Modeling & Simulation of Discrete Event Systems

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Lecture – 29
Confidence Intervals for Comparing More than Two Systems

Welcome to the lecture on confidence intervals for comparing more than two systems. So, in the last lecture we discussed that how to compare the 2 alternative system configurations and we discussed the 2 approaches and in one of the approach the assumption was that the number of replications were same whereas, in the other one the approach was that even the number of replications are not same how can we compare and we can say that they are whether same or different. So, they can be said to be different if there the intervals do not contain 0, I mean if the parameter which is for which we compute the interval that is difference of their performance measures if finally, the interval which we suggest if that does not contain 0 we can say that they differ and if they contain 0, we may say that they do not differ.

So, in that case, there may be cases when you have more than 2 systems. So, there are many systems and you have to compare and most likely the cases arise when one of the system can be considered to be the standard one and others are the simulation readings. So, in that case you have to say that which of the readings which of the replication or which of the values I mean run values they are they can be compare to the standard ones and which cannot be. So, in that case; so, let us see if you have more than 2 systems.

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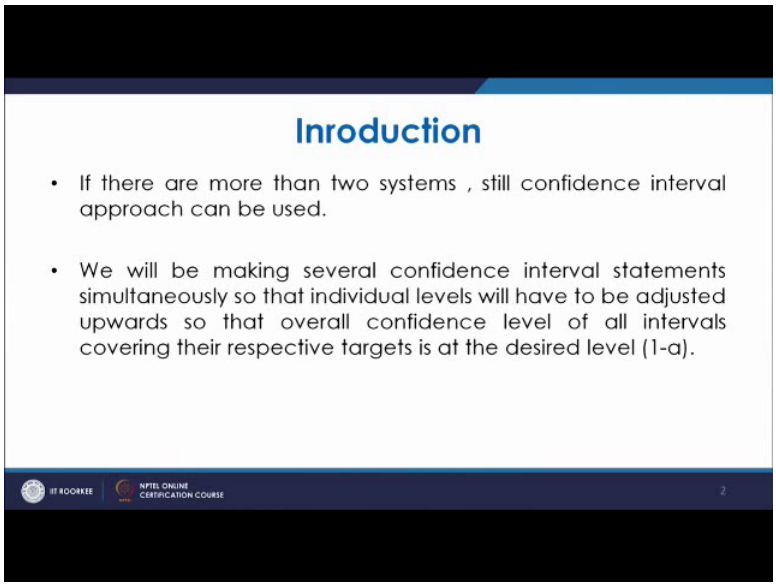

Even that confidence interval approach can be used, but the question is that you know over all we are saying that we have to have the confidence level of certain percentage.

So, suppose we tell that you with 90 percent confidence you have to say that you you have to get the interval which should contain certain quantity. Now in that we get the 1 minus alpha by 2 level ultimately. Now in these cases; what happens now? So, when you have 1 minus alpha is that percentage in that case we find t n minus 1 and 1 minus alpha by 2. Now in this, case what happens that since you have many intervals to be made suppose you are you are having k systems. So, if you have k systems and out of the k system the first one is a standard and then you have to compare every system with the standard one. So, second we used to be compared against first.

So, mu 2 minus mu 1; similarly third is to be compare to first; so, mu 3 minus mu 1 mu 4 minus mu 1. So, ultimately mu k minus mu 1; so, ultimately you have to have the k minus 1 number of intervals now in that case the you see that overall you have to have that target level of 1 minus alpha. So, desired level is 1 minus alpha, but when you have to make k minus 1 number of intervals in that case the individual level in the in in case of individual interval making that confidence level which we have to maintain will be 1 minus alpha by k minus 1.

So, that is basically suggested by the Bonferroni inequality.

