Modelling & Simulation of Discrete Event Systems
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Lecture – 27
Obtaining a Specified Precision

Welcome to the lecture on obtaining a specified precision. So, in the last lecture we discussed about the output analysis for a single system, and we have seen that when we estimate the parameter, there we have estimated about the mean. So, that was the point estimate x bar n, and then we have to depending upon the confidence level what is given, you have to say that if the value is coming, if the value is coming from this particular mean to left and right certain quantity, then in that case we say that with that much of confidence we can say that the value will be closed to that mean value.

Now, the thing is that how we compute it. So, let us see that how we compute such kind of problems.

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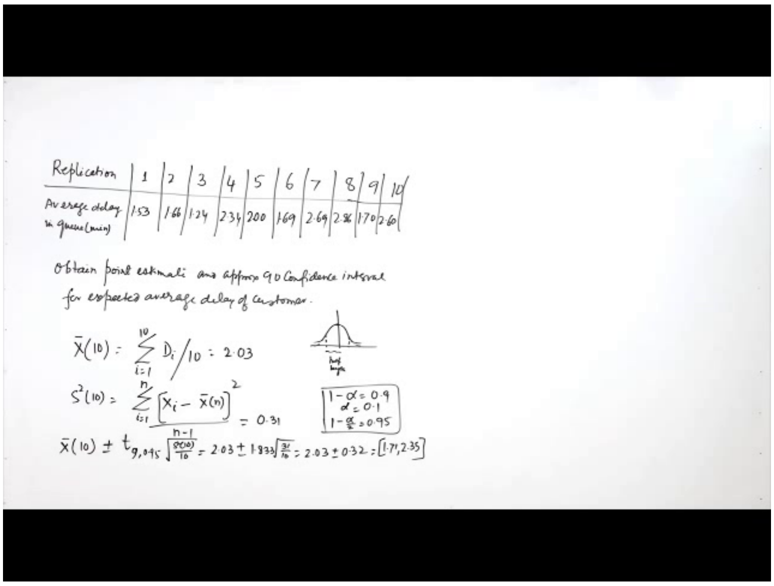

So, suppose you are given for a bank, you have the, you do the different replication, different runs you are doing, and in every run you are finding the average delay in queue. So, it will be in minutes. So, you are doing 10 replications 1 2 3 4 5 6 7 8 9 and 10. So, you on the 10 readings, you have got the average delay values as 1.53 you have 1.66 1.241 2.34 2.00 1.69 2.69 2.86 1.70 and 2.60

So, these are basically we did 10 independent replications, and in that we got the average delay values as this. So, we have to estimate the parameter, first of all the parameter is the mean of this average delay value, and then we have to find the obtain point estimate. So, we have to obtain point estimate, and approximately 90 percent confidence interval for expected average delay. So, of customer. So, this way you may be told that you have to have 1 interval, in that if you get the value, you can say in 90 percent with 90 of confidence that this value will be under that estimate.

So, as we discussed we have to find in that the sample mean. So, sample mean will be computed as the sum of these values x i I will be varying from 1 to 10. So, it will be summation of these delay values, and delay i varying from 1 to 10 and this divided by 10. So, once we add them after that if we divide it by 10, you are getting as 2.03. Now we have to find the confidence interval for this value that what should be to the, what should be the half length, I mean before it, and what should be the half length after it. So, you will have a confidence interval on both the sides. So, basically what we do is, if we have got the moon mean. So, we will have like this and with certain confidence, we are telling that this will be basically half length.

So, that way we are getting this half length calculation, and for that we know that we get x bar n plus minus t n minus 1 1 minus alpha by 2, and then we are getting S square n by n. So, first of all we will find the S square n. So, S square 10 it will be summation of i equal to 1 to n, and then we have the delay values. So, we will have x i minus x bar n, and then this is square upon n minus. So, once we do that. So, every time we will do. So, that will be 1.53 minus 2.03 and its square and plus 1.66 minus 2.03 its square. So, the all these will be squared, and then divided by 9 because n is 10. So, once we do that this value comes out to be 0.31. So, this value is coming out to be 0.31.

