Modeling & Simulation of Discrete Event Systems
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Lecture - 20
Problem Solving on Random Number and Random Variate Generation

Welcome to the lecture on problem solving based on random number and random variate generation. So, in the last lecture we have discussed about how to generate the random numbers and also the random variates of different types of distribution functions. So, in this lecture we will try to solve some of the problems. In the case of random number generation, we would like to solve the problems where we will find the random numbers based on congruential generators and we will also solve problems based on the uniformity and dependence taste for the random number generated.

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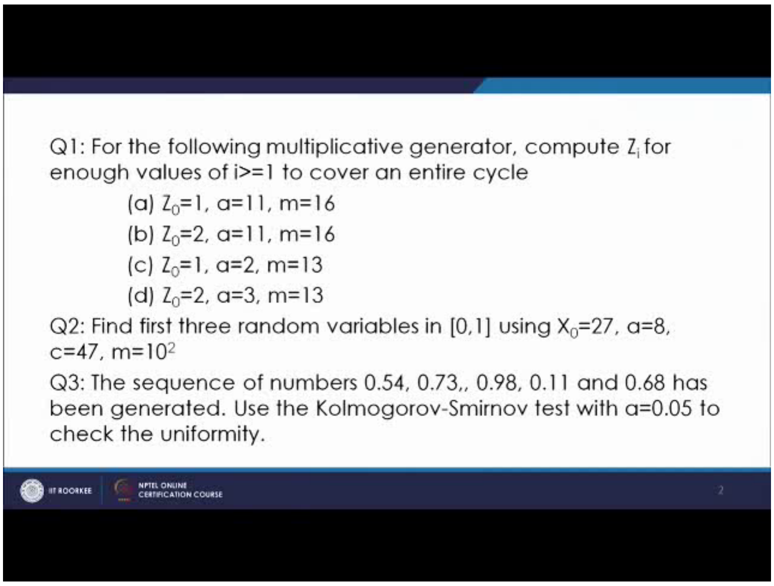

So, the first question is that for a multiplicative generator. We have to find Zi for enough values of i greater than equal to 1 to cover the entire cycle. So, this is a case of multiplicative generator congruential generator. Because you do not have the value of c the expression is a multiplied by Zi minus 1 plus c mod m. So, the c is 0. That is why it is known as multiplicative conventional generator. So, in this case we will see that how the period is varying for the different cases. Because we see that we have 2 values of the seed that is 1 and 2. And we have the initial value a, a is also different differently taken and m is also differently taken. So, this way we will try to see how to generate first of all the random numbers. And then how to see that how the period is varying when we change these parameters.

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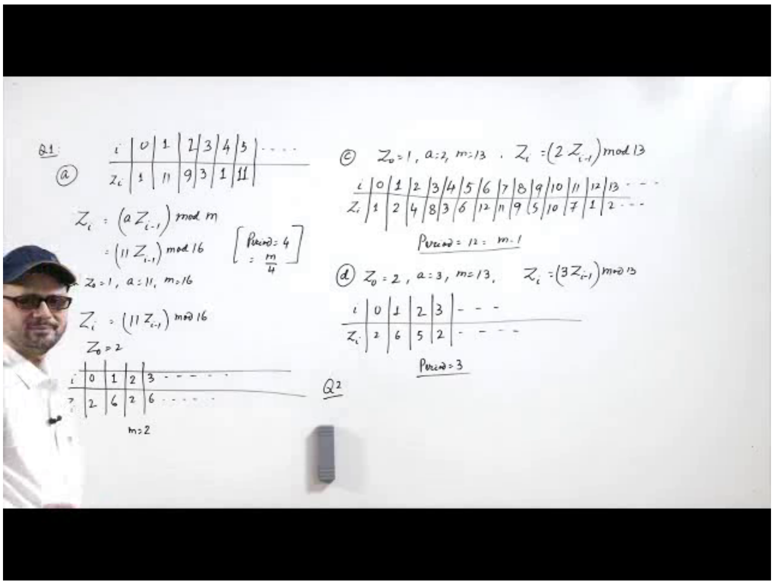

So, let us solve the first problem. The first problem is that we have a is Z naught is 1, a is 11, m is 16. So, question 1, now in this case we have i and Z i. So, let us see the Z naught is 1. So, when i is 0, it is 1. The expression is expression will be Zi will be a into Zi minus 1 into mod m. So, in this case you have a as 11 m is 16. So, 11 Zi minus 1 into mod 16. Now once we have i is 0 Z naught is given as 1. So, Z naught given is Z naught as 1, a as 11 and m as 16.

