Modeling & Simulation of Discrete Event Systems
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Lecture – 28
Comparison of Alternative System Configurations

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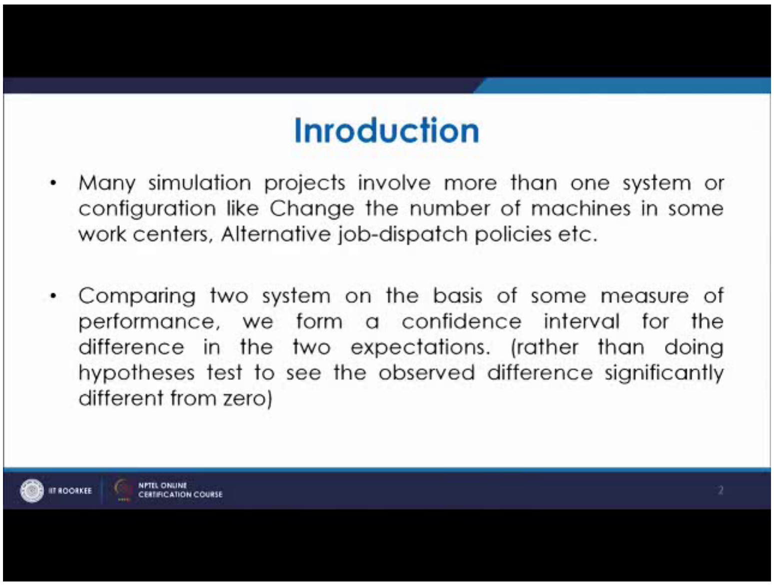

Welcome to the lecture on comparison of alternative system configurations. So, in this lecture, we will try to compare two alternative systems and we will try to see and we will try to find the confidence interval of the difference between the system performances of the two systems. So, many times as we know that many simulations involve more than one system or configuration like change the number of machines in some work centers, alternative job dispatch policies and all that. So, we have to compare this two systems on the basis of certain measure of performance.

So, what we do is we form a confidence interval for the difference in the two expectations. Rather than we find the output performance measures individually for the two systems and getting the difference what we do is we normally take the difference of their performance measures and try to find the confidence interval for that difference itself and then we see whether it contains zero or not. And based on that we go for choosing the hypothesis whether we can say that the two systems are comparable or not. So, in that the first method is known as parity approach.

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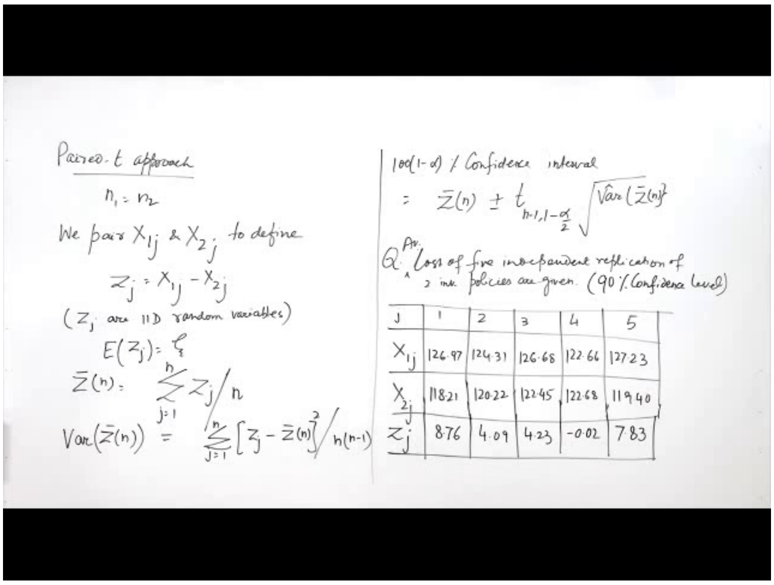

So, the parity approach as the name indicates the parity approach is for those cases when you have same number of observations. So, you have two approaches basically parity approach and modified parity approach. The two systems differ in the sense that in the first case the number of replication for the two are same, so n 1 is equal to n 2. So, in those cases you use the parity approach because they are paired the number of observations are same. So, you get the difference of the output performance measures and based on that again similar on the line we have done in the past, so we make the confidence interval.

