Modeling & Simulation of Discrete Event Systems
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Lecture - 10
Problem Solving on Statistical Models in Simulation

Welcome to the lecture on Problem Solving on Statistical Models in Simulation. So far, we have discussed about various kinds of discrete and continuous distributions. And as we know that the processes will come across many kinds of events which has to follow either continuous or discrete type of distribution function and you have to calculate the different values. So, we will discuss few problems and we will just have an idea how to solve these problems.

So, let us come to first problem.

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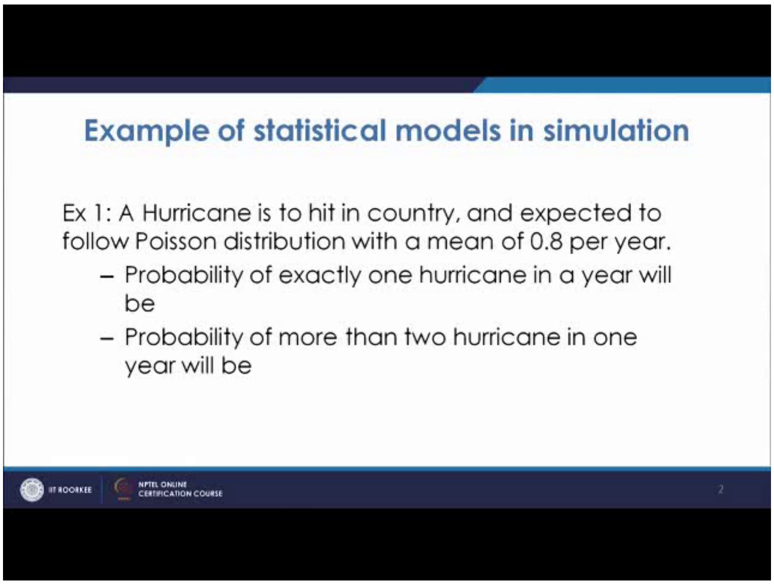

The first problem is regarding the hurricane which is to hit in a country. And it is expected to follow Poisson distribution with mean of 0.8 per year. So, you have to find the probability of exactly one hurricane in a year and then probability of more than two hurricanes in a year. So, in the one case first case you have to simply find f x in the second case you may use the cumulative distribution function value and then you can find the; you know value of having more than two hurricanes in the year.

So, if you take the first case the value of having one hurricane probability of exactly one hurricane in the year.

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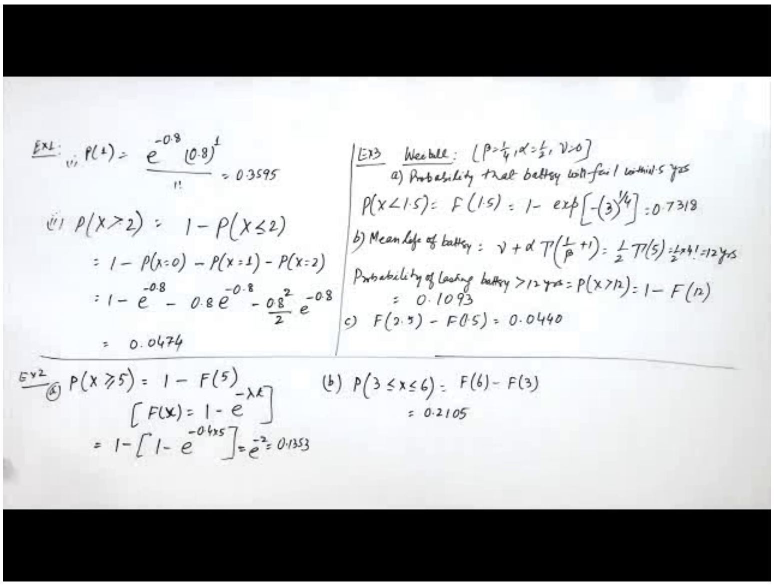

So, in that case, it will be; so, it will our solving example one and that will be simply by putting the value in I mean in that. So, you have to have one. So, p one you have to find and it will be p one. So, as we know this is Poisson distributed and you know the values e raise to the power minus alpha and. So, it will be e raised to the power minus 0.8 minus alpha; alpha raise to the power x. So, 0.8 raise to the power 1 and divided by x factorial. So, so this is one factorial and that will be given as 0 point 3 5 nine 5

So, that is how you find the probability of having the hurricane exactly one hurricane in the year now probability of more than 2 hurricanes. So, you have to find the probability when hurricane value is more than 2. So, it will be nothing, but 1 minus probability of hurricane less than equal to 2. So, either you have no hurricane I mean that is value of 0 or 1 or 2. So, in that case you will have 1 minus p when x is 0, then p when x is one and p x as 2.

