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Modeling & Simulation of Discrete Event Systems
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Lecture – 19
Generation of Random Variates

Welcome to the lecture on generation of random variates. So, in the last lecture we had seen that how to generate the random numbers. And these random numbers which were generated they were of uniform distribution. So, we had to see that the random number which we are generating they are having uniformity they are independent and for that we also had the checks and tests for checking the uniformity and independence of these samples.

But many a times you need the random variates. We should follow a certain kind of distribution. So, for that we need. So, this method, I mean in this lecture we will deal with the procedures.

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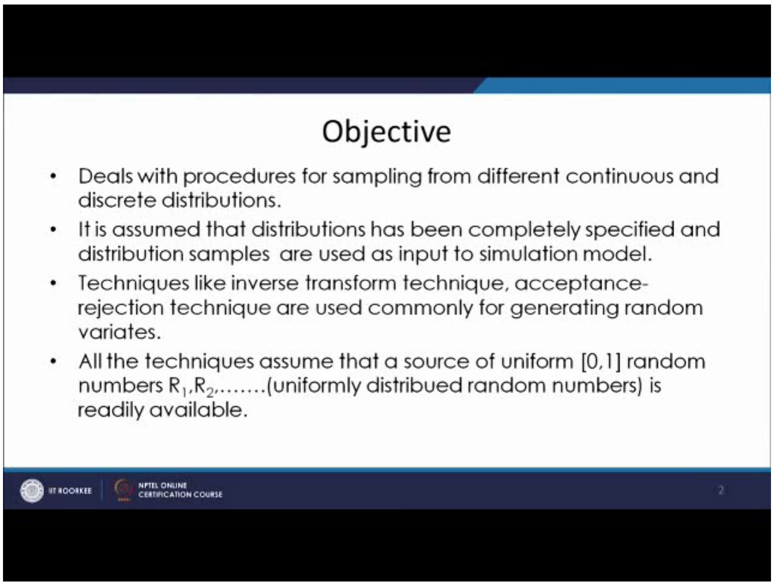

For sampling from different continuous and distribute you know discrete distributions. So, what we will see that you have the random numbers, you have the random generated samples. From there you are going to have the variates. So, you are going to have the variates which are of certain distribution type. So, the it is assumed that distribution has been completely specified. So, you are going to be told that you have to generate the numbers which are of certain kind of distribution, it may be vibal it may be exponential it may be Poisson it may be triangular or so. So that is known to you and are used as the input to the simulation model.

Now, techniques. So, you have different kinds of techniques which are used for such generation of random variates. And one of the method which is normally used is the inverse transform method. So, you have the particular distribution function for that you have certain cumulative distribution function equation. From there you are going to, so, that equation is going to use the, I mean to be used and from there you are going to get the variate which are going to be of following that particular type of distribution.

Now, there are techniques like inverse transform technique, acceptance rejection technique these are the techniques normally used we are going to discuss mostly on the inverse transform technique how it works. So, it is basically you have the equation the cumulative distribution function in that whatever you are getting that is R. So, this R will be taken as input. So, now, earlier you generate the random numbers which are uniform. Now we are using the equation and you are transforming in a inverse manner. This R will be used as input and the output will be those numbers which are following this exponential distribution or Weibull distribution or so. So the techniques we will they are all assuming that you have source of uniform random numbers is available.

So, you must have the random numbers available at source. This number can be generated by different kinds of generators you can use the standard softwares like excel or so also. So, if you put the rand function it will give you the number of generated they are normally uniform distributed numbers. Now from there you have to calculate the exponential suppose variates. So, for that you we are going to have the you know inverse transformation. So, what is the inverse transform technique.