Now, in the earlier case what we used the formula for finding the estimate. Now that can be computed. So, we are getting that x bar 10 plus minus, then you have t n minus 1 n is 9 and 1 minus alpha by 2. So, here you have 90 percent confidence interval. So, 1 minus alpha is 0.9. So, alpha will be 0.1. So, 1 minus alpha by 2 will be 0.95. So, we are getting t n minus 1, and 1 minus alpha by 2; that is 0.95, and then multiplied with under root S square root 10 by 10.

So, that is how we are getting the result. So, we will get, this will be equal to. So, S bar 10 we have got 2.03 and plus minus. Now t 9 and 0.95 so that we can get from this table.

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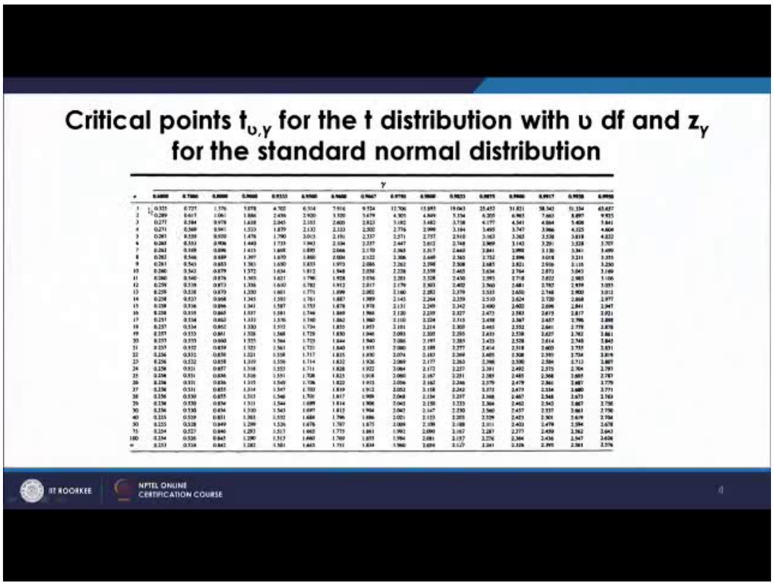

Now, from this table if you look at you have 9. So, t 9 and then you have 0.95, you are coming in this side. So, it is coming out to be 1.833. So, if you, if we look at this, it will be coming as 1.833. So, it will be 1.833 under root square n, S square n is coming as 0.31 divided by 10. So, if we get these values, it will be basically 2.03 and plus minus 0.32. So, we are getting interval of 1.71 and 2.35 minutes. This is the interval which we get and we say that with 90 percent confidence that the value will be in this interval. Now the thing is that, here you are having the mean value, sample mean is 2.03 and we are having the half length of 0.32.

So, certainly this is the half length. So, this much, it has to vary either to the left or to the right.

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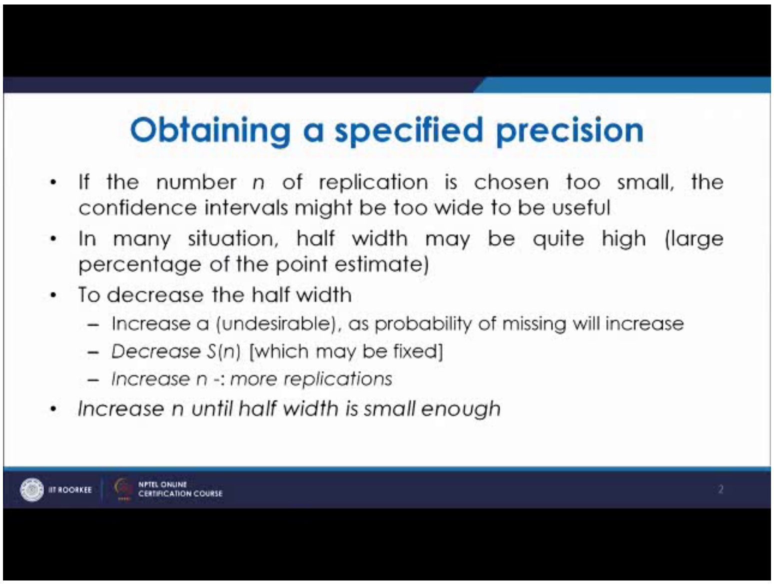