In the other method, it is the modified parity approach that is also known as Welch approach. So, in that not necessarily you know every time we have n 1 equal to n 2. So, may be that we have n 1 and n 2 different, like many a times we validate. So, we validate against certain results, which have only small number of readings. And we do the experiment or we do the simulation which have large number of readings, how to compare them how to compare these two alternative system configurations. So, for that we have the modified parity approach where n 1 is not necessarily equal to n 2. So, the two methods we will discuss and we will see that how they are used to compare the two alternative systems.

So, first of all we have parity approach. Now, in the parity approach, what we see is we have n 1 equal to n 2. So, what we can do is that even if somewhat it is so may be that we have few more or less few may be different, so we have to discard. So, we have to make n 1 equal to n 2, so that we can use these parity approach. Now what we do is we pair X 1 j and X 2 j, so to define Z j equal to X 1 j minus X 2 j. So, basically these are the sample of the observation x is. So, this is for the first system this is for the second system this is a jth observation. So, this way you define the difference between the first systems observation and the second systems observation and then for that we try to find the confidence interval.

So, what we do is, so in that case what happens this Z js this is IID random variables even if the X i js are not these Z js are normally IID random variables. So, what we do is now E Z j we define as one parameter. Now, what we do we find Z bar n. So, this we get as summation of Z j, and j will be varying from 1 to n and then divided by n. So, this way as we know we have to find the sample mean. So, we are finding the sample mean for this quantity Z j.

Then after that we will also find variants of Z n. So, again variance of Z n will be found by standard formula that will be summation j equal to 1 to n and then it will be Z j minus Z bar n and that will be squared and then all divided by n into n minus 1. So, this way you are getting the variance Z j. Then we are getting the 100 1 minus alpha percent confidence interval. So, we are making, so we have to find the confidence interval for this quantity. So, 100 into 1 minus alpha percent confidence interval so again that will be same way used. So, it will be equal to Z prime n and then we have same thing t n minus 1 1 minus alpha by 2; and further you have variance Z bar n. So, this way we are using the formula and we are getting the values.

Now, what we need to do is in that case because we are taking the difference of two system performance measures; and ideally, we feel that they should be same. So, it means when they are same their difference will be zero. So, ideally when we try to get the confidence interval, we expect that this interval must contain zero; it means they will be treated as the same kind of system configurations otherwise they will be different. So, in that case mu 1 will not be taken as mu 2 or so, so this way we find the we compare the two systems.

Let us see how we can solve any problem based on such you know problems. So, a problem is there where you have the cost of five independent replication. So, this is average cost of five independent replication of two inventory policies are given. So, we have to basically compare the two inventory policies. How the policies look like? Now, you have one is the number of replication, so that is replication number you have 1, 2, 3, 4, 5, 1, 2, 3, 4, and 5. Now, for the first policy the average cost which is coming is 126.97 units similarly the second is 124.31 units third is 124.68 units 122.66 units and 127.23 units. So, these are the values given for the first policy. Now, for the second one which will be X 2 j and this value is given as 118.21, then you have 120.22, further you have 122.45, we have again 122.68 and then in the end you have 119.40. So, these are the two inventory policies for which you have been given X 1 j and X 2 j.

Now you have to compare these two and as we see that n is same, you have five independent replication, and you have the results for these five replication. And since n is five we can use this parity approach to steady the systems alternative systems. So, as discussed we will define this Z j. So, Z j will be X 1 j minus X 2 j. So, once we get the difference of these two, we will have the values here. So, it will be 8.76, further you are here it will be 4.09 then this will be 1. So, this is 126.97 minus 118.21, so that is why it is coming as 8.76, this is the difference between these two performance measures for the two policies.

So, and then similarly you will have the difference of this minus this that is 4.09; further this minus this, so it will be 4.23. Then you have again 122.66 minus 122.68, so it will be minus of 0.02; and then in the end you have 127.23 and 119.40, so it will be 7.83. So, we got this Z j as these five values.