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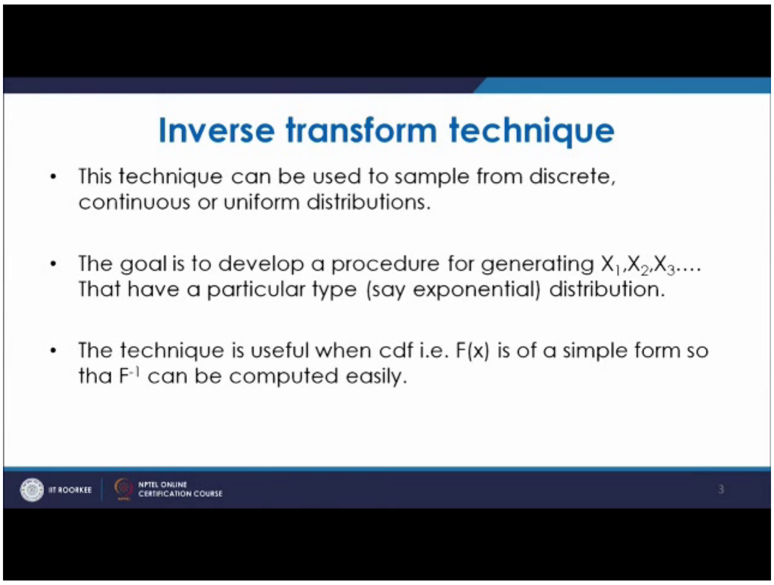

So, this technique is used to sample from discrete continuous or uniform type of distribution. So, our empirical kind of distributions, and the goal is to develop a procedure for generating x1 X 2 X 3 all that which have a particular kind of distribution function.

So, you have for a particular type say any anything like Weibull exponential or gamma or so. So and the technique is useful when cdf that is F X is of simple form. So, what we have is you have a particular you know cumulative distribution function expression that expression is used for in the inverse transfer and that must be simple. So, that the inverse can be calculated in a simple way otherwise this technique will not be so, useful.

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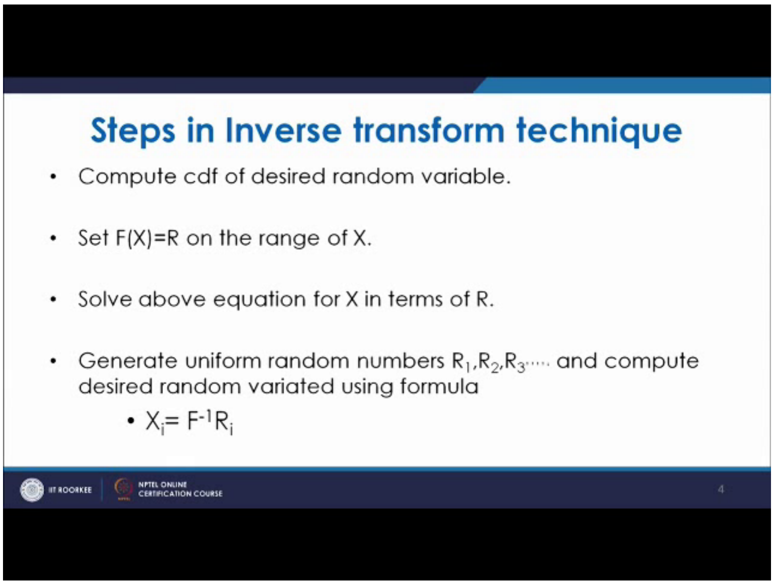

Now what is done in the case of this inverse transform technique. So first of all you will compute the cdf of desired random variables. And then you are going to have set F X equal to R on the range of x. And now so once you get F X equal to R. So, F X has certain expression and X will be F inverse R.

So, once you have the F X equal to R you are going to solve that equation. So, suppose it is exponential equation you are asked to generate the exponential variates. In those cases, you have the cumulative function 1 minus e raise to the power minus lambda x. So, like that that will be equated to R i and then from there. So, if you there will be equated to R and from there R you are going to get the solution for this X in terms of R. So, that will be solved. So, generate the uniform random numbers R 1 R 2 R 3 that is input itself. So, you are going to generate it you will have that in hand and then you are computing the desired random variates. So, that is variate basically not variated it is random variate. So, this you are going to generate the random variate using this this formula xi will be F inverse R i.

