Modeling & Simulation of Discrete Event Systems
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Lecture - 14
Simulation of a Single Sever Queueing System

Welcome to the lecture on simulation of single server queueing system. So, in the last lectures, we have discussed about the basic you know derivations related to single channel queueing system. Now, we will see that how the simulation proceeds before that, in this lecture, we will see that how we will find the measures of performances and how they are basically represented in the graphical terms.

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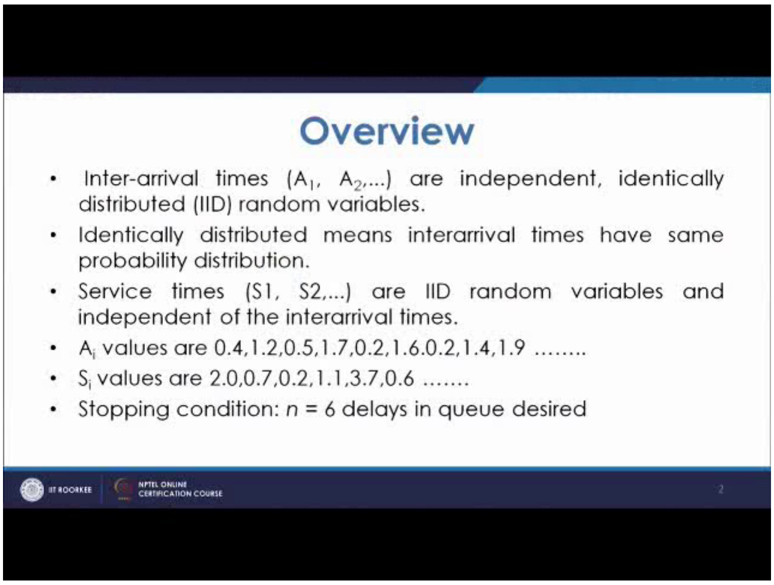

So, let us see as we know that the inter arrival times are independent and identically distributed random variables. So, they are known as I I Ds.

As we know that these inter arrival times, they are you know exponentially, distributed and they are Poisson, you know distributed or they are Poisson arrivals, if we talk about the number of arrivals in time T and then time D t, also then you have, similarly, identically distributed and independent random variables, that is service times, also now, what does it mean, it means that they have same probability distribution, then we will discuss for a case, for the problem, where the A i values are given, inter arrival times are given and we will try to find the certain measures of performance. How to find it? How to represent it on? You know on a paper and how to find like average number of customers in the system or in the queue like average delays, how can be computed? How it can be computed or how can we also find that, what is the percentage utilization of the server?

So, all that is basically computed, if you have these two data, now as we discussed that before the simulation proceeds, we will have a condition, which will tell that when the simulation has to stop. So, that is stopping condition. So, in this condition we are going to have the condition like whenever the 6th customer comes into the system, then we are going to stop it. So, when there is 6 delays in the queue. So, I am, I am 5, services are over and the 6 customer comes into the queue. So, he has completed his delay. So, that is basically, the stopping condition.

So, at that time, you will stop it and we will see that what will be the measures of performances, how we compute it.

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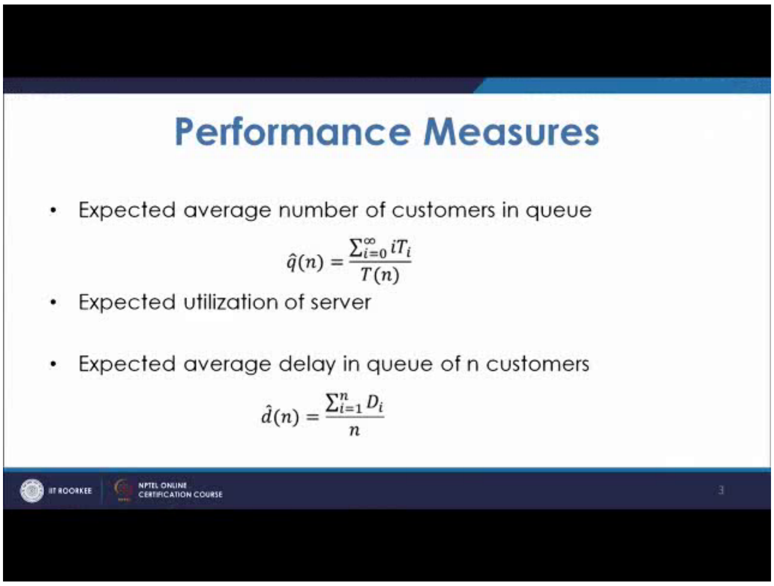