Now, the thing is that, in many cases it may go very high. So, if the number of replication n is small, then the confidence intervals might be too wide. Now that can be seen by looking at the table, that if the n is smaller, if the n is smaller then you see for any value, this value goes on increasing as we move up in the table. So, if that basically will go on increasing, in that case the half length will go on increasing. So, this interval will go on increasing, if the n is small as n will be larger, it will be going on smaller and smaller.

So, the thing is that, that is not basically going to be useful. So, what happens that in many situations, we are specifying that what should be this half length, we are basically specifying the precision; we say that, you tell me now the thing is, that to decrease this half length we have certain options. We have some options, how to decrease this half length. Now if you look at this expression, the expression of x bar n plus minus t n minus 1 1 minus alpha, and under root S square n by n in that case you have three ways; one is that if we increase the alpha.

So, if we the increase the alpha, you know this half length half width will be reducing, but then increasing alpha means this probability value is increasing. In that case the chances of the number going beyond that will be more. So, that is not basically advisable. Now we can decrease the S square n. So, that also basically we can do, but then we have to take it as a fixed one, normally that is a fixed quantity. So, then the last option is to increase the value of n. Now if we increase the n value continuously, and we keep the S square n value fixed in that case. So, we have the expression, the expression is like x bar n plus minus t n minus 1 1 minus alpha by 2 under root S square n upon n.

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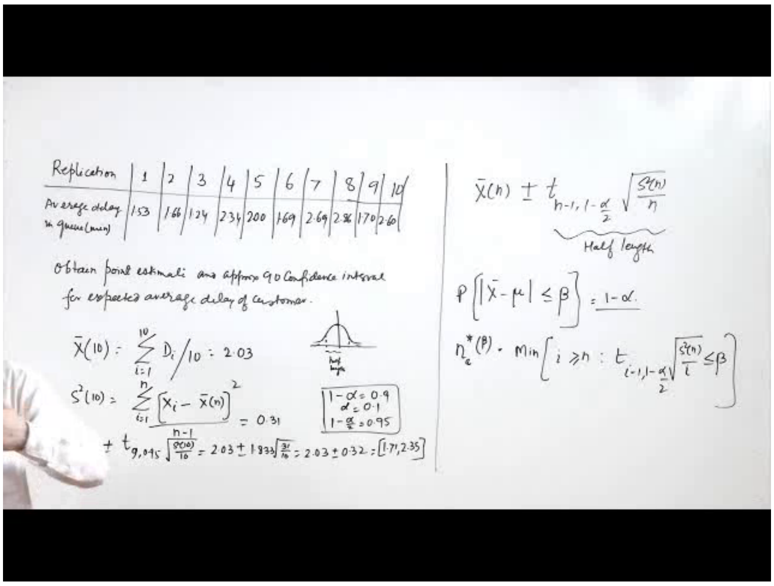

So, in this case now this is the half length, and what we see if we have seen this problem, we see that it is approximately about 15 to 16 percent of this x bar 10 value. So, we see that this 32.32, it is something close to 15 to 16 percent and in many cases it may go quite as high as 40 percent. So, basically that is not desirable. So, in many cases what we say that, we specify this that it should be how much, it should be what percent, it should be what value. Suppose we say that it should not be more than 0.2 or more than 0.15. So, in that case as we discussed the cases. Now we have either we can increase alpha, either we can decrease this S square 10.