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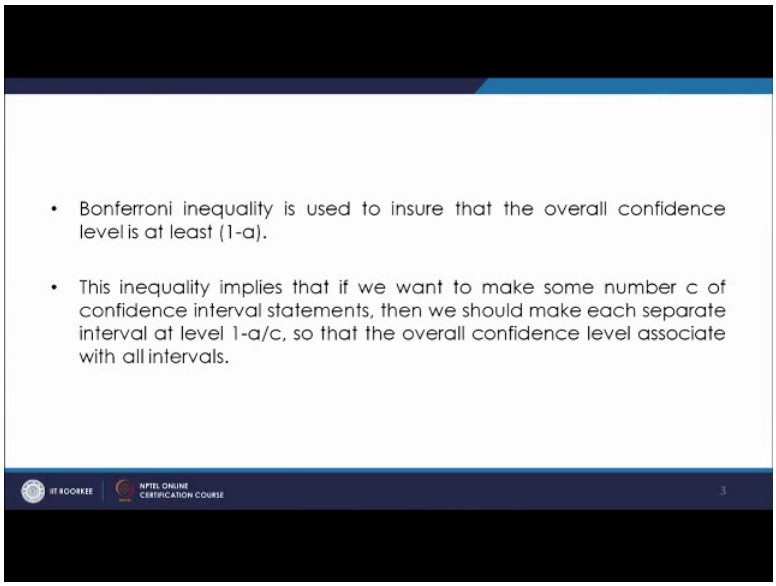

And it tells it is used to ensure that the overall confidence level is at least 1 minus alpha. So, for that; so, this inequity implies that if we want to make some numbers c of confidence interval statements then we would make each separate interval at level 1 minus alpha by c. So, if we suppose want to have 90 percent of confidence and if we have we are making suppose 2 more. So, or we have 4 more; so, in that case, 1 minus alpha by 4. So, alpha basically is 0.1. So, 0.14. So, that is 0.025.

So, in that case every level every interval has to be constructed at confidence level of hundred minus 2.5 that is 97.5 percent confidence level should be there for every individual level, then overall you will have the confidence level of 90 percent. So, what we see that in that case for every confidence for every level we are making it wider because once we go for the larger confidence level we are making that wider. So, that is the case, but that is how it is made and then in that case it is seen that which of those comparisons. So, that contains 0. So, that way we can say that it is not differing from the standard and if that do not contain 0 then we say that we it differs from the standards. So, this is how the comparison is made.

Now, let us see that if you the 5 independent replications for inventory policy and we are getting these values X 1 j, X 2 j, X 3 j, X 4 j and X 5 j.

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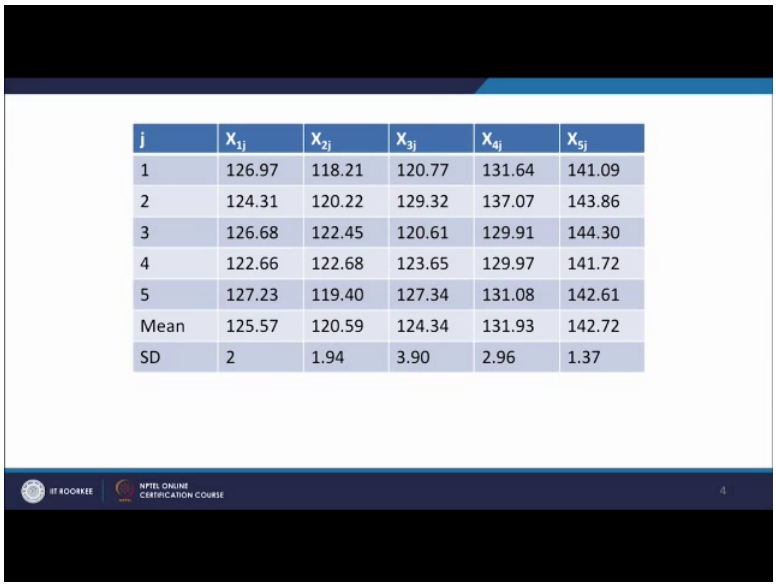

So, for these 5 readings, we had seen in the last problem that we have these; these 2 readings this was the first and this was the second. So, we checked with the 2 methods parity approach and also modified t confidence approach. So, in that basically the first one was the standard and the second one we did. So, we did X 1 minus X 2 and in that case we had found. So, X 2 minus X 1 or X 1 minus X 2 we will do. So, we do that. So, we do X 2 minus X 1 and then we found the confidence level or confidence level at 90 percent and then we found the interval.