Now we have to find the confidence interval for this Z j now we have. So, we are going to use this equation again. So, first of all we will find the Z bar n. So, we are going to get the summation of this and to divide it by 5, so that will be Z bar 5. Then we are going to see the value of t n minus 1 1 minus alpha by t 2. So, that will be t of 4 and this is I think we have hundred. So, this we are going to choose at 90 percent confidence interval. So, here we are taking 90 percent confidence level. So, we are having 1 minus alpha as 0.9. So, 1 minus alpha by 2 will be 0.95. So, we are going to see how to get it.

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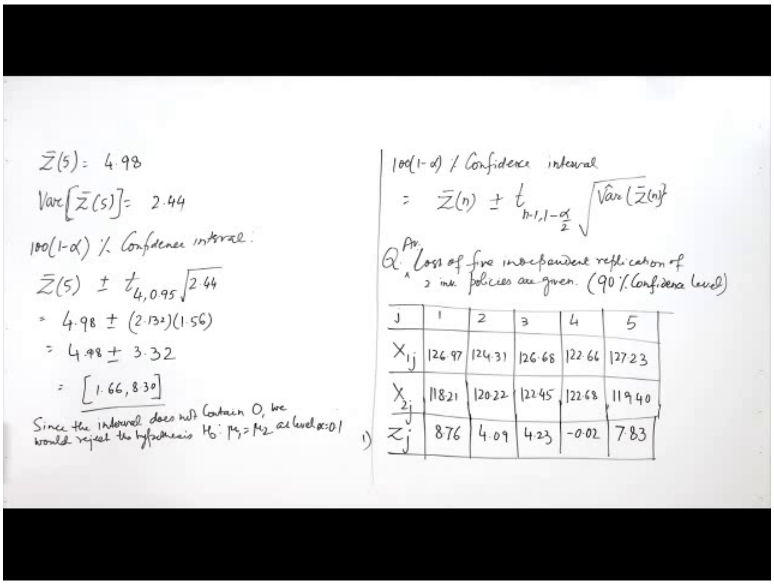

So, we have to get first of all the Z prime 5, so that will be basically all the sums of these and then divided by 5, so that comes out to be 4.98 that is what we need here. Apart from that we need the variance Z prime n. So, again variance Z prime 5 will be equal to, so we have to again further use it like. So, 8.76 minus 4.98 whole square, so it will be something like 3.78 square then plus again four point. So, 4.09 minus 4.98, so it will be minus of 0.89, so that will be square. So, plus again 4.23 minus 4.98, so it will be again four point minus of point seven five that squared. Again minus 0.02 minus of 4.98, so it will be minus 5, so that square. And again plus 7.83 and minus 4.98, so it will be something like 2.85, so that squared then all divided by n into n minus 1, so that will be 5 into 4, so that will be your variance Z prime n. So, once we calculate that that comes out to be 2.44, so that computation you can do and you can get these results.

So, then you have to find the confidence interval. So, for that what we get is so 100 into 1 minus alpha percent confidence interval, it will be coming as Z bar 5 plus minus t and you have to have this parameter it will be t n is 5. So, we are getting 4 and 1 minus alpha by 2 will be again since it is 90 percent confidence level, so you have alpha as 0.1. So, alpha by 2 is 0.05. So, 1 minus alpha by 2 will be 0.95 and then we have under root 2.44.

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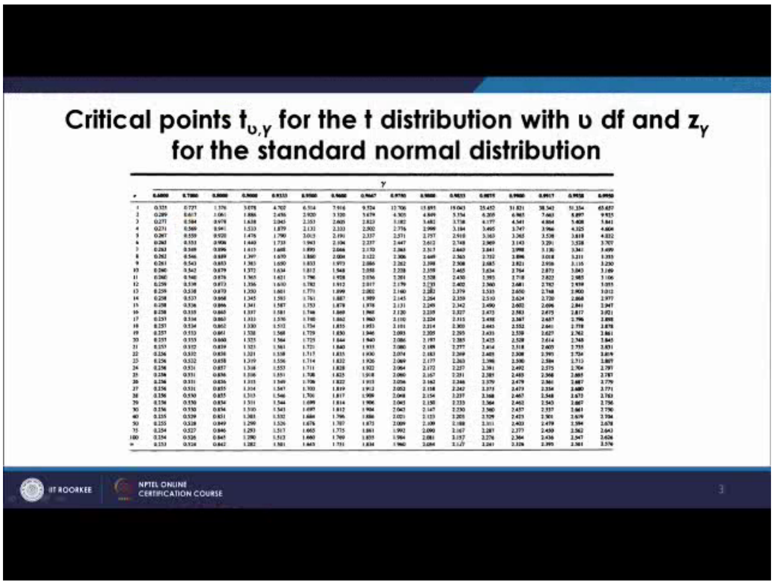