So, that is basically kept fixed in most of the cases then we have. So, as we discussed that we cannot increase alpha, because that will giving us more chance that the estimation will be outside that limit it may go. So, because it will be coming quite narrow, and then we can increase this n. So, we can increase this n. So, as we increase n for a fixed S square n in those cases. We can see that we have this lesser value of the half length. So, that is what is known as obtaining the specified precision. So, increase n until half width is small enough. So, we have to increase this value of n and it iteratively we have to go, and we have to see that when this half length will be coming, I mean to a certain value.

So, for again we will see that how it will be done, and then we can solve through a problem. So, what we see here is that, in this case you have x minus mu p. So, in that we have the priority value x minus mu is less than equal to beta. So, this beta is nothing, but this value is x bar minus mu that value should be basically beta, and this is specified, it should be less than equal to beta. So, this probability value is basically equal to 1 minus alpha.

So, we are telling. So, we will get the expression that or we will have the equation, like yeah you go for that many number of replications. So, that you are getting that specified precision. So, ultimately you are going to have the number of runs; that is for specified beta, and it should be minimum of that run, where I will be more than equal to n. You have already run for i after that how long you should run. So, that the half length reduces to that particular value. So, for that t i minus 1 1 minus alpha by 2 under root S square n by n i it should be less than equal to beta. So, basically we are every time changing the n or i and then we are seeing that, when this value will come to that particular value.

Now, let us see that we have the problem again we are told that in this case, we need to have the, you know half length lesser than certain specified value. Now suppose we got here this half length as 0.32 and we need to get it to 0.25, we want to reduce this point 0.32 half length to 0.25. So, for that we have to increase the n. So, i is will be increasing and for every increase we have to then. So, basically in that case, this parameter will be changing. So, we have go for 10, if we go to 11 this parameter will be P 10 and 1 minus alpha by 2, and then this will be again S square n is already finished. So, it will be by 11.

So, similarly for 12 13 14 and 15 16 or. So, we have to go till we get this result. So, suppose for the same bank model.

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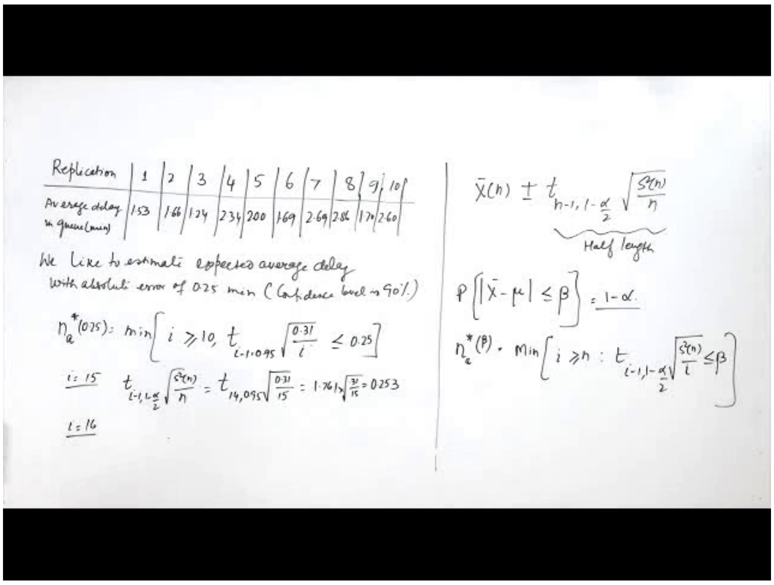

If we are to find, we like to estimate expected average delay with absolute error of 0.25 minutes and confidence level is 90 percent. So, what we had seen in the earlier case, we had the values and based on that we found the sample mean, and then we found the confidence interval for the 90 percent confidence level. Now in that we want to check the half length. So, that is error, basically that is why it is error. So, half length both on the left and right side. So, that is basically known as the absolute error. Now that is to the basically confined to 0.25. So, 0.25 minutes in the earlier case, it was 0.31 or so.