Now, in this case, we have the 5 independent replications and if we assume that this is basically the standard one. So, we have basically k readings and if we take the; if we try to compare these you know readings these replications output. So, in that case you have basically 4 values. So, c will be we have to have the 4 intervals you have to make 4 separate intervals. So, it; one will be like for mu 2 minus mu 1. So, for this minus this; another will be for mu 3 minus mu 1. So, next will be mu 4 minus mu 1 and then this will be mu 5 minus mu 1. So, like that you will have the values.

Now, what will happen suppose we are getting the values? So, what we will get here here we will get suppose X 2 minus X 1 X 2 j minus X 1 j. So, it will be 1; 18.21 minus 126.97. So, we will see that this value will come out to be something like 8.76; something like that hopefully that. So, it will be suppose minus of 8.76. Similarly this

will be 120.22 minus 124.31. So, if we take that. So, once we have that table. So, we can have this table.

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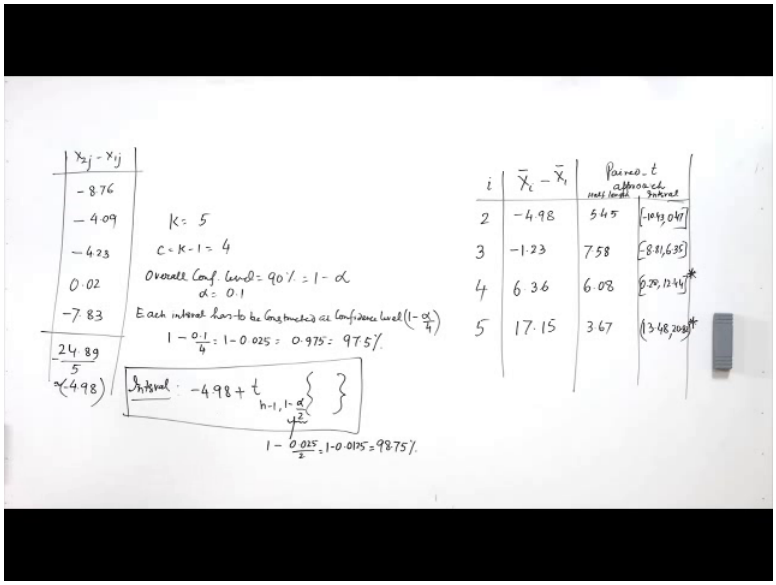

So, what we get is we get in the first one; we get X 2 j minus X 1 j. now X 2 j minus X 1 j. So, that is basically the parameter for which we have to make the confidence interval.

So, once we are doing that it will be 8.76 and that will be minus. So, it will minus of 8.76, similarly you have the value that is 120.22 minus 124.31. So, it will be minus of 4.09. So, it will be minus of 4.09, then further you have the 122.45 minus 126.68. So, that will be 4.23 and that is minus. So, it will be minus of 4.23, then we have 122.68 minus 122.66. So, it will be 0.02 and similarly you will have 119.40 minus 127.23. So, it will be something like 7 point. So, 127.23 minus 119.40; so, 7.83; so, it will be 7.83 minus. So, this is the value which is computed as the. So, once we have to compare we have to compare by taking the difference between their values.