Now, So, once we go further we can just go and get the values. So, if it is 1, it will be 11. Then i will be 2. So, once i is 2, Z 2 will be 11 into 11. So, 121 mod 16. So, 16 into 7 plus 9. So, it will be 9 go to 3, 9 into 11 99 mod 16. So, 6 into 16 plus 3. So, it will be 3 4 3 into 11 33 mod 16. So, it will be 1. 5 1 into 11 mod 16. So, it is 11. So, what we see is this 11 has come here. It means the period is only for 11 9 3 1. And then 11 has come. So, what we see is that the period is you know 16 by 4. So, we had seen that in one case the maximum period is 2 raised to the power b. So, b is anyway here 4 and in some case it will be 2 raised to the power b minus 2.

So, in this case we are seeing that this model, I mean period is 4. So, this will go on repeat. So, what we see is that in this case period is 4, that is m by 4 because it m is 16. So, 16 by 4 it is 4. So, this is the a part. Let us go to the b part. In case of b it will be Zi will be a into Zi minus 1. So, a is 11 and then here the mod is 16 and Z 0. What is given as 2. So, in this case you will go like this. You have i and you have Z i. So, once i is 0 it is 2. Now how it will move.

So, in the first case what will be there. So, in the first case 2 into 11. So, it is 22 mod 16. So, it will be 6, Z 2 for Z 2 this 6 has come. So, 11 into 6 66 mod 16. So, it will be 2 it means further it will go to 6 and so on. So, here we see that only the period is 2, you are getting only 2 random numbers. And after that the series is recycling or cycling. So, this you are cycling you see only after 2 entries. So, m is 2. Next question, c part. In c you have Z naught as 1 and a has 2 m as 13.

Now, in this case what will happen? You have i and Z i. So, Z naught is given as 1. So, if it is 0, it is 1. So, c part Z naught as 1 in this case the expression will be Zi will be a into Zi minus 1. So, 2 into Zi minus 1 and then mod 13. So, once the first value is 1, when i is 1 in that case 2 times 1. So, it will be 2 in the second case 2 into 2 4 mod 13. So, 4 3 4 into 2 8 mod 13. So, 8 4 8 into 2 16 mod 13. So, 3 16 about 13 will be 3 where 13 into 1 plus 3. So, it will be 3, 5 is 6. 6 will be 12. 7 will be 12 into 2 24. So, it will be 11, 8 will be 11 into 2 22. So, it will be 9 9 will be 9 into 2 18. So, 5 10 10 will be 5 into 2 10 11 will be 10 into 2 mod 13 7 12 7 into 2 14 mod 13. So, it will be 1.

So, it will be further going towards to end all that. So, what we see is that you have the period going from 1 to 12 and then it is repeating. So, what we see is in this case period is 12 that is m minus 1. So, in this case we see that the maximum period is achieved. So, that is 12 equal to m minus 1. We take the last part D part of the question number 1 in question number d1 D Z 0 is 2 a is 3 and m is 13. So, so Zi will be a is 3. So, 3 Zi minus 1 mod 13. So, again in this case i and Zi at 0 it is 2. So, then with 1. It will be 2 times 3 6 mod 13. So, it will be 6 then 6 into 3 18 mod 13. So, then you have 2, 6 into 3 18 mod 13. So, it will be 5 in third case 5 into 3 15 mod 13 it will be 2. So, further it will be going.