So, in that case what will happen you have to find for some value of number of replications, and where you have to get the value of 0.25, it will be minimum of the number of replications, where it will be more than 10 already, we have started from 10. So, after that you have to go, because at 10 you are seeing that you have that absolute error close to 0.3 or so, but it you have to commit down bring it down to 0.25. Now for that the value once we change the I will be. So, it will be i minus 1 1 minus alpha by 2 and 1 minus alpha by 2 for 90 percent, it will be basically 0.95. So, you have to write it as 0.95 , then you have under root S square n by n. So, S square n we are taking at fixed. So, which is 0.31 and then this will be changing that is i.

Now, that should be less than equal to 0.25. So, after 10 you have to check it. Now suppose after 10 you have to see for every i you have to check it. So, supposedly you take i equal to 15, i equal to 15 suppose you are taking. So, what will happen at i equal to

15 this value will be t. So, this t i minus 1 1 minus alpha by 2 under root S square n by n that will be basically equal t, this is 15. So, this will be 14.95 under root S square n by n. So, that is 0.31 by 15.

So, t 14 0.95 we have to look from the table. Now if you look at the table in that t 14 and 0.95. So, it is coming out to be 1.761. So, it will be 1.761. So, this will be 1.761 and in to under root 0.31 by 15. So, it is coming out to be 0.253. So, if you go to 16 it will be lesser if you go to 16, then suppose you go to. So, further. So, certainly it is bound. So, it is just coming to 0.253. So, it will be coming to. So, it is not, still coming 0.25, it is 0.253. So, if you to 16 it will be done just below that you can compute i equal to 16, and you will get some values at 16 which will be just below 0.25. So, you can say that if you go for n equal to 16.

So, that much run you have to complete and. So, that you guess you must have that many numbers. So, that you have the space specified precision of 0.25 is achieved now. So, what we see is that, either you can specify a particular value of beta; that is beta is greater than 0. So, for that you are going for big enough value of n. So, that the half length is less than the beta, many a times we are also given the relative precision that this precision what we get that error, there the in the relative terms this value is given like this value divided by x bar n; that fraction is specified.

So, many a times either you have absolute value given or you have the relative precision given. So, what do you mean by that relative precisions. So, in that case x bar minus mu divided by x bar, that value is given in terms certain decimal.

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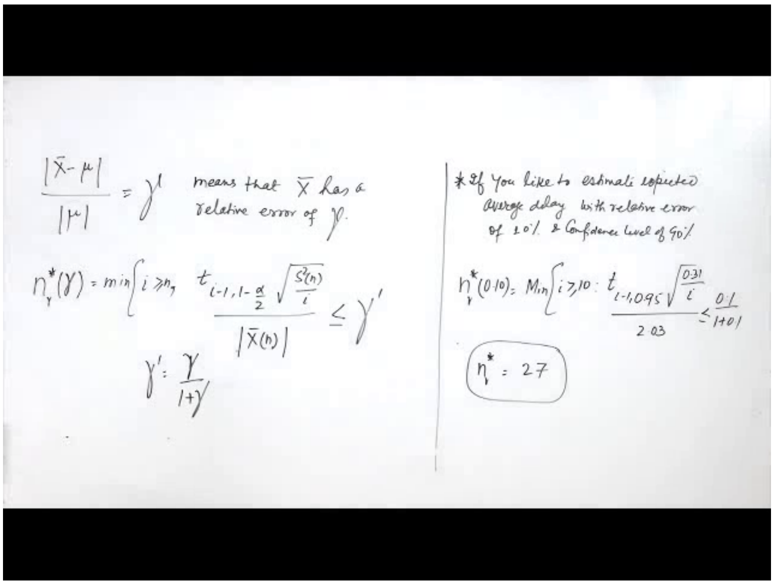

So, basically what we say is that, x bar minus mu and divided by basically x bar. So, that basically a we tell it as a value that is specified. So, this is the relative, you know precision. So, in those cases we can, say that this means that this x bar has a relative error of mu.