So, once we add. So, it will be 412.85, then you have 16 or 17.08 and then it is 24.91 and. So, 24.89; so, the mean will be 24.89 by 5. So, it will be something like 4.98 and this is minus. So, this value we get as minus of 4.98; now how to get the confidence interval. So, we have 5 values and for that we have to find suppose we take the; so, if we take the parity approach. So, this is known as paired comparison method. So, we are basically comparing with the standard. So, this is a case of comparison with standard now in that case if we use. So, what we get is this; this is the X bar value and then plus

minus. So, the interval how you will make interval will be minus of 4.98 and then plus t n minus 1 1 minus alpha by 2 and then you will have s square n by n. So, now, for these values we can have the calculation of standard deviation or variance. So, once we compute that. So, once we have those values then we will have use here.

Now, the most important thing is that how to get these values. So, once we have these values from where we can find the s square n and then what we have to do is how to take this. So, any way this will be four, but then what will be 1 minus alpha by 2. So, as we have seen that you have k as 5. So, you have k as 5. So, you have number of intervals will be 4. So, c will be k minus 1 that is 4. Now what will happen this overall confidence level is suppose 90 percent. So, that is 1 minus alpha. So, in that case your alpha is 0.1 because 1 minus alpha is point nine. So, alpha is 0.1.

Now, in this case this alpha because we are making c as 4. So, this alpha will be nothing, but here it will be this by 4. So, ultimately each interval each interval has to be constructed at confidence level 1 minus alpha by 4 because we have 4 intervals we are comparing because we have 5 ultimately systems that is 5 and we are comparing against one standard. So, we have 4 intervals and the bonferroni inequality tells that for maintaining overall a level of confidence when you have that many intervals making every interval must be maintained at a higher level of confidence.

Now, that higher level of confidence is obtained by dividing this alpha by the number of intervals we are making. So, in this case it will be 1 minus this is 0 point one by four. So, it will become 1 minus 0.025 and that becomes 0.975. So, it becomes 97.5 percent confidence level this is for individual level. Now once we use this formula where the variance term will come here in that now what should be this alpha. So, what should be this 1 minus alpha by 2 values now the thing is that overall we require 97.5 percent every individual interval has to be 97.5 percent in that case now the alpha becomes here the value of 0.025.

So, what happens 1 minus alpha by 2 will be basically 1 minus 0.025 upon 2. So, it will be you know 1 minus 0.0125. So, it will be 98; 75 percent. So, basically what we see that in in the earlier case when we had to compare with only one system in that case when we have to find we have to have the confidence level of 90 percent then we see here this term comes out to be 0.95 whereas, if we are comparing with I mean if there is

comparison and we are making 4 individual intervals. So, that overall confidence level is 90 percent then this comes out to be 97.5 percent confidence level for each interval and that is why here it will be 98.75.

So, you will see from the table t of 4 and 0.9875 and this is how how you will calculate the values of that. So, if you compute the values what you get you can solve this and you can get certain values like if you have I and then you have X I prime minus X 1 prime and once you do with parity approach you can do with any approach like Welch approach also. So, we can do with parity and we can also test with the Welch approach also. So, in that case what we see is for the second system we have seen that this mean is coming out to be minus of 4.98.

Now, in this case what is the half length? So, this quantity which we have seen here what we compute this becomes the half length that can be computed and half length is computed to be something 5.45. So, once we have the half length we can compute the interval. So, this is basically for the second system means the performance measure is the value of second system second replication minus the replication that is first one was the standard one.

Now, this is coming out this this was the mean which we got here similarly in the table for the third one. So, once we have got this half length means what will be the interval first will be the lower line limit will be minus 4.98 minus 5.45. So, it will be minus ten point and it will be 4 3 and another will be minus 4.98 plus 5.45. So, it will be 0.47. So, this is how the confidence interval is found out what we see here that in this confidence interval the interval which we get we are getting 0 into it; it starts with minus of ten point 4 3 and it goes to 0.47. So, we have 0 in it.