So, this way once you solve this equation, you are going to get the variates which are of that typical distribution function. So, that is suppose exponential or so. So let us see by example if we say we are doing for exponential distribution. So, as we know the in case of exponential distribution.

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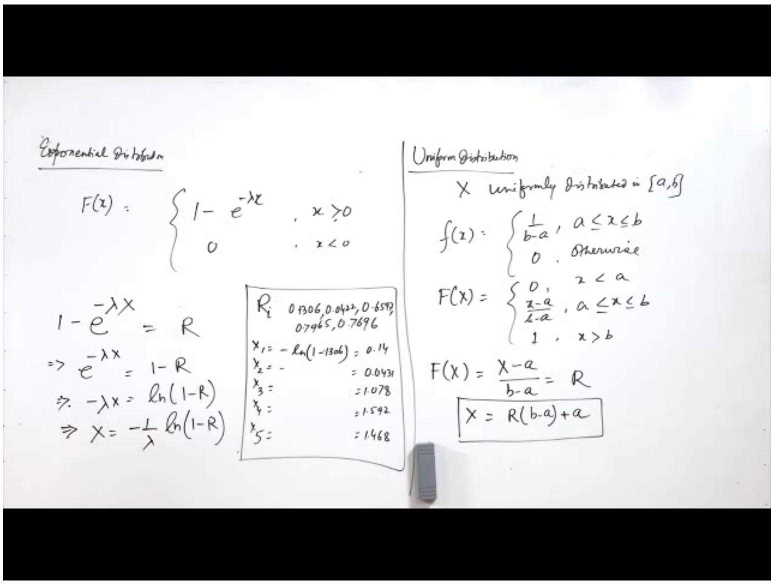

Suppose you have the exponential distribution. In that case as we know that the cumulative distribution function for exponential function is 1 minus e raise to the power minus lambda x. So, that is 1 minus e raise to the power minus lambda x. So, for X more than equal to 0 lambda is the mean and then it will be 0 otherwise. So, if X is less than 0. So, that is what we get in the case of exponential distribution.

Now, what we see by the steps you have to compute the cdf. So, this is a cdf. So, F X will be 1 minus e raise to the power minus lambda x. So, 1 minus e raise to the power minus lambda and this will be the random variable this will be now equated to R. So, once you request that to R. So, that will be you have to solve this, you have to find now this R is known to you yours R is given to you these R these are the random variables of uniform distribution. So, now, anyway the meaning is that even if you are getting the uniform distribution numbers or you are getting any number which is typically of you know you some certain type of distribution that has to be uniform. So, from there you are getting this number in uniform manners. So, it is just the inverse transfer.

Now, in this case once you solve this, what you get is e raise to the power minus lambda X will be. So, this will go this side and this will be like 1 minus R. So, you are getting minus lambda X will be l n of 1 minus R and X will be minus 1 by lambda of l n of 1 minus R. Now this Ri is known to you. So, once you know this Ri you have different value of R i. So, once you have in a particular column Ri you can get generate the xi. In the next column. So, in that case and you get the different Xis.

Now, let us see for example, if one example is given that you have R is given as 0.1306 0.0422 0.6597 0.7965 and 0.7696. Suppose these numbers of 5 numbers of random numbers are given to you. In that case you are simply using the inverse transformation. You are going to get this Xis. So, x1 X 2 X 3 X 4 and X 5 you can calculate. So, x1 will be minus 1 by lambda. So, if the lambda is taken as mean as one in that case you are getting minus of l n 1 minus R. So, 1 minus 0.1306.

Similarly, minus of l n. So, whatever is value is there you can calculate it and if this is computed out to be something like 0.14, 0.04301, 1.078 then 1.592 1.468 or so. So this way once you have the random number of uniform distribution, you can generate the exponential variates or any kind of variates, that does not matter you are using you are calculating that using the inverse transformation method. Let us see how it works for other kind of you know distribution functions.