So, this is how you can find the probability of having more than 2 hurricanes in a year and if you compute these values it will be 1 minus e raise to the power minus 0.8 then further 0.8 into e raise to the power minus 0.8 and then 0.8 square upon 2 factorial. So, it will be 2 and e raise to the power minus 0.8. So, it is solved these 2 p one will be this and p 2 will be this. So, this if you compute this value comes out to be 0.0474. So, this is how this kind of problems can be solved.

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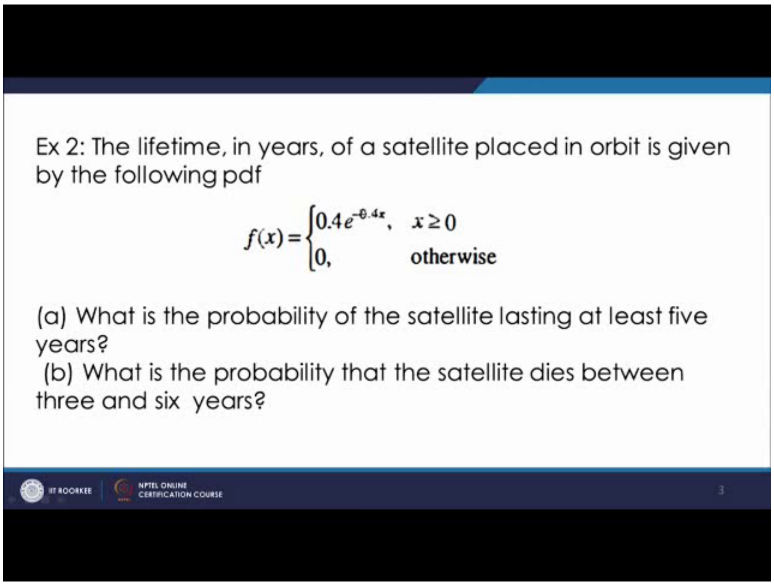

Next problem is the life time in years of a satellite placed in orbit is given by the following pdf f x is given as 0.4 e raise to the power minus 0.4 x when x is more than equal to 0 and 0 otherwise. So, here you have to find the probability of the satellite lasting at least 5 years now in this case the probability is to be found for at least 5 years, it has it should you know the stay and for that probability x more than equal to 5, I means it must be. So, it is written that at least 5 years means 5 years or more now for that you will have 1 minus f of 5.

So, f 5 will be up to 5 years now more than; that means, it will be 1 minus f of 5 and as we know f x is defined as 1 minus e raise to the power minus lambda t you have been given the expression 0.4 as the lambda; lambda e raise to the power minus lambda t. So, that is how your lambda is defined. So, this value will come out to be 1 minus 1 minus e raise to the power minus 0.4 into t is 5 years. So, it will be e raised to the power minus 2. So, it will be e raise to the power minus 2 that is 0.1353. So, this is how you can find the probability of the satellite lasting at least 5 years.

Now, what is the probability that the satellite dies between 3 and 6 years. So, for that you have to find the cumulative function value at 6 and cumulative function value at 3 and then you have to subtract it. So, if you try to find. So, this is a part and the b part will be.

So, b part will be F 6; capital F 6 minus f 3 or you can write it in terms of probability value. So, probability value when its life is 3 to 6 it will be f 6 minus f 3. So, that is 1 minus e raised to the power minus lambda t.

So, it will be 1 minus e raise to the power minus lambda into 6 minus 1 minus minus of in bracket 1 minus e raise to the power minus 3 lambda. So, that way it will be and it will be coming as 0.2105. So, once you calculate these values on calculation you will get the value of point 2 one 0 5. So, that is how you can compute the problems I mean you can solve these kinds of problems.