Now, this parameter is having the value that we have to see from the table. So, if we go to the table we see that this is degree of freedom is 4; and from here you go to 95 percent. So, it is coming as 2.132 which it is coming like this one, so 2. So, here it is coming as 95 percent this is 2.132 against this 4 this is the degree of freedom n minus 1, so you have coming at 2.132. So, you are multiplying this 2.132 and Z prime 5 is already given as 4.98 plus minus 2.132 into this will be something like 1.56.

So, you are getting 4.98 and plus minus 3.32. So, this is how you are getting this value. So, your if you look at this it will be 1.66 and then other sides, so it will be something like in the range of 1.66, 4.98 minus 3.32, so it will be 1.66. And if you add them. So, it will be 8.30. So, this is the range I mean interval which you say that what you get because of the 90 percent confidence level. Now from here what we see that this interval does not contain 0. So, since the interval does not contain 0, so we would reject the hypothesis which tells us null hypothesis which tells that mu 1 equal to mu 2 at level of alpha is 0.1, so that is what you get the final you know outcome from this. So, once you compare, you can say that they cannot be compare, they are not comparable that way. So, once you have compared, you can say that we are rejecting this hypothesis null hypothesis which tells that mu 1 equal to mu 2. So, this is the parity approach, where when the n 1 will be equal to n 2 then we use this formula.

Next approach is the modified parity approach, which is also known as Welch approach. So, there we have not this condition that n 1 equal to n 2, many a times as we discussed n 1 and n 2 may be different. So, how to compare the two systems when n 1 and n 2 are different. So, we will see that.

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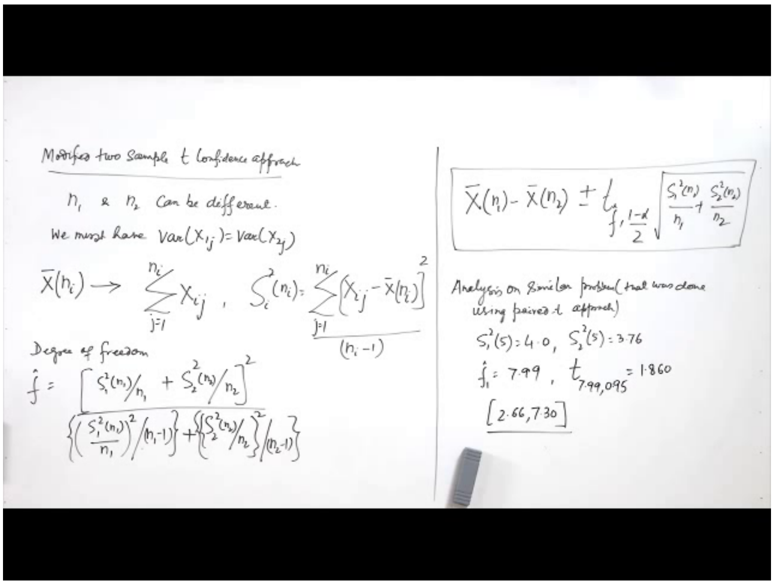

So, next is modified two sample t confidence approach. Now, in this case as we have discussed that not necessarily always the two systems, which are to be compared they both have the similar number of replications or readings. So, when that is the case in that case how to you know compare them, how to find the confidence interval, and how to compare the two systems. So, in that case, what we see is n 1 and n 2 can be different. Now, in this case, the variance is considered to be equal because what happens that we must have variance X 1 j equal to variance X 2 j. Otherwise, this is one requirement in this approach because otherwise may lead to serious coverage degradation, so that we have to assume in this case that the variance does not change for the two distribution. So, variance X 1 j will be same as variance of X 2 j.