In this case what we see is that you have 6 5 2 mod. So, the 3 random members you are getting in after that it is getting repeated. So, here the period is 3. This is how you calculate the periods of the generator, which have different values of the seed multipliers and the increment as well as the modulus modular values. We will go to next question. Question number 2. Question number 2 is that you have to generate 3 random variables in 0 one using the seed value as 27 x naught a is 8 c 47 and m is 10 raise to the power 2.

So, the concept is that you are going to use the linear congruential generators formula. That is xi will be a into xi minus 1 plus c mod m. So, in that basis expression will be xi will be a, a is your value here 8.

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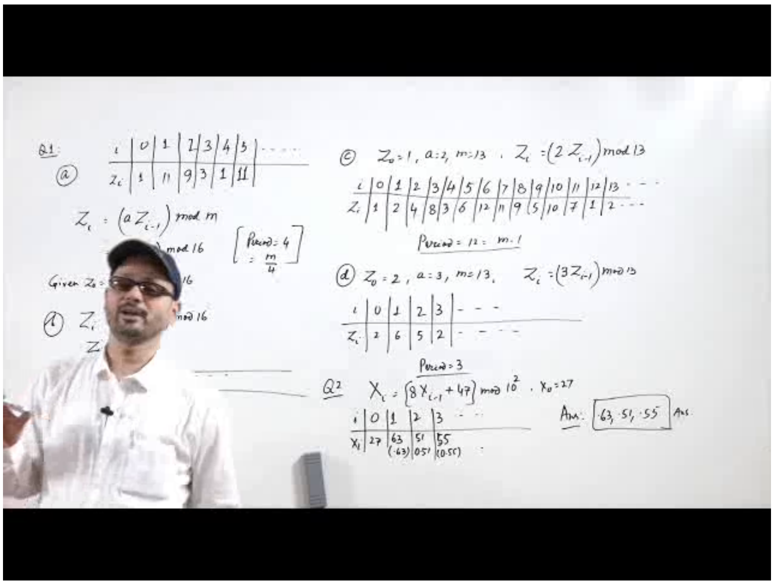

So, 8 into xi minus 1 plus c, c is 47. And then you have mod 10 raise to the power 2. So, again here also we can generate the numbers i and xi and x naught or x 0 is given as 27. So, when i is 0 it is 27. When i will be 1 in that case 8 into 27, 2 1 6 plus 47. So, that is 263 mod 100. So, last 2 digits will be taken. So, it will be 63.

Now, in this case you have to generate the variates. So, in the in finding the variates, you are going to divide it by m. Because the maximum number is m you are getting the model mod of m. So, you are going to divide it by m that is 10 raise to the power 2. So, your number will come as 0.63. Similarly, 2. So, again 63 into 8 500 4 plus 47. So, 551. So, 551 has and then further mod of that as 100. So, it will be 51. So, the variate will be 0.51 then in the third case the third number will be 51 into 8 400 8 plus 47. So, that will be 455. So, 455 divided and mod of that with 100. So, that will give you 55. So, you will get the variate as 0.55and this way you can go. So, what you see that 3 variates in 0 to 1 are answer is 0.63 0.51 and 0.55 this is the answer.

So, whenever we have to generate the variate in certain domain in that case in between 0 to 1 in that case the number, which you are getting using the linear congruential generator or any generator. In that case and you do the modulo operation in that case you are going to divide it by the mod value. And then you are getting then the variates in between 0 and 1. So, this is how it is solved. The next question is regarding the finding of uniformity, whether the number which is sequence in certain sequence, whether it is uniform or not and you are given certain value of alpha. That is 0.05 to check the uniformity test with the form with that test method of Kolmogorov Simonov test.

So, for that we will try to have the data. And we will see we have seen how to do it. So, in that question 2. We have the data as 0.68. So, the data is 0.54 0.73 0.98 0.11 and 0.68. Now in that case. So, you have alpha as 0.05. So, in this what we do is, we are first of all writing the numbers in ascending order. And then we are making a table we are finding I by n ri minus. So, do you have different parameters based on that we are finding D plus and D minus and then D is calculated as maximum of t plus and D minus. And then further we are checking it with that chi square value from a particular table, the critical value and then we have to see that the value which we are getting for this sample. It should be less than the value which we are getting from the table.

