1 minus 3 and then we will get 1 minus 4. So, suppose we do it for the three values. So, if you look at this 63.72 minus 63.06, so it will be 0.66 and. So, it will be 0.66

Similarly, the second number, so second number will be again 32.24 minus 31.78. So, it will be 0.46. So, all other values can be computed and that will come out as minus 0.04, minus 0.77, minus 0.5, minus 0.99, 0.71, 0.44, 1.62, and 6.41. Now, this is the difference between the responses of the two systems first and second. Similarly, we have to get between first and third then we have to get between first and fourth. So, suppose we get first and third difference, it will be 63.72 minus 57.74. So, it will be something like 5.98. So, it will be 5.98. Similarly, 32.24 minus 29.65, so it will be two point again 2.59, so this way it will go on for that, so that can be computed here. And, similarly 1 minus 4 will be computed if the time permits we will do for this also. But let us see how to find the interval for this. So, we will have the average of that and average is coming out to be 0.8, this is the sample average sample mean.

So, what we saw that now from this value we have to find the sample variance. So, sample variance will be again this minus mean sample mean A square plus this minus sample mean A square. So, this way we will have the sample variance and that summation all divided by n minus 1. So, that we will get s square n and that will be divided by again by 10, so that that will be further divided by 10. So, if you do that calculation, so if you say we get the 0.66. So, if we take this as A this as B and this as C, so for calculation what we get is we get A minus, so I suppose A I minus 0.8 and this will be square and we have the summation. So, we will sum it any way we will sum it later in the end. So, A I minus 0.8 square.

Now, see 0.66 minus 0.8 square, so it will be minus of 0.14 square, so it will be 0.0196. Similarly, 0.46 minus 0.8 square, so it will be 0.34 minus of 0.34 square, it will be .1156. So, minus of 0.04 minus, so minus 0.84 square, so it will be 0.7056. Again minus 0.77 minus 0.8, so it will be minus of one-fifth minus of 1.57 square, so it will be basically it will be 2.4649, so yeah it will 2.4649. Similarly minus 0.5 minus 0.85, so it will be minus 1.3 square, so minus 1.69. So, it will 1.69 not minus into all we plus. So, minus again minus of 0.99 minus 8, so it will be minus 1.79 and that square will be again

3.2041. So, it will be 3.204 sorry it was minus of 0.99. So, it will be again 3.2041, it is right. So, this way we can find all the values 0.0081, so that is for the seventh value. This is the sixth value.

Then further you have 0.1296, then you have 0.6724, and last is you have 31.4721. Now, we have to sum them and after summing them we have to get the basically divided by 9 into 10 – 90, so that we will do. And in that case and then its square. So, its square once we do that that we get as 0.67. So, if you sum them, so we have to sum them, sum of this A I minus 0.8 square and then what we do is we get. So, confidence interval once we try to find, it will be again the mean value sample mean 0.8 plus minus. Now, at we saw it will be t 9 and 0.9917 and then under root this is the summation we will get here and summation of a I minus 0.8 square and it will be divided by 9 into 10 - 90. So, it will be divided by 9 into 10.

So, this value we are getting if you compute you will get it as 0.67. And if you look at this table t 9 and 0.9917. So, again you will go to the table for 9, we have to go to 0.9917 here. So, if you do that this is this is the value 2.936, we are getting here against the nine degree of freedom value and then you have this 0.9917 in the top row. So, for that it is coming as 2.936. So, it will be 2.936. So, what we get is 0.8 plus minus, you get it as 1.96. So, you have if you take it will be from it will be minus. So, minus of 1.16 and then once you added it will be 2.76. So, basically what we see that this interval contains zero. So, you can say that they can be they are comparable they are mu’s are comparable. So, you can go for such systems you can say that, so what we are asked that which of this differ from the current design this can be said that does not differ from the current design because this contains zero in this interval. So, this again can be done for these two values. So, for these two, if you do you can get the values, and you can further check them and you can see whether they differ or not. So, this way you go and get the problem solved.

Thank you very much.