

Steam and Gas Power Systems
Prof. Ravi Kumar
Department of Mechanical & Industrial Engineering
Indian Institute of Technology – Roorkee

Module No# 06
Lecture No # 25
Problem Solving (Impulse Steam Turbine)

I welcome you all in this course on this steam and gas power systems. Today we will solve numerical's on impulse turbine.

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For a single stage 3000 rpm impulse turbine, the mean diameter of the blades is 105 cm. The nozzle angle is 20° , the ratio of blade speed to steam speed is 0.45 and the ratio of the relative velocity at outlet from the blades to that at inlet is 0.85. The outlet angle of the blade 3° less than the inlet angle. The mass flow rate of steam is 7 kg/s. Draw the velocity diagram for the blades and find:

- tangential thrust on the blades
- axial thrust on the blades
- power developed in the blades
- blade efficiency
- resultant thrust on the blades

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$N = 3000 \text{ rpm}$
 $D = 105 \text{ cm}$
 $\alpha_1 = 20^\circ$
 $\frac{u}{v_1} = 0.45$
 $\frac{v_2}{v_1} = 0.85$
 $\beta_2 = \beta_1 - 3$

The diagram illustrates the velocity triangles for an impulse turbine. It shows the inlet velocity triangle with absolute velocity v_1 at angle α_1 to the axial direction, blade velocity u , and relative velocity v_{r1} . The outlet velocity triangle shows relative velocity v_{r2} at angle β_2 to the axial direction, blade velocity u , and absolute velocity v_2 . A schematic of the turbine shows the mean diameter D and rotation at 3000 rpm . A small diagram shows the blade profile with a curved leading edge.

This is first numerical which states that for a single stage 3000 RPM impulse turbine. So there is an impulse turbine with $RPM = 3000$. The mean diameter of the blade is 105 centimeters. So the diameter is 105 centimeter.

In turbines, the blades are fixed on the surface of the rotor. If you look at the end view the blades will look like this. When the steam is flowing perpendicular to the direction of this port. Now if you look at here, why it is stated that the mean blade diameter because this rotor is moving in certain speed that is 3000 RPM, right. So when we are moving in radial direction, the peripheral velocity is changing.

In velocity diagrams, if you look at, we have always considered mean diameter. But in actual practice, this U is changing when you are moving along the radius of the rotor. Or if you draw a velocity diagram, for impulse turbine this is absolute velocity V_1 , blade inlet angle sorry, nozzle inlet angle, this is blade inlet angle, u . This is relative velocity at the entry now here, this is blade outlet angle, nozzle outlet angle, this is relative velocity at the exit, absolute velocity that is it.

So when the u changes, this is U peripheral velocity when there is a change in u , the diagram will change. So in fact we are going to have different velocity diagrams for different positions, right that is one thing. So the calculations are done for an average value of the diameter that is if this is D_1 , this is D_2 then average is going to be $D_1 + D_2$ by 2.

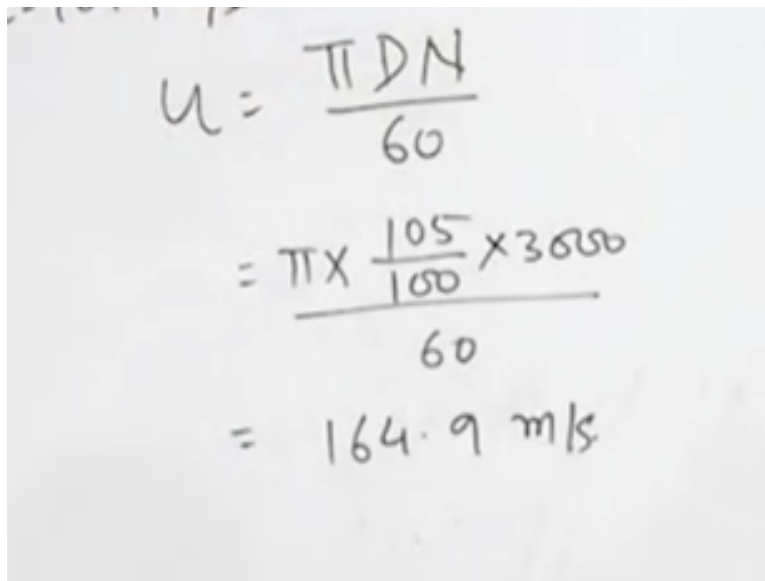
Now if we change the U , suppose u is increased if you change the U , same inlet velocity and blade inlet angle sorry the nozzle inlet angle, the blade angle will change. It means when we are moving in this direction, the blade inlet outlet angles will be altered. That is why the blades of the turbine are twisted. The blades of the turbines are twisted because we have to ensure that steam should glide over the blades, it should not strike the blades.