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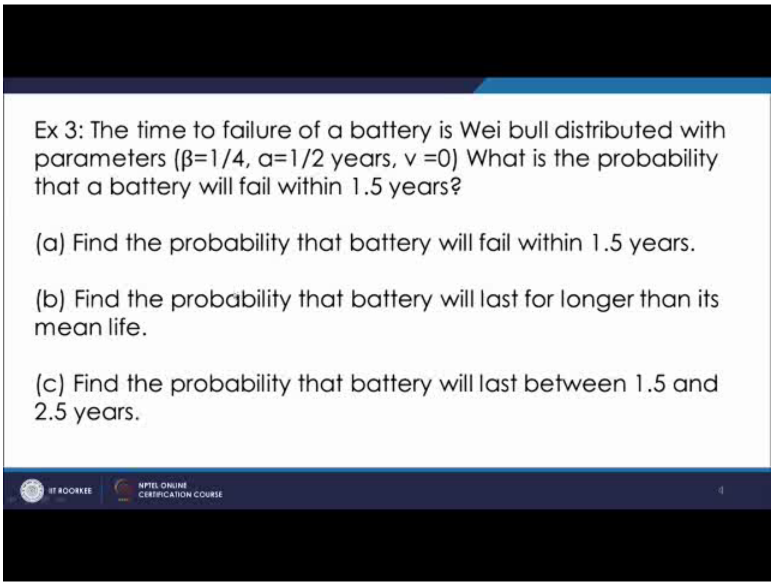

The next problem is the time to failure of a battery is Wei bull distributed with parameters beta is 1 by 4 alpha is 1 by 2 years and nu is 0. So, this is another distribution that is we have already discussed about it Wei bull distribution which has 3 parameters alpha beta and nu; nu is given as 0. So, it will start from 0 and then beta 1 by 4 and alpha is 1 by 2 years. Now what is the probability that a battery will fail within you know 1.5 years. So, find the probability that battery will fail within 1.5 years. So, in that case as we know for the Wei bull distribution function the expression for f x is beta by alpha x minus nu by alpha. So, when nu is not there x by alpha raised to the power beta minus 1 into exponential of minus of x by alpha raised to the power beta. So, that is how the f x is calculated.

Now, find the probability that battery will fail within 1.5 years. So, it means the probability you have to find. So, this is example 3. So, within one point 3 years means it will talk about the cumulative distribution function value up to 1.5 years. Now for that as we know the cumulative for Wei bull, we have already discussed the cumulative distribution function value f x will be 1 minus exponential minus of x minus nu by alpha raise to the power beta. So, that is how the cumulative probability can be found out.

Now, in this case what we have is probability that battery will fail within 5 years within 1.5 years. So, it will be probability x less than 1.5. So, that is nothing, but f value 1.5 and that value will be computed as 1 minus exponential and in that case you have minus of x by alpha raised to the power beta. So, x is 1.5. So, it will be minus of 1.5 by alpha. So, alpha is in that case 1 by 2; so 1.5 by 1 by 2.

So, 1 by 2 that is 3; so, 3 raised to the power beta and beta value is 1 by 4. So, 3 raise to the power 1 by 4. So, this is how. So, if you solve this you will get the value as 0.7318. So, things are clearing this you know alpha you know beta you know nu you must know the formula and in the formula the cumulative function for f x will be 1 minus exponential minus of x minus nu by alpha raise to the power beta. So, nu is 0 here. So, x by alpha raised to the power beta x is 1.5 and beta is 1 by 2, I mean alpha is 1 by 2. So, an 1.5 by 0.5 that is 3 raise to the power beta that is 1 by 4 and you get this value. So, the probability that the battery will fail within 1.5 years will be you know 0.7318.

Then b part is the b part is to find the probability that battery will last for longer than its mean life. So, for that you have to first find the mean life of the battery. So, the mean life of the battery will be calculated using the expression for mean. So, mean life of battery it is calculated. So, its expression is it is given as nu plus alpha gamma of 1 by beta plus one now this is the mean life of the battery beta is 1 by 4 and alpha is 1 by 2. So, beta 1 by 4 alpha is 1 by 2 and nu is 0 that is how the values are given if you find the mean life of the battery nu is 0 then it comes alpha; alpha is 1 by 2 and gamma of 1 by beta 4 plus 1. So, 5; so, that is 1 by 2 of 4 factorial. So, this is 12 years. So, the mean life will be 12 years

Now, the probability that battery will last for longer than its mean life; so, battery has to has the life of more than 12 years. So, probability of lasting battery more than 12 years that is p x is more than 12. So, that will be 1 minus p x is less than equal to 12 and that will be basically the cumulative distribution function. So, it will be 1 minus cumulative function of the value x equal to 12. So, it is, but 1 minus p x is less than equal to 12 and x is less than equal to 12 up to that that will be F, I mean capital F value that is cumulative value.