So, it means that it does not differ from the standard we can save we can infer that the second system does not differ from the standard suppose you go to the third system in the third system if you do you will get minus of 1.23 how you get it you have to get this value compute you have to compute this 120 minus 126.97. So, that will be coming something close to 6.20 and that will be minus 6.20 plus 4. So, this will be this minus this. So, plus 5.01 plus again, it will be minus of 120. So, it will be 6.07. So, this way you are going to have the X 3 j minus X 1 j. So, you will get the 5 values you are going to get the mean of it and that value comes out to be minus of 1.23.

Now, for all these 5 values you are going to get the variance and once you get the variance sample variance you are going to further use in this formula again you are going to have the same thing. So, if this value will be same t of 4 and 0.9875. So, you will have the parameters here from.

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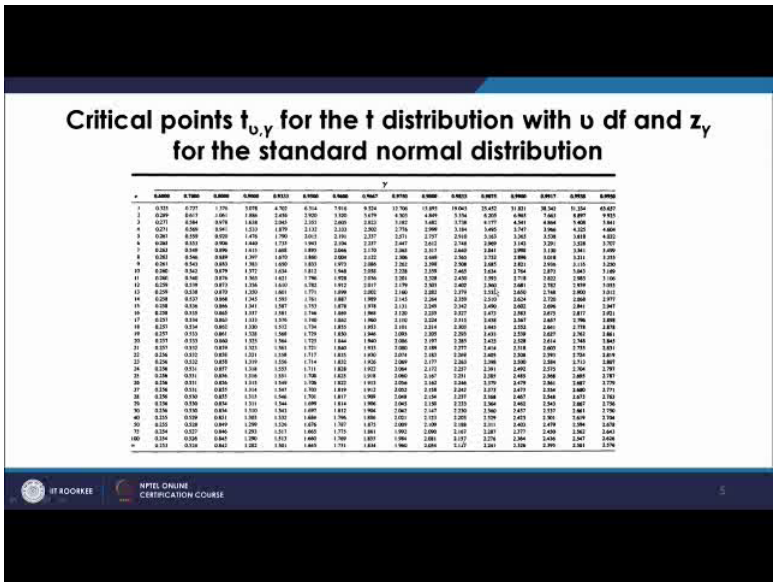

You can use any value like t 4 and then once you go above you will have all these values. So, you can use from here these parameter values. So, that comes out to be here you get the half length as 7.58. So, what we see minus 1.23 minus 7.58. So, it will be minus of 8 point; it will be 81 and then if you do plus it will be minus 1.23 plus 7.58. So, it will be 6.35.

So, for again for the fourth system; if it is found to be 6.36, we get 6.08. So, you will have the confidence interval 0.28 and this will be twelve point 4 four and for the fifth system you will have the mean value as. So, difference mean of the difference is query 17.15 and this is coming out to be 3.67. So, you have half length. So, your interval comes 13.48 and you have 20.82. So, what we see that in this case, we get these confidence interval values. Now in this case, we you see that for these 2 systems here these asterisk mark these readings they do not contain 0. So, we can say that they differ from the standard.

So, this is how you compare the more than 2 systems. Now this was the case when we had to compare we had one standard replication and we compared it with that now there

may be a situation when we have to compare one from everyone and every comparison has to be taken into account here we have 2; 2 I mean you have replication 2 minus 1. So, you have mu 2, mu 1, mu 3 minus mu 1, mu 4 minus mu 1 and mu 5 minus mu 1, but then we can have case where we have all pair wise comparison. So, there we have the comparison with the standard, but we can have all pair wise comparison.