When the blade angle is changing in that case we will have to twist or these blades have to be twisted in order to ensure that there is a no shock entry or the steam simply glides over the blade surface. Which is prime requirement of a impulse turbine right. So here the mean blade speed is given 105 centimeters. The nozzle angle is 20 that is nozzle inlet angle α_1 is 20 degrees. The ratio of blade speed to steam speed is, steam speed means V_1 .

So this rho is 0.45 Rho is nothing but U by V1. So it is point four five ok and the ratio of the relative velocity at outlet from the blades to the inlet is 0.85. It means $VR2$ by $VR1 = 0.85$. The outlet angle of the blade is 3 degree less than the inlet angle. The mass flow rate of, so outlet angle $\beta_2 = \beta_1 - 3$. The mass flow rate of steam is 7 kgs per second. So mass flow rate is 7 kg per second.

Draw the velocity diagram for the blade and find, so first of all in this case we have to draw the velocity diagram. Now first of all with the help of these two informations we will calculate the average peripheral velocity.

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The image shows a handwritten calculation for the peripheral velocity U. The formula is $U = \frac{\pi D N}{60}$. The values are substituted as $U = \frac{\pi \times \frac{105}{100} \times 3000}{60}$, resulting in $U = 164.9 \text{ m/s}$.

So $U = \frac{\pi D N}{60}$ we have taken because this is RPM. So this RPM is converted into rounds per second. So it is going to be π into 105 divided by 100, converting into meter, into 3000 divided by 60 and U is going to be = 146.9 meters per second.

So this is the value of U is equal to we will put here U is equal to. Because we will be frequently needing this information. So I am noting it down here, 164.9 meters per second.

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$$\begin{aligned}
 X &= V_1 \cos \alpha_1 - u \\
 Y &= V_1 \sin \alpha_1 \\
 \tan \beta_1 &= \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} \\
 &= \frac{366.4 \sin 20}{366.4 \cos 20 - 164.9} \\
 &= 0.6985
 \end{aligned}$$

This is the value of U so in a velocity diagram we can always draw U and direction of U is this. Now blade inlet angle sorry this nozzle inlet angle is 20 degree. At 20 degree a line will be drawn and the length of this line is equivalent to V_1 , V_1 is not given ok but we have the value of ρ . $\rho = U$ by V_1 so $V_1 = \rho$ by, so sorry, U by $\rho = U$ by ρ .

And that is 164.9 divided by 0.45 it is 366.4 meters per second. So you V_1 we can also note it down, 366.4 meters per second. Now we have the value of absolute velocity of steam which is entering the blade.

This is nozzle inlet angle α_1 , and α_1 is given here 20 degree. So now geometrically we can draw this triangle because we have the value of U is 164.9. And this $V_1 = 366.4$ meters per second. So definitely we can draw, geometrically we can draw, this triangle. But normally we prefer analytical solutions because analytical solutions are more correct.

In geometrical solutions, the values are not that correct because we have to scale down, we have to take a uniform value of X for V_1 and U and then draw a triangle. So there is a possibility of error incurring the results so now this is VR_1 right. So in order to find that β_1 , in order to find β_1 we can always take sorry $V_1 = 366.4$.

Now this is X and this is Y so $X = V_1 \cos \alpha_1 - U$, V_1 projection in this direction $-U$ is going to give you X . And $Y = V_1 \sin \alpha_1$. We know we have all the values. We have value of V_1 , we have value of α_1 we have value of U .

So tan beta one, this is blade inlet angle, is $V_1 \sin \alpha_1$ divided by $V_1 \cos \alpha_1 - U$. Now we will put the values, V_1 is $366.4 \sin 20$ divided by $366.4 \cos 20 - 164.9$. Now if you solve this, you are going to get tan beta 1 is 0.6985. And this gives the value of beta 1 as 34.9 degree. So now we have the value of beta 1 also right.