Now, the performance measures, which are normally of interest, one is the expected average number of customers, in the queue. So, it is basically queue, if that is estimator n. So, this is steady state value basically that. So, that having n customers in the queue and it is basically, summation of you know i varying from 0 to infinity i T i upon T n. So, it is in that time frame T n then expected utilization of the server, similarly, we can find the expected average delay in queue of n customers. So, that can be again found by summation i on to n D i upon n.

So, this way you know expected average delay of n customers, in queue can be found, you know these are you know, we can have the time integrated averages. So, time integrated averages sometimes. So, if we take it will be some integration integral of 0 to infinity iTi. So, basically, time integrated average that way once, we use the integration, a sin that basically tells you when we get any graph or any shape the area under that curve that tells. So, that even tells you more accurately, the values here, you have the discrete points from here, you are calculating the values. So, let us see that there is a problem which is given where the arrival is occurring at you know inter arrival time is given.

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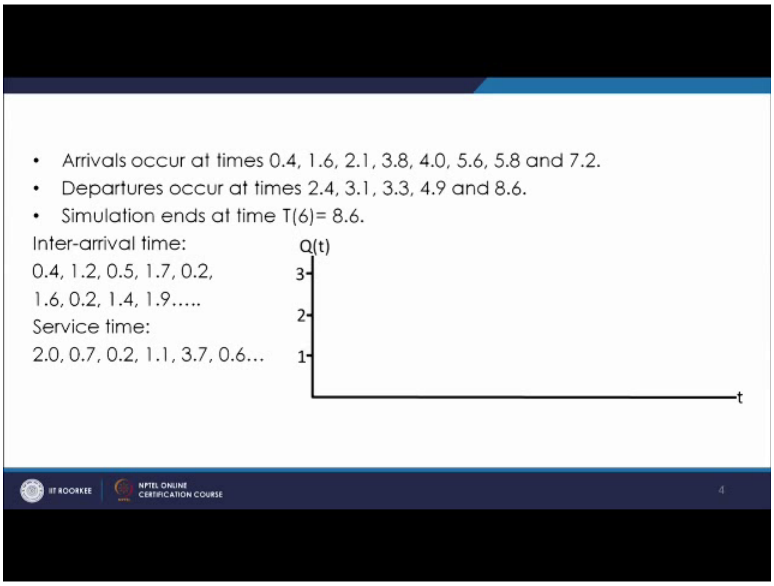

So, the thing, which will be given to you will be this, 0.4, 1.2, 0.5, 1.7, 0.2, 1.6, 0.2, 1.4, 1.9. So, these are the interval times, which are given, means the first customer will come at a 0.4 time. So, when, if we start the simulation at 0 time, the first customer is going to come at 0.4 time, then the time between the first and the second customer which is 1.2.

So, certainly the second customer is going to come at 1.6. So, in the case of arrivals, there is no, you know not much of complexity. Simply, you have to add. So, you will get the time of arrival of the customers. So, you are getting 0.4 1.6 to 0.1, then 3.8 4.0 5.6 5.8 7.2 and so on. So, this way you are going to get the different arrival time, anyway, then this next is next you are given the service time. So, service time again as you know, it is are the IID variables, random variables.

So, for the first customer the service time is 2. So, this customer I mean first customer, is going to take 2 minutes of time for it, service the units, if they are, may you know assume to be in minutes. So, it will take 2 minutes of time. So, the first customer, we will leave the queue at 2.4, then the second customer, when it will leave the queue. So, second customer. So, as he leaves the first customer, which is going to join the queue, there are first customer; however, he comes at 1.6, but he is going to join the queue at 2.4.