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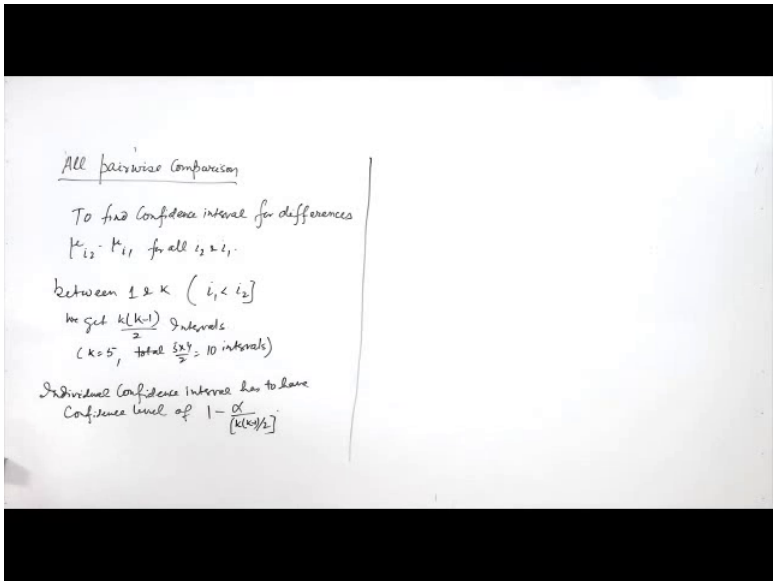

So, in that case we compare each one with other one and then we can try to have the confidence interval and there also we can see after making the interval whether it contains 0 or not and we can identify that which of the one how they are you know they are having the similarity with standards or. So, so what we have into say in that that you have to find the; so, to find confidence interval for differences mu i 2 minus mu i 1 for all i 2 and i 1. So, in that case you had one standard every time you had to simply do the difference from second to one first third to first fourth to first and fifth to first, but in this case we are going to have the comparison of every one with other, but then we are going to see that this this one is on the higher side. So, we are doing to have the comparison one is mu 2 minus mu one then we are doing to have 3 minus 1 and 3 minus 2.

Similarly, you have going to have 4 minus 1, 4 minus 2 and 4 minus 3 and then 5 minus 1, 5 minus 2, 5 minus 3 and 5 minus 4. So, this way you have some number of the intervals that is form. So, what we see is between 1 and k. So, when if we take i 1 as less than i 2 in that case what we get is we get k into k minus 1 by 2 intervals. So, we get k

into k minus 1 by 2 intervals. So, if you look at this in that case if you start from the second data you have one in the third case you will have 2 in the fourth I mean mu 4 minus mu 1 mu 2 mu 3 or. So, so in that case 3 and then you have 4. So, 1 plus 2 plus 3 plus 4 1. So, it will be 10. So, it will be 5 into 4 by 2. So, that is why when k is k is 5 there will be total 5 into 4 by 2 that is ten intervals.

Now, in that case again since we are making ten intervals and overall you must have a confidence level of 90 percent. So, again we will use the bonferroni equal inequality and in that case the individual interval must be maintained at a confidence level of 1 minus alpha by k into k minus 1 by 2. So, individual confidence interval has to have confidence level of 1 minus alpha by k into k minus 1 by 2. So, this will be the individual confidence level of individual interval.

Now, if you see the earlier case in the earlier case, if you look at from here what we saw that when we had the comparison from the standard you had to make 4 intervals and when you had 90 percent of overall confidence level in that the individual confidence interval was to be made using the individual confidence level of 97.5 percent. So, you made t of n minus 1 and 1 minus alpha by 2. So, this 1 minus alpha by 2 came out to be 0.9875 because your alpha is basically 0.025. So, 1 minus alpha is 97.5 percent. Now in this case it will be 1 minus alpha by 10. So, this value becomes 10. So, in that case your individual confidence individual intervals confidence level has to be at 99 percent.