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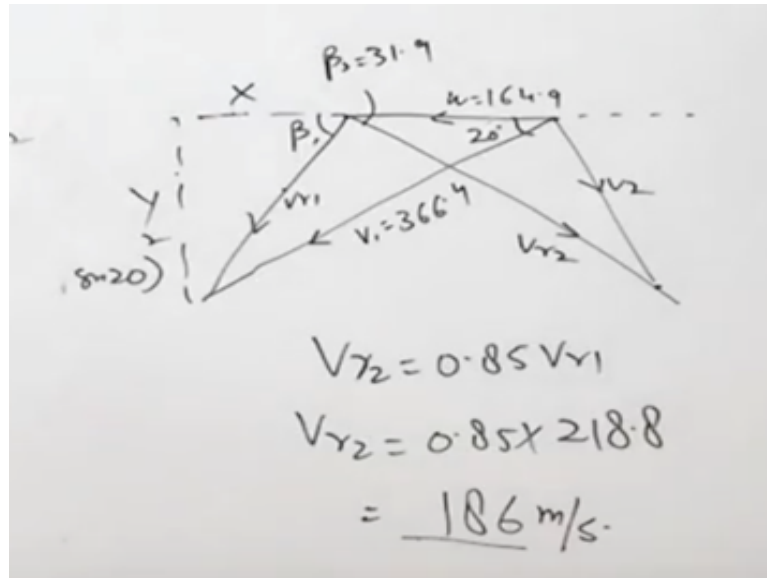
$N = 3050 \text{ rpm}$ $u = 164.9 \text{ m/s}$
 $D = 105 \text{ cm}$ $V_1 = 366.4 \text{ m/s}$
 $\alpha_1 = 20^\circ$ $\beta_1 = 34.9^\circ$
 $\beta_2 = 31.9^\circ$
 $p = 0.45$
 $\frac{V_2}{V_1} = 0.85$
 $\beta_2 = \beta_1 - 3$
 $m = 7 \text{ kg/s}$
 $V_2 = 218.8 \text{ m/s}$

Once we have the value of beta 1 we can always have the value of beta 2. Beta 2 is beta 1 - 3 degree. So it is going to be 31.9 degree. Now we have found the value of beta 2 as well, but we do not know the value of V_2 and V_{R2} right. But we have the constant, this V_{R2} by V_{R1} as 0.85 but for this purpose we need the value of V_{R1} .

Now V_{R1} can be taken from here because we know the value of X , we know the value of Y . So $V_{R1}^2 = (V_1 \cos \alpha_1 - U)^2 + (V_1 \sin \alpha_1)^2$ right. And this will give now V_1 is 366.4 - U is 164.9, sorry $\cos 20 - 164.9$ whole square + $366.4 \sin 20$ whole square.

Now this 366.4 this 1 will give 125.3 whole square + 179.4 whole square because this is a right end triangle. So this square = this square + this square right. And then we get $V_{R1}^2 = 218.8$ meter per second and here also because V_1 is greater than V_{R1} . We can see from here also and numerically also we are getting V_{R1} as 218.8 meter per second.

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So VR1 is 218.8 meters per second but once we have the value of VR1 and this ratio the relative velocity at outlet from the blades to the inlet of the blades VR by VR1 is .85. So VR2 = 0.85 VR1 so VR2 = 0.85 multiplied by VR1 that is 218.8 and we will get the value of relative velocity at 2 is 186 meters per second right.

Now we have blade outlet angle here sorry blade outlet angle come here so blade outlet angle is 31.9 right. So we know the direction of relative velocity of the steam which is leaving the blades and we know the magnitude as well, VR2. So this beta 2 is 31.9 and 186 this is the value of VR2.

We can draw this line once we have this line we can definitely complete the triangle and as we have found the value of, if it is required, then as we have found the value of VR1, we can find the value of V2.