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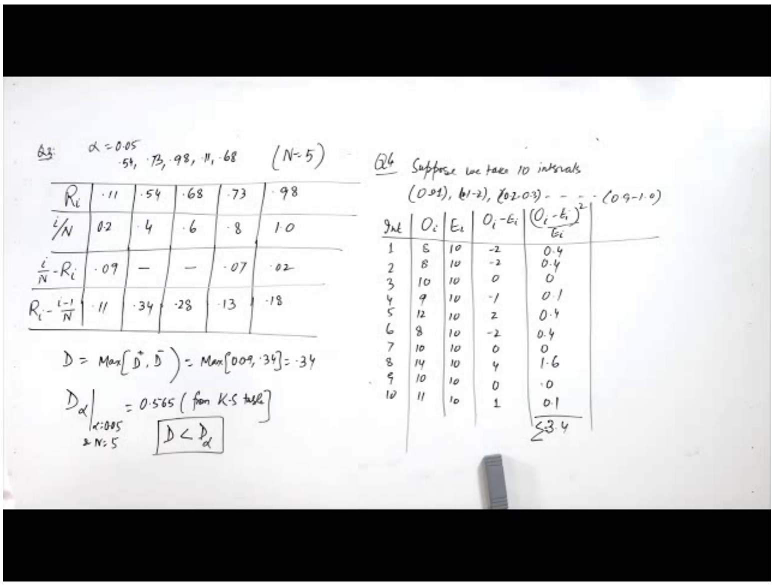

In that case we can say that it is uniform. So, let us find. So, what we see is you have ri and if we write it in the ascending order. So, Ri value will be coming like this. So, in the value we have value as 0.45, 0.54 0.73 0.98 and then 0.11 and 0.68. So, your first value will be here 0.11. Then we will have the next value as 0.54. Then we have 0.68 then we have 0.73 and then we have 0.98. Where i is 1 2 3 4 and 5. Now n is 5 here. So, what we see is n is 5. Now the next parameter which we are going to calculate is i upon n. So, this in this case i is 1 and this is 2 3 4 5 like that. So, I by n will be 1 by 5, that is 0.2 0.4 0.6 0.8 and 1.

The next parameter which we have to find is i by N minus R i. So, we are going to have the difference of this minus this. That will be 0.09. This will be minus of 0.14. So, that we are not taking. Then this will again be minus this will be 0.07 and this will be 0.02. Then ri minus i minus 1 upon N. Now in this case the ri will be subtracted with i minus 1 upon n. So, this will be 0.11. Now in this case 0.54 will be subtracted with 0.2, i minus 1 by N will be this one, for this value it will be this one. So, 0.54 minus 0.2 4 0.68 minus 0.4 0.2 8 0.73 minus 0.6 0.13 and 0.98 minus 0.8 0.18.

Now, after finding this we have to find D plus and D minus and then d. So, D will be D will be basically maximum of D plus and D minus. Now the D plus will be basically maximum of this value. So, D plus will be maximum of this row value that is 0.09. So, it will be maximum of 0.09 and maximum and D minus is this maximum of this. So, in this the maximum is 0.34. So, that is 0.34. So, we are getting D as maximum of 0.09 and 0.34 that is 0.34. Now we have to find D alpha. So, you have 5 (Refer Time: 22:48) and D alpha.

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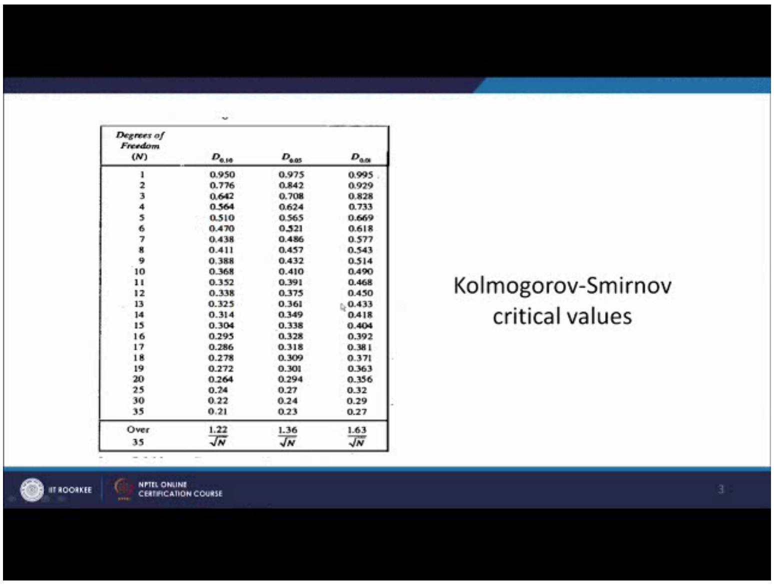