So, that is 1 minus f of 12 and once we get the f of 12 and then you can get this value. So, this is we computed further and it is coming out to be 0.1093. So, f of 12 can be computed by 1 minus exponential minus of you know x by alpha; so 12 by alpha and then on that. So, 12 by alpha 24 raise to the power 1 by 4. So, this way the value can be computed and that comes out to be 0.1093. So, this way this probability of battery which is will last more than its mean life will be 0.1093.

The last part is the find the probability that battery will last between 1.5 and 2.5 years again such problems can be solved by finding the cumulative distribution function value at the 2 x 2 points that is 2.5 and 1.5. So, f of capital F of 2.5 minus capital F of 1.5; so c value will be capital F of 2.5 minus capital F of 1.5 means the battery will last up to 1.5 years it will be f capital F 1.5 battery will last up to 2 point years it will be capital F or cdf function value of 2.5 for the Wei bull distribution. So, in between cdf will be I mean the life will be.

So, the probability value will be f of 2.5 minus f of 1.5 and if you compute this value it is coming as 0.0440. So, this you can further solve I mean you can find this by placing the value x as 2.5 in the expression in x as 1.5 in the expression and then further subtracting it and getting the values. So, this way you can solve such problems.

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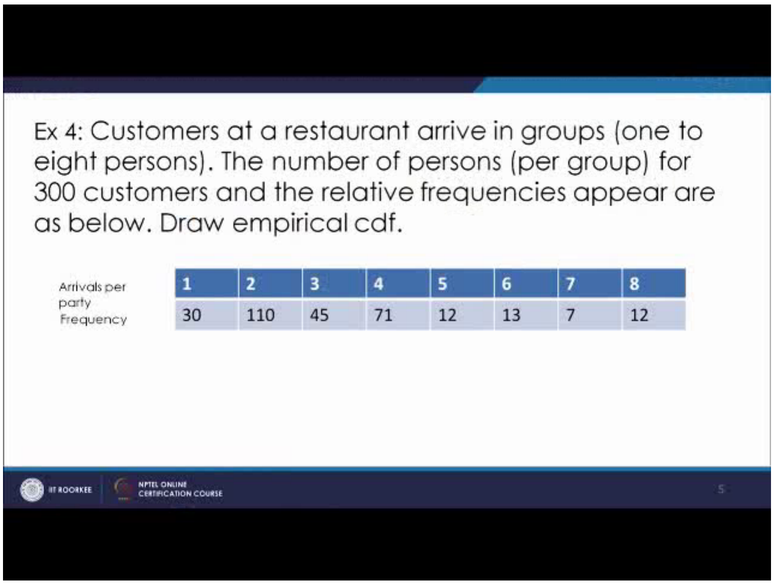

Next problem is related to the empirical distribution function. So, as we discussed you may face certain data which are not necessarily you know matching with certain standard distribution functions. So, then they are known as empirical distribution functions you have certain data from there you have to find the histogram you have to find the frequencies you have to find the cumulative frequencies. And then you have to find the graph and then maybe you can see there; how much it is matching or how it looks. So, such are such types of analysis are done using this empirical distribution function. So, here the in the empirical distribution function you have the case where the customers at a restaurant arrive in groups.

So, customers at a restaurant are arriving either alone or with 2 persons or 3 persons or 4 persons may be up to 8 persons it may go they may go in the groups and I mean it has been seen that they are 3, I mean there has been cases for 300 persons and it was seen that for 2 persons. So, 55 times it was seen that with one person 30 numbers of occasions were seen with 2 persons 110 and so, for the last 300 you know for 300 customers it was seen and for 2 it was 1. So, for 2 batch of you know 2; we have we are seen to go 55 number of times a total one 10 customers went in the group of 2 in the group of 3 total 45 customers have gone in a group of four.