So, let us see for uniform distribution. So, for uniform distribution as we know, you have x uniformly distributed in a b suppose. So, as we know that in the case of uniform distribution the probability F X small F X it will be F X will be 1 by b minus a, if x is in between a and b and it will be 0. If you if it is less than a or more than b. So, that is what we know and if we try to find the cdf. The cdf will be 0 if x is less than a. And it will be X minus a upon d minus a, if it is between 0 to I mean a to b. And then if it is more than b it is 1. So, in that case you know the cumulative distribution function. And once we know the cumulative distribution function then we can have the inverse transformation. So, for uniform distribution what we get is, we know that we get F X as one by b minus a when a to b x varies and 0 otherwise. So, we get the cumulative distribution function like it will be 0, if x is less than a, it will be X minus a upon b minus a, if a is x is in between a and b. And it is 1 if x is more than b.

So, F X will be x minus a upon b minus a that is basically the cumulative distribution function between when x is in between a and b and we are to find the uniformly distributed x is between a and b. So, we are going to express this expression in and equate it to R and then we are going to have the inverse transformation. So, what we get is F X will be we are getting x minus a upon a minus b, b minus a basically. So, it will be b minus a and this is taken as R. So, once we get that then you can solve this equation. Now once we get you are getting the this R. So, you have to get x. So, what we get X will be equal to R into b minus a and plus a. So, once you have rs and you know that you have to find the X between a and b. In that case in between a and b you can use this formula to find the different X is.

Now, let us see for another distribution if you have the Weibull distribution.

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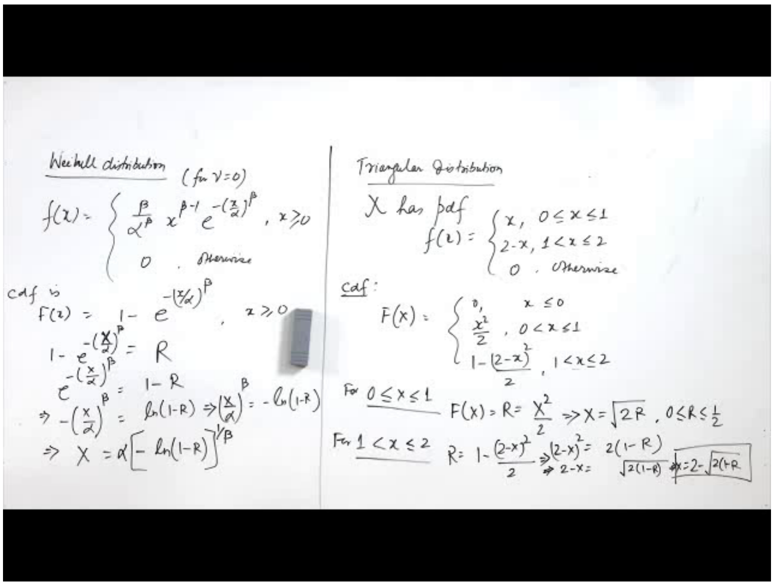

In case of Weibull distribution again we will have to find the cumulative distribution function and we know that in the Weibull distribution you have 3 parameters alpha beta and nu. And the nu location parameter we have to set it to 0. So, what we say normally for the time to failure this Weibull, I mean transformation Weibull distribution is used. So, in that as we know we get F X as. So, for nu equal to 0 the location parameters we are taking as 0, we are getting F X as beta by alpha raise to the power beta, X raised to the power beta minus 1. And e raise to the power minus of x by alpha raise to the power beta.

So, this is for X greater than equal to 0 and 0 otherwise. So, this is the pdf of Weibull distribution for alpha and beta greater than equal to 0. Now for that we have already seen the F X comes out to be. So, cdf is F X equal to 1 minus e raised to the power minus of x by alpha raise to the power beta, for X more than equal to 0. Now when we go for inverse transformation technique in this formula again. So, that is why we are going to equate it to R. So, 1 minus e raise to the power minus of x by alpha beta will be equal to R. Now we have to find. So, this X will be basically taken as the capital X we are going to get this X is.