Degrees of Freedom (N)	$D_{0.10}$	$D_{0.05}$	$D_{0.01}$
1	0.950	0.975	0.995
2	0.776	0.842	0.929
3	0.642	0.708	0.828
4	0.564	0.624	0.733
5	0.510	0.565	0.669
6	0.470	0.521	0.618
7	0.438	0.486	0.577
8	0.411	0.457	0.543
9	0.388	0.432	0.514
10	0.368	0.410	0.490
11	0.352	0.391	0.468
12	0.338	0.375	0.450
13	0.325	0.361	0.433
14	0.314	0.349	0.418
15	0.304	0.338	0.404
16	0.295	0.328	0.392
17	0.286	0.318	0.381
18	0.278	0.309	0.371
19	0.272	0.301	0.363
20	0.264	0.294	0.356
25	0.24	0.27	0.32
30	0.22	0.24	0.29
35	0.21	0.23	0.27
Over 35	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

If you look at. So, we have to look at the table we will see the Kolmogorov Simonov critical value table and in that basically for this d, alpha value of0.05 this is the 0.565 value with 5 entries you have D alpha is coming as 0.565. So, D alpha for alpha as0.05 and n as 5 it is given as 0.565 from Kolmogorov Simonov table.

Now, since this D is less than D alpha. So, the hypothesis is not rejected. We can say that this table the values pass the uniformity test as per this Kolmogorov Simonov conditions. This the samples are from the uniform are you uniformly distributed. Next question is that we have a table, we have the values and for the following data we have to use the chi square test with alpha as 0.05 to check the uniformity.

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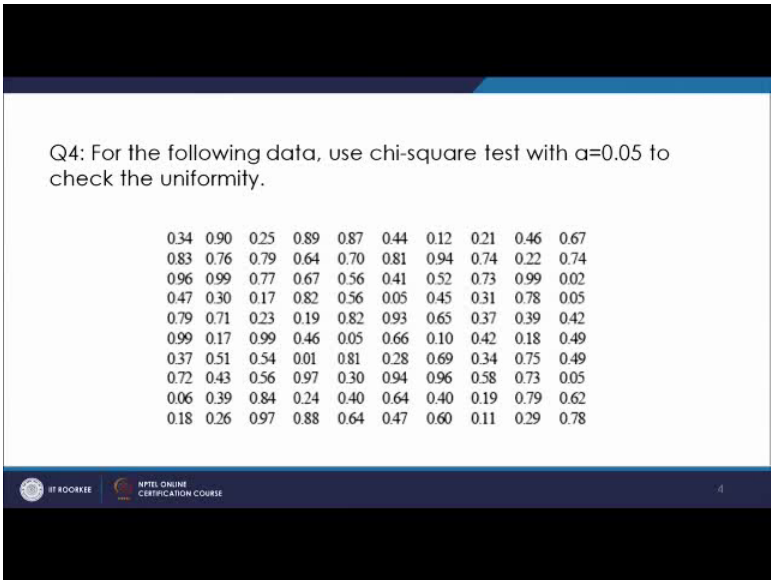

Now, in this case we have already understood that when the data is limited in number. In that case we are going to use the Kolmogorov Simonov test; however, when the data becomes quite large in those cases the chi square test is carried out.