Now, what we do is we have two values n 1 as well as n 2. So, the process will be that you find X bar so for n i that will be basically summation X i j and this will be j equal to one to n i. And similarly we will find S i that is square n i and that will be equal to again we have the make formula. So, j equal to 1 to n i, and then you have X i j minus x prime n i and that will be squared divided by n i minus 1. So, this way you are getting the values X prime n i and S i square n i and then we are using the formula. So, what we do in this case as the n 1 and n 2 is different. So, we need to calculate the degree of freedom part because as we have understood that in normal cases we have the degree of freedom as n minus 1, but here the n 1 and n 2 are different. So, we need to calculate the degree of freedom part and for that the degree of freedom is computed, degree of freedom that is f estimator. And this f estimator is calculated using a formula that is S square of n 1 divided by n 1 plus S square. So, this is s one square and this is S 2 square n 2 divided by n 2 and whole square divided by then S 1 square n 1 by n 1 square divided by n 1 minus 1 and then plus S 2 square n 2 divided by n 2 square divided by n 2 minus 1 .

So, this is how it looks like in the denominator. So, basically once we have the sample for the two you know X 1 j and X 2 j, for that we are going to separately compute these values and once we know these values, from here we are going to compute the degree of freedom f estimator. And this f estimator will further be used to compute, so that will be coming in that term where we use the t n minus 1 and 1 minus alpha by 2. So, basically for t n minus 1, so for n minus this f estimator will come.

Now, let us see what will come here. So, in that case, we are going to have this confidence interval as X prime n 1 minus X prime n 2. So, we will have the average values for this. And then we will have the difference of them then plus minus and further we will have t, this is the degree of freedom f estimator which we calculated from here and then we will have 1 minus alpha by 2. And then we will have under root S 1 square n 1 by n 1 plus S 2 square n 2 by n 2. So, this is how you get this confidence interval for when we compare the two systems which are basically which are having different n 1 as well as n 2. So, this process is known as modified two sample t confidence approach.

Now if we try to solve the similar problem, which we have done using the parity approach, so any way we have same n 1 and n 2. So, we can do the analysis based on that problem itself although. So, if we do the analysis on the earlier problem on similar problem that was done for parity approach. So, using parity approach, so in that case what we see is we have the values. So, what we see is once we calculate the S 1 square 5 and that is computed out to be 4; similarly S 2 square is computed out to be 3.76 then we calculate the f 1 star.

So, once we have S 1 square 5 and then S 2 square 5, because we are taking the n 1 and n 2 same as 5. So, once we have these two again same in that case we can compute these putting these values here we can compute the f that is degree of freedom. Now, this degree of freedom is coming out to be 7.99. So, we have to find this t 7.99; and 1 minus alpha by 2 again the confidence level is taken as 90 percent. So, in that case alpha is 0.1, so that will be 1 minus alpha by 2 will be 0.95. So, this value its very close to 8.

So, anyway if we look at here, so that if you for 8, it is 1.670. So, we do some what the interpolation then we will get it as 1.860. So, it is taken as because it is close to 8. So, as we see from for 8, it will be 1.860. So, if we have suppose 7.5, you may have anything in between the two numbers. So, in those cases you have to interpolate between the numbers, but here we get as with respect to 8, we are getting at 95 percent we are getting 1.860. So, this is coming as 1.860.

So, in that case, we find the interval and this interval comes out to be 2.66 and 7.30. So, what we see is that we have the different results using the two you know approaches, but in the both the approaches we see that they do not contain zero inside again here. So, that that hypothesis is again rejected here because we do not see mu 1 similar as mu 2. So, this is how these two methods are used to compare two alternative system configurations. Now, the thing is that when to use it, so many a times as we discussed that many a times we have the cases when you have real world observations, we have also the experimental results, we have the simulation results. So, those cases so we have once we have the real world results one is your simulation result. So, first system can be taken as the standard one and second system can be compared. So, this way the analysis is done, and we can get the results and we can compare the two systems.

Thank you very much.