Now, this X we have to find the expression. So, what we see you will take that that side. So, 1 minus R. So, e raise to the power minus of x by alpha raise to the power beta will be 1 minus R. So, minus of x by alpha raise to the power beta will be l n of 1 minus R. This implies that x by alpha raise to the power beta will be minus of l n 1 minus R. So, X will be minus of l n, 1 minus R raised to the power 1 by beta. And this alpha will go that side it will be multiplied. So, alpha multiplied by minus of l n, 1 minus R raised to the power one by beta.

So, this way once you have to calculate the Weibull variates. And you are given the R is that is random numbers of uniform distribution. Then using the transformation, you have the parameter F alpha given you have the parameter beta given you have the R is being given once you know that you can get Xis. So, once you have R is on a on a column in the excel or so we can directly use a formula to calculate these Xis. So, this way the Weibull variates can be calculated. Now we will do the inverse transform technique for the triangular distribution. So, all it is done for triangular distribution.

Now, we know that in the case of triangular distribution, you have the limits or parameters is set between a b and c. So, you have 3 limits. So, here suppose the variate has the pdf that is F X equal to it will be varying between 0 to 1 and 1 to 2. So, it is X if X is between 0 and 1. Then it is 2 minus X if the X is more than 1 and less than equal to 2 and then it is 0 otherwise. So, this is the pdf for the triangular distribution. So, you will get the cumulative distribution function. In the cumulative distribution function, we can further write F X as 0 if it is less than 0 if X is less than equal to 0. It will be X square by 2 for 0 to X to 1 and it will be 1 minus 2 minus X square by 2 for 1 to X to 2.

So, 1 X is less more than 1 and less than 2 it will be 1 minus 2 minus X square upon 2. So, once we get that now you have the 2 places. You will have to have the expression put in and then that will be set equal to R. And then you will get the inverse transformation equation being used. So, what you do is you are for 0 to 1, you have to find. So, for X between 0 to 1 you have this formula. So, this will be F X that is equated to R. This will be X square by 2. So, if you do the universe transformation X will be equal to 2 R under root. So, X will be 2 R under root and R will be varying from 0 to 1 by 2.

So, X is varying from 0 to 1. So, R will be varying from 0 to 1 by 2. Similarly, when the X value is between 1 and 2 then you have this as the cdf. So, for 1 2 X is more than 1 and less than equal to 2. You have R equal to 1 minus 2 minus X square upon 2. So, now, again this equation is to be solved using the inverse transformation method. So, that from there you can say you get it as 2 minus X square will be equal to 2 into 1 minus R. So, 2 minus X will be. So, you can 2 minus X will be equal to under root 2 into 1 minus R. And X will be equal to 2 minus under root 2 into 1 minus R. So, this is the inverse transformation expression which tells that, once you know the R in a triangular distribution from there you can get the different values of Xis.

So, as the technique indicates that whichever distribution is given to you can always go for getting the cdf, first once you get the cdf then in that case from that cdf you are equating that cdf expression to that random number generated. And once you have that random number generator expression then you have to just solve for x. So, in the cdf the capital X will come which are the random variates which we have to find and the inverse equation will tell you the or the values which you get that will be the variates for that particular distribution. Now there is. This can method can be applied even to the discrete distributions even to the empirical distributions and so on.

So, this is the method which is used more popularly. Now there is another method which is used is the acceptance rejection technique. So, in the acceptance rejection technique what we do there is you have, suppose a uniform distribution in that you have to have the variates between 1 by 4 and 1. So, in that if you are basically predicting any number and if that falls in that range of 1 by 4 is to 11. You are going to accept it if not you are going to reject it.

So, that acceptance and rejection condition is given and it has to satisfy that condition if it satisfies, if it in that range we are going to take it and otherwise you are going to reject it. So, that is another technique which is also used you can read it from the standard book of either banks or from the book of flow and Kelton or any book standard book of modeling and simulation of discrete event systems.

Thank you very much.