Now what do you mean by that. So, basically either it is expressed in terms of a particular absolute value or we also tell it in the terms of relative error value. This is said to be the x bar is has relative error of gamma; basically it is which respect to this mu. So, we will have mu here, because x bar how much it is this, this deviation who, much it is deviating from that actual mu, and what is the relative you know difference. So, that is how the x bar minus mu; that is defined as the gamma, and in that case you have the expressive further, and in those cases what we get is that, we are getting the.

So, for getting certain that relative value we have the expression. So, for a particular relative error of value gamma will be equal to minimum i is more than equal to n. So, because it is relative error of y with respect to mu and we have to see with respect to x bar. So, in case of x bar, it becomes gamma by 1 plus gamma. So, that is why the expression comes like this t i minus 1 1 minus alpha by 2 under root S square n by i, and again by you have x bar n. So, this will be less than equal to gamma dash.

So, because we are relatively looking with respect to;, we know x bar n. So, accordingly we can see with rate of sum of the arithmetic operations, this gamma prime comes, this gamma prime comes out to be gamma by 1 plus gamma. So, this gamma prime is coming as gamma by 1 plus gamma. So, if it is 10 percent, suppose we get 10 percent of relative error. So, if a gamma is 0.1 in that case 0.1 by 1 plus 0.9. So, it will be something close to 0.9. So, for 0.9 we have to get these values.

Now, if suppose we have the same condition in case of the earlier problem, which we discussed and we are given that we like to. So, if you like to estimate expected average delay, with relative error of 10 percent of 10 percent, and confidence level of 19 percent 90 percent. So, what will happen you have to go for n more than 10? So, that this relative error comes out to be only 10 percent to. In that case what we see is n r. So, we have to go for 0.1 we have only 10 percent is specified, and for that we are going to see for all the values. So, we will go on increasing that. So, I will be more than equal to 10 and for that t i minus 1, and then 1 minus alpha by 2 again. So, if the 1 minus alpha is 0.9. So, 1 minus alpha by 2 will be 0.95. So, it will be 0.95, and then further S square i by i. So, it will be 0.3 by i and then divided by x bar. So, x bar i will be something 2.03.

So, x bar n we know that. So, that is 2.03, and that has to be less than 0.1 by 1 plus 0.1. So, we are going to do it, you are going to iterate this process continuously, and if we do that we are getting this n r. So, for relative, certain relative error value we are going to get it when we e will going to 27. So, what we see that when we do the less number of replications as we have seen that, where we tried initially for 10 number of replications and was 10 for that with the mean value, sample mean value of about 2.01 you had the half length coming out to be 0.32. Then we were told that we have to decrease this half length.

So, half length was basically decrease to 0.25, because any a times you say that you go for last number of replications. So, that you can say with that much confidence that the half length is reduced the area that is basically. So, as you go on increasing the number of n, this half length will be going on reducing. So, we saw that when the n was increased to 25, I mean n was initially increased to 16. So, at 15 we checked and at 15, it was just coming to 0.253. So, when that was specified as 0.25, then we assumed that at 16 number of iterations, we can come to that value of 0.25, and then relative precision was also set. So, we told, we were told that you have to have the relative error of 10 percent maximum.

So, in those cases you have again to increase. So, in that case once you increase this n 2 27 as we see that, once you increase this number of runs to 27. In that case you are getting this condition full filled that you have relative error of only 10 percent. So, many a times when we do the simulation, we are said that the relative error should not be more than this, and you have to run for that much of time, or how much should be the n, what should be the number of values of this n. So, that you get a particular value of this relative error.

So, this is for the analysis of different type of such situations. Now apart from that we have already studied that you have a steady state behavior also. So, this was for the terminating type of simulations, and when we have non terminating type of simulations also. And now in that when we discuss with the study state type of behavior in those cases, you have some way. Basically you are specifying the initial conditions initially that for what conditions, up to what time, basically you have to neglect or you have to delete the values. So, in those cases you have to come to that condition, and then you have to take the data, and then further you have to analyze. So, that happens in the case of the steady state analysis. Hence, and then the same process applies once you have the mean values, you find again the, you know half length, and from there you can estimate the different parameters.

Thank you very much.