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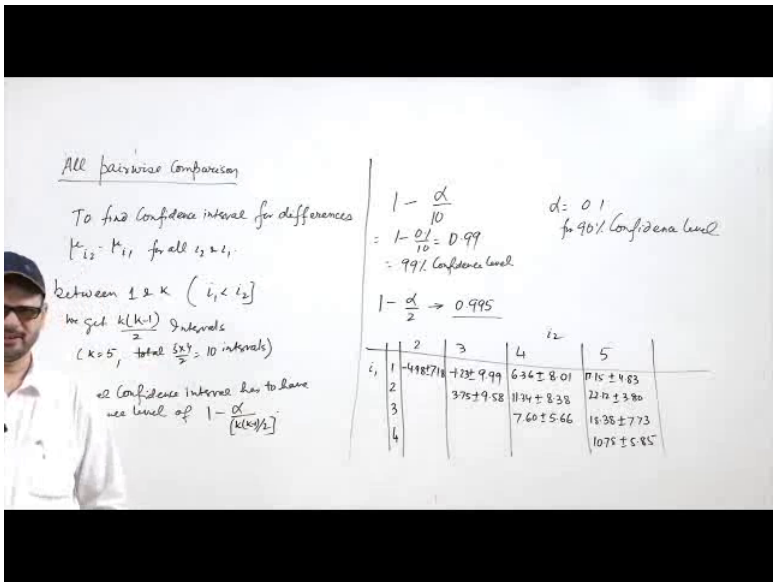

So, what we see is that 1 minus alpha by 10 and alpha was basically 0.1 for 90 percent confidence level.

So, what we get is here we get 1 minus 0.1 by 10. So, that will be 0.99. So, it will be 1 minus 0.01 that is 0.99. So, you have to make 99 percent confidence level at 99 percent confidence level you have to make the confidence interval for individual you know when we are whenever you are making individual for individual class you are making this at 99 percent confidence level. So, 1 minus alpha by 2 will be equal to 0.995.

So, basically you are going to look into parameter that is 0.995. So, against the degree of freedom part you are going towards this value of 0.995 and you are coming here in this column you have to take that particular value for the t new alpha. So, if we take its readings as we see you will have i 1 and then if you do the all pair wise comparison in that case, i 2 will be either the second system third system fourth system or fifth one. So, in that case you will have I one coming as. So, 1, 2, 3, 4; so, for the second one when we take the mu 2 minus mu 1 that value; so, difference will be basically its mean; mean of the differences of the second replication minus first replication. So, that comes out to be minus 4.98 and then it will be coming plus minus 7.18. So, you can check it that in this time because the confidence every time you see that this half length is changing this half length is changing because of the value of alpha changing because of the number of intervals every time it is changing.

So, when we go for third because we are getting 2 minus 1. Now for the third one we will have 2 values third minus 1 and third minus 2. So, it will be minus of 1.23 and plus minus 9.99 and then further; we will get third minus second it will be getting 3.75 plus minus 9.58. So, we can see that in this we are getting 0. So, it will be minus and plus. So, it will be containing 0 now in this case it will have minus and then it will also be having 0 into it here again you will have 0. Now we got to 4 minus 1 and 4 minus 2. So, 4 minus 1 will be 6.36 plus minus 8.01 and it will be 11.34 plus minus 8.38 and similarly it will be 7.60 plus minus 5.66.

So, it is left upon you to as an exercise that how you find it for fifth one, it will be 17.15 plus minus for 0.83, then 22.12 plus minus 3.80; 18.38 plus minus 7.73 and 10.78 plus minus 5.85. This is how you get this confidence intervals at basically 99 percent of confidence level when because you had the ten intervals you had to make. So, with that t

nu and 1 minus alpha by 2 you had to compare this and out of that you can see which wherever you get the interval having 0, there can be said to be you know similar to the standard one and otherwise if we does not contain 0 then they appear to differ from the standard one. So, this is how you compare the all pair wise comparison you can practice more and more questions on that and you will be having the confidence how to find these confidence intervals you can refer the book of law and Kelton for such analysis as well as the book of banks.

Thank you very much.