So, this way you have a case and you and it was seen that you have this many cases. So, I mean as we see you have 1 percent group 30 times 2 person group, 110 times 3 persons

45; 4 persons 71 or so there are you have 300 groups which are there and this is altogether coming out to be 300 and for that now suppose you have to find the empirical cdf for such cases. So, for such cases what you have to do is you have to make a table and the table will talk about the relative frequency the cumulative frequency and then finally, you can draw these you know gurf; I mean graph. So, for such cases as we will see how can we draw the graph. So, what we see is you have arrivals per party.

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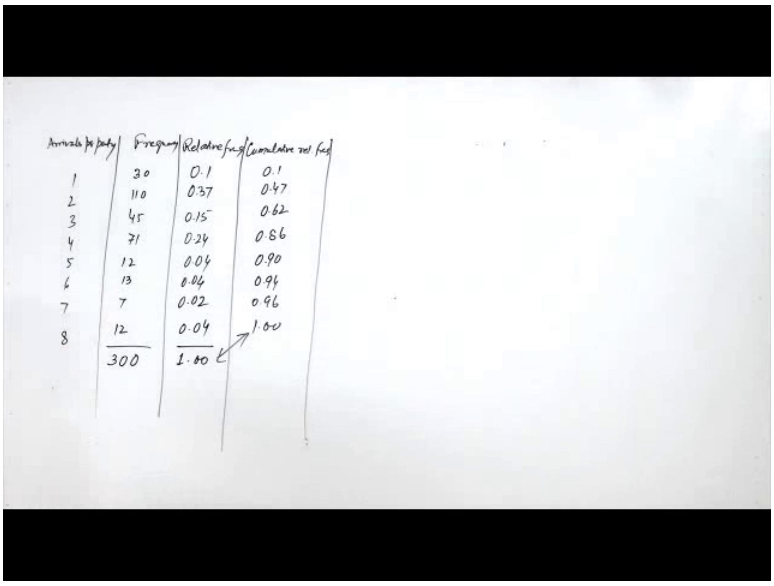

Arrivals per party	Frequency	Relative freq	Cumulative rel. freq
1	30	0.1	0.1
2	110	0.37	0.47
3	45	0.15	0.62
4	71	0.24	0.86
5	12	0.04	0.90
6	13	0.04	0.94
7	7	0.02	0.96
8	12	0.04	1.00
	300	1.00	

So, it is varying from 1 to 8; 1, 2, 3, 4, 5, 6, 7 and 8, then the frequency is given frequency is given as 30, 110, 45, 71, 30, 110, 45, 71 and then further 12, 13, 7 and 12.

So, basically mystically, it was said that you have 55 groups or so, it is not that you have we have discussed about the groups which are coming. So, 2 persons per party and they are coming 100 times, 10 times; similarly 3 persons coming; they are coming 45 times. So, all together it is coming out to be 14 plus 4, 18, 25, 26, 27. So, this way 28 and it will be coming as 300. So, it will be coming as 300. So, you have 300; 300 observations have been you know captured and the 8 number group has been seen 12 times 7 number group have been 7 times 6 number group have been 13 times like that. So, the 2 number group has been seen hundred 10 times.

Now, this is the frequency. So, we will talk about the relative frequency relative frequency will be thirty by 300. So, it will be 0.1. Similarly, it will be 0.37; it will be 0.15, it will be 0.24, it will be 0.04, it will be again 0.04, it will be 0.02 and it will be

0.04. So, altogether it comes out to be one how it will come as one that you can see from the cumulative relative frequency cumulative relative frequency and if you compute that it will be point one it will be 0.47, it will be 0.62, it will be 0.86, it will be 0.90, it will be 0.94, it will be 0.96 and this will be 1. So, this way you see that the cumulative distribution function cumulative relative frequency can be calculated.

Now, based on that you have to find the graph; so, you can find first the histogram now if you find that what you see is now we have to find the histogram which will talk about the relative frequency and which will also from where there will calculate the cumulative you know. So, cumulative frequency anyway is not required in that case. So, once we try it. So, let us try to find that histogram.