So, he will take 0.7 time, minutes of time, to complete 2 2 2, I mean, his service will take about 0.7 minutes of time. So, he joins the queue at 2.4, the second customer and leaves at 3.1, because 0.7 is the time taken. Now, at 3.1. So, now, at 3.1, further the time of service is 0.2. So, it will be 3.3. Now, from 3.3, 2.0, 3.3, 8.0. We see that there is no customer.

So, then further, it will take 1.1 minute. So, it will be 4.9 and then lastly, it will go to 8.6. So, it will be take 3.7 minutes. So, 8.6; so, this way what we see is that you have the different, you know events occurring, you have 2 types of events, one is arrival, another is departure. So, whenever there is arrival, you know there will be queue formation, when they will be departure, the number of customers, in the queue will decrease by 1. So, we will find the number of customers in queue. In queue means as he goes on to the service counter, he is not in the queue. So, he is getting the service, he is said to be in the system not in the queue. So, as long, he is not getting the service, he is in the queue.

So, if we see, we have, we will see by drawing the graph and try to see, how we can get these values. Now, let us draw it on a line.

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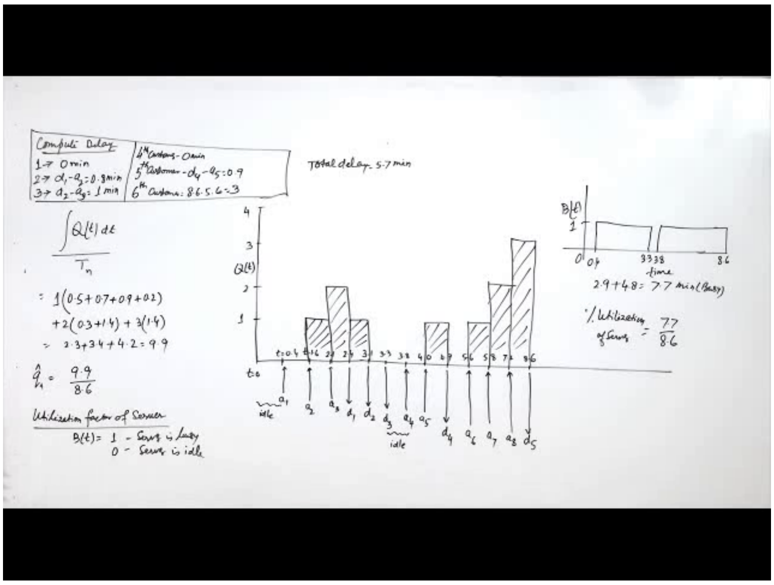

So, let us see this line now on this line, what we see is if we take it as, this is as 0 t equal to 0. Now, the arrival is coming first, arrival is coming at time 0.4. So, this is t equal to 0.4 and here, you have 1 arrival. So, this is the arrival and we tell it as a1. So, this is arrival 1.

First arrive customer, arrives at time 0.4, again next is the arrival. So, second customer comes at time 1.6. So, the second event, which occurs is at t equal to 1.6 and then here again you have arrival. So, this is the second event and it is a2, then now the next event. So, next again, we see that the arrival is at 2.1. So, the next event will be at 2.1 and it will be again another arrival.

So, it will be a3. Now, after a3, what we see the service of. So, the customer who comes at 0.4, he gets the service and goes at time 2.4. So, after this you will have the event at time 2.4 and here you have the event departure. Now, what we see is that once we come at this point, you have one customer in the system, you are coming here you have two customer in the system.

However at this point you have one customer in the queue. Similarly, at this point you have three customer in the system, but two customer in the queue and at this point you will have two customers in the system and one customer again in the queue, because there is another departure. So, we are seeing these events. Now, next event is at 3.1, which is further the departure after that you will have 3.3, whether you have further the departure. Now, we can see from here that you have 0.4 1.6 2.1 then 2.4 3.1 and 3.3.

So, the three customers who have come, they have all left by the time 3.3, the next arrival is at 3.8. So, from the time 3.3 to 3.8 there is no customer in the system, leave aside the queue, there is no customer in the system. So, next time is 3.8 and here you have the arrival of the fourth customer that is a 4. So, what we see is from 3.3 to 3.8 you have the server idle 3.3 to 3.8

Next you have again the arrival at 4. So, what we see here again by looking at this, you have the arrival. So, you have the arrival at 3.8, and then further arrival at fifth arrival is at 4 time of 4. Now we have seen that there is a break here, the three customers will have come all together, they have left the system. The fourth customer has come at 3.8 time and for the forth customer, the service time is 1.1.