Now, in this case what we do is once we have such data. First of all, we have to convert them we have to make table we will have the intervals and then for the interval we will have the frequency and also. So, we will have the frequency. So, here we have 100 data. So, with 100 data we know the frequency. And we will also have the frequency in ideal case, that is ei as we know that in this case we are having the chi square value being calculated and that is basically computed as. So, question number 4. This is question number 3. And then we are coming to question 4. In that case we are making in such cases you are finding the interval. So, here we are making 10 intervals. And we will start with 0 to 0.1, 0.1 to 0.2 0.2 to 0.3. So, we are suppose we are taking and interval 10 suppose we take 10 intervals. So, interval will start from 0 to 1, 1 to 2, 2 to 0 to 0.1 sorry 0.1 to 0.2, 0.2 to 0.3 and so on. So, last will be 0.9 to 1.

So, you will have 10 intervals. So, if we draw table you have interval. So, you have interval as number 1 2 3 4 5 6 7 8 9 and 10. Now in this case you have the observed frequency observed frequency. Once we if you try to see this table and you calculate that what is the frequency of these numbers which fall in between this interval, then you will get this values like 8, 10, 9, 12, 8 ,10, 14, 10 and 11. So, this is the observed frequency in this interval. Then you go to find the expected frequency. Ideally what it should be because it as to be uniformly distributed. And since there are 10 intervals in every interval there must be ideally 10 numbers. So, it should be 10. So, it should be 10 in every interval.

Then what we do is further we are getting ei oi minus e i. So, we get minus 2 minus 2 0 minus 1 then 2 minus 2 0 4 0 and 1. So, ultimately you have to find oi minus ei square upon e i. So, this will be 4 upon 10. So, it will be 0.4. It will be 0.4, it will be 0. It will be 1 upon. So, it will be 0.1, it will be 4 upon 10, 0.4 0.4 it will be 0. So, it will be again 0.1 6 and then you will have 0 and ultimately it is 0.1. So, so once we get that. So, we have to add it. In fact, there is a mistake here now in this case it will be 4 into 4, 16 divided by 10. So, it will be one 0.6 in fact.

So, this will be one 0.6 and once we add them it will be 3.4. So, summation will be 3.4 summation of this oi minus ei square upon ei, where i is varying from 1 to n that is 10. In that case this is summation is coming as 3.4. Now this is your for the chi square value for this sample. Now we have to see from the problem now in the table.

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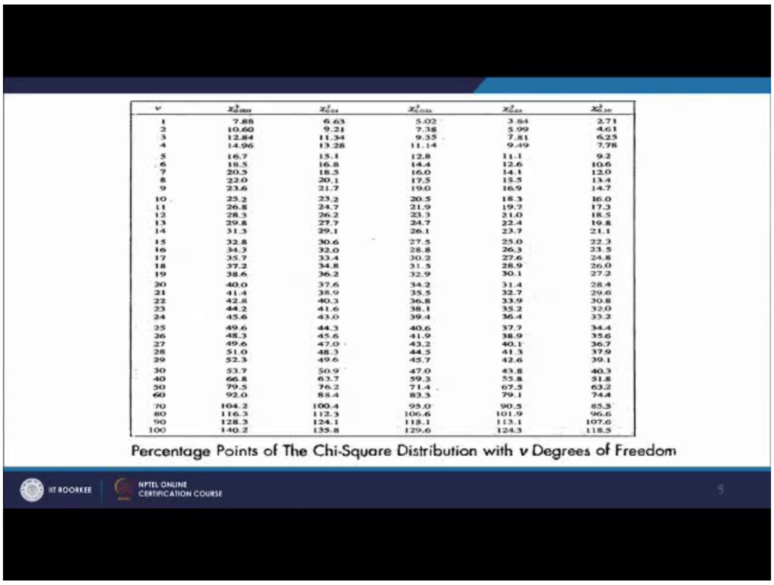