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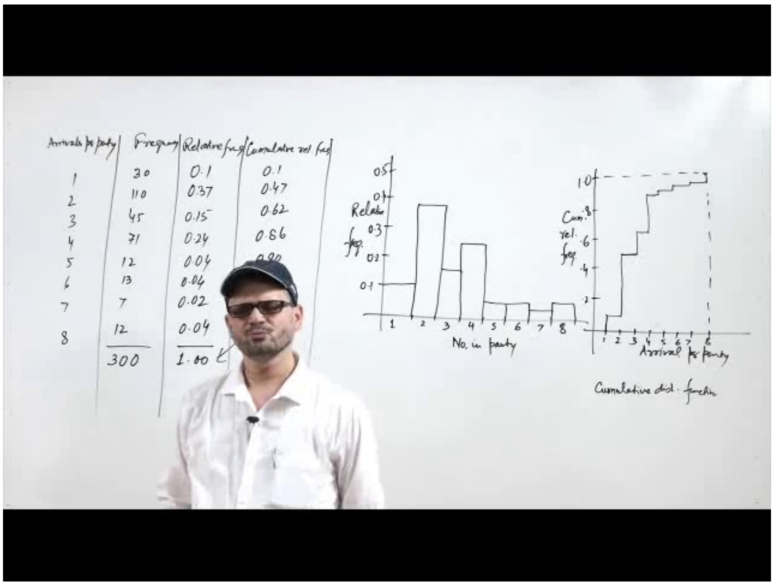

And for that you have in y axis you have relative frequency which will be the value which is occurring in this table and in this you have number in a party or in a group.

So, in the first one; so this will be one this will be 2 this will be 3 this will be 4, 5, 6, 7 and 8. Now further you have probability values as 0.1, 0.2, 0.3, 0.4 and 0.5. So, what we see is for the person who are going alone they are relative frequencies point one. So, it will come as this then with 2, it is 0.37. So, it will be here 0.37 will be somewhere here it will be matching with 2. So, this will talk about the people who are in the party in a group of 2, they have been going for hundred 10 times whose relative frequency is point 3 7 then for 3 it is 0.15. So, this will be point one 5 and it will be 3.

Similarly, for 4; it is 0.24. So, 0.24 will be again somewhere here for 4 for 5; it is 0.04. So, now, it is less 0.04. So, this is for 5 for 6 again it is 0.04. So, it will be again 0.04; 7 is 0.02. So, it will be further here and then for 8 it will be 0.04. So, this is how you can compute the probability mass function for such empirical cases as empirical distribution function then you have to find the cumulative distribution function that will be known as empirical cumulative distribution function. So, for that you have again arrival per party. So, for that you have arrival per party. So, this is cumulative distribution function. So, this is arrival per party and you have cumulative relative frequency. So, in this you will have 0.2, 0.4, 0.6, 0.8, and ultimately you have to go up to one in this case.

So, being the discrete type of distribution function you will have the discrete values 1, 2, 3, 4, 5, 6, 7 and 8 so on; one it is 0.1. So, we will go up to 0.1, then on 2, it is going to 0.47. So, on 2 it will go to. So, this is 1, 2, 3, 4, 5, 6, 7, 8 on 2 it is going 2.47, it is coming like here, then on 3 it goes up to 0.62. So, here again it will go to 0.62, then further on 4, it is going up to 0.86. So, further it will go up to 0.86, then on 5, it will go up to 0.90. So, on 5 it will go to 0.90 and then 0.946, it will be 0.947 will be 0.96 and 8 will be ultimately one. So, this will be one.

So, this is the cumulative distribution function for such empirical distribution functions similarly you may be dealing with cases where there may be a situation when there are certain frequencies given for certain intervals maybe some of the; in some of the cases in the time event itself in the time interval itself number of failures which occur maybe from 0 to 1 minute, 1 to 2 minutes, 2 to 3 minutes and the number of failures are given. So, in that case such are the cases of continuous distribution functions in those cases those in the time limits you have go in the order of increasing time interval I mean increasing time itself the interval anyway is specified. So, the frequency is given relative frequency can be calculated than cumulative relative frequency can be calculated.

Based on that you will have a continuous type of distribution function; so, here you have the discrete kind of distribution function whereas, in those cases you will have the ranges here and in the ranges you will have a graph which you will have some value. And in that case you will have a continuous kind of graph which will talk about the continuous cumulative distribution function for such cases. So, this way in the case of cumulative distribution functions also you can solve the problems.

So, we have discussed about few of the distribution function examples there can be many cases of many cases we may have the cases in case of Bernoulli distribution, we may have the cases related to gamma distribution where you have the parameters told and the event maybe told to follow the gamma distribution. So, you may have to find the desired you know parameters or desired values and you can use those respectable values for finding those things. So, then this is how you compute or you go on computing the parameters required in such cases.

Thank you.