So, the customer who is coming at 3.8 he is going to leave at 4.9. Whereas, the next arrival is 5.6. So, out of 5.6 and 4.9, 4.9 being less, there will be a departure further. So, at 4.9 you have a departure, and this will be of the fourth customer. So, that is departure. Now the next eminent event which is going to occur is, because you have the fourth customer departed and the fifth customer has joined the queue at 4.9.

Now, for the fifth customer that time for the service is 3.7 minutes. So, it means he will be leaving the system at 4.9 plus 3.7 8.6 minutes, before that you have many arrivals. So, you have arrival; that is sixth arrival at time 5.6. So, this is a 6, then next arrival is at 5.8. So, a 7, further a 7 is 5.8 and if 8 will be seen 0.2, then the next event which is going to come will be the departure of the fifth customer.

So, it will be at 8.6. So, it will be the departure of the fifth customer and as soon as the fifth customer departs, the sixth customer is going to be in the queue. So, that is the stopping condition, where the simulation is automatically going to stop. So, this is how you can see that you have different times at which you have, you know different events occurring. Now see, how it is the, how the queue number of customers in the queue is changing. So, the number of customers in the queue will be changing, as if we know that the time starts from 0 at 0.4, one customer comes into the system.

But as the server is idle. So, he has not to it. So, he is not in the queue. Basically, he does not have, he does not experience any delay, he immediately joins the queue. So, the queue length is basically, if you take Q t number of persons in the queue and if it is 1 2 and 3 . So, if it is like this, if your problem is like this, you have this is Q t, in that case this 0 goes up to this point also and when it comes here at 1.6, the number person in the queue. So, the if it is 1 and this is 2 and that is 3.

So, in that case at 1.6, it becomes 1. Now, at 2.1, you have further one arrival. So, it will go to 2 , but at 2.4, you have another departure. So, at 2.4, it becomes again 1 less and then at 3.3, it becomes further. So, from 3.1 to 3.3, you have a customer in the system, but not, but he is getting the service. So, Q t is basically, 0 from 3.1 to 3.3 from 3.3 to 3.8, the again queue is idle.

So, you will have the value Q t as 0 at 3.8, one person comes in the queue . So, again it will be 0 up to 4.9, but then at 4.9, the person who comes at 4, he leaves the system. So, you will have again 0 in this range. So, again you go to time 5.6. Now, at 5.6. So, now, let us see at 3.8, no there was a mistake at 3.8, the customer who comes at 4. So, at 3.8 to 4, you have one customer in the system, but Q t is 0, because he is getting the service. So, at 4, there is the next event is arrival.

So, from 4 to 4.9, you will have one customer in the queue, because there is next arrival here. So, the queue length will go to 1 and that remains as 1 up to this point. Now, at this point, this customer a4 that d4 is living and again. It will be 0 in this case. So, the queue is not empty, the server is not idle, but the person in queue is not there Q t is not you know 0, I mean Q t will be 0, there will be no person waiting. So, in the system, there will be low person system, there will be one person and no person, we will be waiting for getting the service.

So, at 5.6, again person comes there is one arrival. So, at this point you will have further from 5.8. So, now, the person who is departing at 4.9 10. So, a4 coming and a5 who is coming. So, he will be getting and then this person, if I will be getting the service. So, at a6 again it will go as 1, the person was there in the queue, this a5 was there in the queue. So, this person and another arrival will bring this Q t value 2 1. This will further go and increase to 2 and that will further increase to 3 and once the person departs at this point, the simulation will stop.

So, this is how, this Q t diagram 2 t versus t, this diagram indicates. So, integral Q t D t will be the area under this curve. Now, if you try to find average number of customers in the queue. So, in that case it will be the area under this curve Q t D t upon total time. So, you can find this values and you can further find other values like the delay. Now, if you try to see here, what you see is you have in the cyst in the queue. You have 0 up to this point, then you have 1, if you see it will be from 2.1 minus 1.6 plus 3.1 minus 2.4 plus 4.9 minus 4 plus 5.8 minus 5.6.