v	$\chi^2_{0.005}$	$\chi^2_{0.01}$	$\chi^2_{0.025}$	$\chi^2_{0.05}$	$\chi^2_{0.10}$
1	7.88	6.63	5.02	3.84	2.71
2	10.60	9.21	7.38	5.99	4.61
3	12.84	11.34	9.35	7.81	6.25
4	14.96	13.28	11.14	9.49	7.78
5	16.7	15.1	12.8	11.1	9.2
6	18.5	16.8	14.4	12.6	10.6
7	20.3	18.5	16.0	14.1	12.0
8	22.0	20.1	17.5	15.5	13.4
9	23.6	21.7	19.0	16.9	14.7
10	25.2	23.2	20.5	18.3	16.0
11	26.8	24.7	21.9	19.7	17.3
12	28.3	26.2	23.3	21.0	18.5
13	29.8	27.7	24.7	22.4	19.8
14	31.3	29.1	26.1	23.7	21.1
15	32.8	30.6	27.5	25.0	22.3
16	34.3	32.0	28.8	26.3	23.5
17	35.7	33.4	30.2	27.6	24.8
18	37.2	34.8	31.5	28.9	26.0
19	38.6	36.2	32.9	30.1	27.2
20	40.0	37.6	34.2	31.4	28.4
21	41.4	38.9	35.5	32.7	29.6
22	42.8	40.3	36.8	33.9	30.8
23	44.2	41.6	38.1	35.2	32.0
24	45.6	43.0	39.4	36.4	33.2
25	49.6	44.3	40.6	37.7	34.4
26	48.3	45.6	41.9	38.9	35.6
27	49.6	47.0	43.2	40.1	36.7
28	51.0	48.3	44.5	41.3	37.9
29	52.3	49.6	45.7	42.6	39.1
30	53.7	50.9	47.0	43.8	40.3
40	66.8	63.7	59.3	55.8	51.8
50	79.5	76.2	71.4	67.5	63.2
60	92.0	88.4	83.3	79.1	74.4
70	104.2	100.4	95.0	90.5	85.5
80	116.3	112.3	106.6	101.9	96.6
90	128.3	124.1	118.1	113.1	107.6
100	140.2	135.8	129.6	124.3	118.5

What we see is in the table we have to see this is 0.05 and your degree of freedom because you have 10 intervals. So, the degree of freedom will be 10 minus 1. So, it will be 9 and corresponding to this 9 it will be 16.9.

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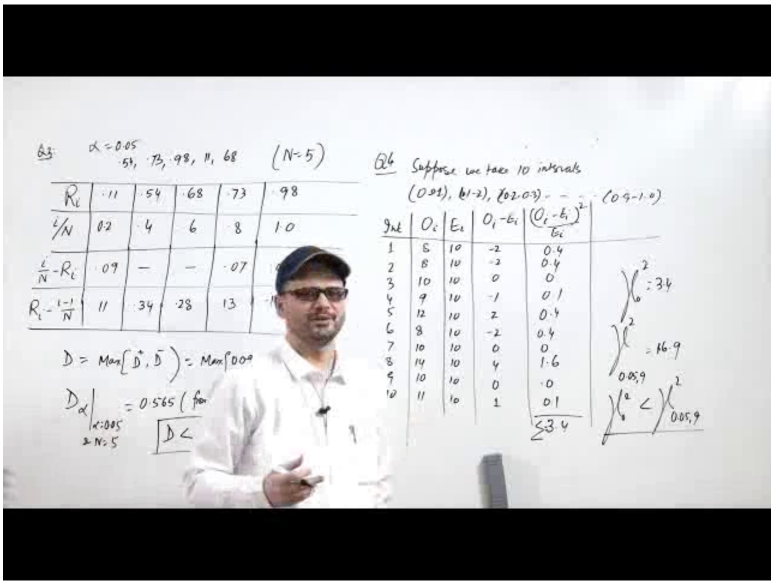

So, what we see is we see that we are getting as 3.4 and for the sample it will be for degree for 0.05 and degree of freedom 9 it is coming as 16.9.

So, what we see is this is basically less than 0.059. So, the hypothesis is not rejected it is assumed that it will pass the uniformity test. So, the sample is from the uniform distribution. So, the hypothesis of the uniformity is not rejected in this case. So, we have seen that this is how the checking of uniformity can be carried out for such samples.

Thank you very much.