So, it will be 1 into that. So, this way then 2 into this time limit 2.4 minus 2.1 plus 7.2 minus 5.8 and then you have 3 times 3 into 8.6 minus 7.2. So, these are the values. So, what we see is you have the expressions and the expressions can be used to calculate this Q t D t. So, you have the Q t Q t D t upon T n. So, T n is 8.6. So, what we see, is average number of customers, in the queue will be integral Q t, D t upon T n.

So, if you try to find that it will be you know 1, 1 is from 2.1 2.0 1.6 that is 0.5 plus 3.1 to 2.4 that is 0.7 plus 4.9, 2.4. So, 0.9 plus 0.2 plus 2 times, 2.4 minus 2.1 that is 0.3 plus 7.2 minus 5.8; so, 1.4 plus 3 times, 8.6 minus 7.2 that is 1.4; so, it will be 2.3 plus 3.4 plus 4.2 9.9. So, we get. So, if you look at that it will be 8 point.

So, this is 9.9. So, this value will be, it will be 9.9 0.9 upon 8.6 T n is 8.6. Similarly, if you want to find the utilization factor of the server. So, if the utilization factor of the server is to be found out. So, this is calculated by assuming a function known as B t, busy time function. So, you have value of B t as 1, if server is busy and 0, if server is idle. So, percentage time utilization of the server will be integral B t D t upon T n. So, if you take integral of Bt Dt, now you see the value, if you find Bt in this domain, what we see is the server is idle only up to this time, then you have idleness here and then the server is busy.

So, in this case you will have server busy from 0.4 2.0 3.3 and then again from 3.8 to 8.6 , if you find the percentage of the time, the server is utilized, it will be. So, B t D t, B t is 1. So, it will be 3.3 minus 0.4. So, 2.9 plus 8.6 minus 3.8. So, it will be 4.8. So, 8.7 minutes, busy out of 8.6 times. So, it is not, it is 7.7, sorry, this is 7.7 minutes.

So, percentage utilization of the server, it will be 7.7 divided by 8.6. So, this way using this, busy time function, you can find the percentage of the time for which the server is utilized. Similarly, another parameter, which you can find is the delay, which is caused or delay, which is observed by the customers. Now, if you try to find the delays compute the delay . So, what we see is delay, means the time for which the customer has to wait before he goes into the service.

Now, first for first customer, he has 0. Now, our second customer, the second customer once he comes and when he joins the queue in that time, that is at that time difference between them, that will be the delay time for the second. So, for second customer comes at a2 and he will join, only when the first customer departs. So, it will be d1 minus a2. So, for second customer, it will be d1 minus a2. So, it will be d1 is 2.4 and a2 is 1.6 d1 is 2.4 a2 is 1. It will be 0.8. Similarly, third customer, third customer is coming at 2.1 and second customer leaves at 3.1.

So, it will be d2 minus a3. So, it will be d2 is 3.1 and a3 is 2.1. So, 1 minute, similarly, fourth customer. Now, we are finding the delays of every customer and for how much time, he gets delayed for fourth customer, it will be. So, for fourth customer basically, he comes at this time and this was idle. So, for him there is 0 delay. So, for fourth customer, it is 0 minute for forth customer, fifth customer will be departure of the fourth customer minus arrival of the fifth customer departure of the fourth customer is 4.9 arrival of the fourth customer fifth customer is 4.

So, it will be 4.9 and for the sixth customer again, he comes at time of 5.6 and he joins at 8.6. So, 8.6 minus 5.6 that is 3. So, if you see the total delay, which is observed will be the summation of all these values, 0.8 plus 1. So, that is 1.8 plus 0.9 2.7 plus 3.0 5.7. So, 5.7 minute is the total delay. So, total delay is 5.7 minutes. So, per customer, if it is average delay per customer, it will be then 5.7 divided by 6. So, that will be the average delay and that can be computed. So, this way all these. So, performance measures values can be computed.

Thank you very much.