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$$V_2^2 = (V_{r2} \cos \beta_2 - u)^2 + (V_{r2} \sin \beta_2)^2$$

So $V_2^2 = V_{r2}^2 \cos^2 \beta_2 - 2u V_{r2} \cos \beta_2 + u^2 + V_{r2}^2 \sin^2 \beta_2$. And from here we can do because we have the V_{r2} , we have the value of β_2 , $\cos \beta_2$, u is also with us so we can easily find the value of, or geometrically also we can do that we will be getting the same result.

Now what is required here because now we have complete; now we can draw the velocity triangle which is the first requirement for any numerical in steam turbines. Now, what is required here tangential thrust on the blade. In order to find tangential thrust on the blade, we have to find the wheel velocity.

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$u = 164.9 \text{ m/s}$ $V_w = 337.4 \text{ m/s}$
 $V_1 = 364.4 \text{ m/s}$
 $\beta_1 = 34.9^\circ$
 $\beta_2 = 31.9^\circ$
 $V_{r1} = 218.8 \text{ m/s}$
 $V_{r2} = 186 \text{ m/s}$
 $V_w = V_1 \cos \alpha_1 + V_2 \cos \alpha_2$
 $V_w = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2$
 $V_w = 218.8 \cos 34.9 + 186 \cos 31.9$
 $= 337.4 \text{ m/s}$

$V_{r2} = K V_{r1}$
 $= 0.85 \times 218.8$

Now wheel velocity of the flow wheel velocity is this one this is wheel velocity right. Now wheel velocity can be expressed either like $V_w = V_1 \cos \alpha_1 + V_2 \cos \alpha_2$. This is

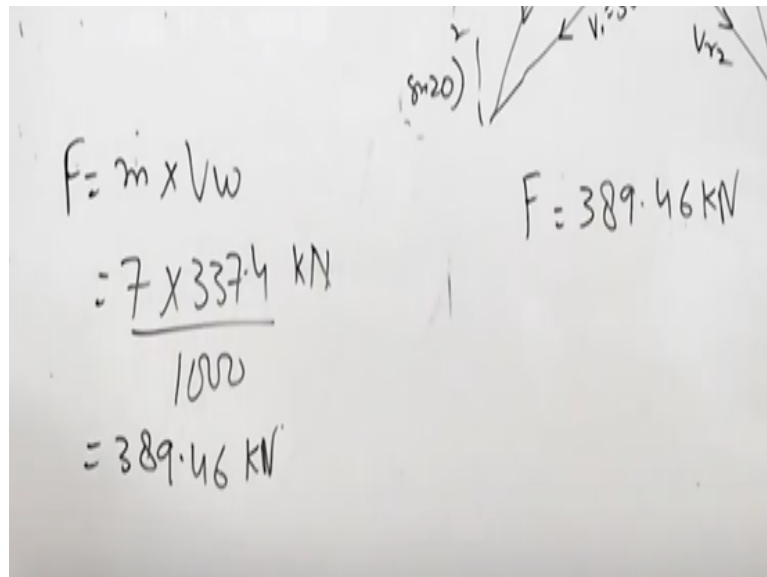
alpha 2 so alpha 2 we have not calculated alpha 2. We have not calculated V2 yet but we can always find the value of V2 and alpha 2 with help of the available information ok. So the wheel velocity is $V_1 \cos \alpha_1$, this is alpha 1 + $V_2 \cos \alpha_1$ or $VW = VR_1 \cos \beta_1 + VR_2 \cos \beta_2$.

This is simple trigonometry, VR1 is this, relative velocity at inlet + beta 1 this length + VR2 Cos beta 2 this length. So if information, let us see whether information regarding this is available with us we have already calculated VR1, beta 1 is also with us, VR2 we have calculated. We have calculated, VR2 is .85 times VR1 so VR2 is equal to, we have calculated VR2, it is 186 meters per second.

So $VR_2 = K VR_1$, and K is 0.85 and VR1 is 218.8. So from here we got the value of VR2 so VR2 is also with us and Cos beta 2 is also with us. So in fact we have all the information. Now we will calculate VW so $VR_1 \cos 34.9 + VR_2 \cos 31.9$.

And this gives the value of VW as 337.4 meters per second. We will note it down here, $VW = 337.4$ meters per second. This is wheel component of the velocity now tangential thrust on the blade force.

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$$F = m \times VW$$

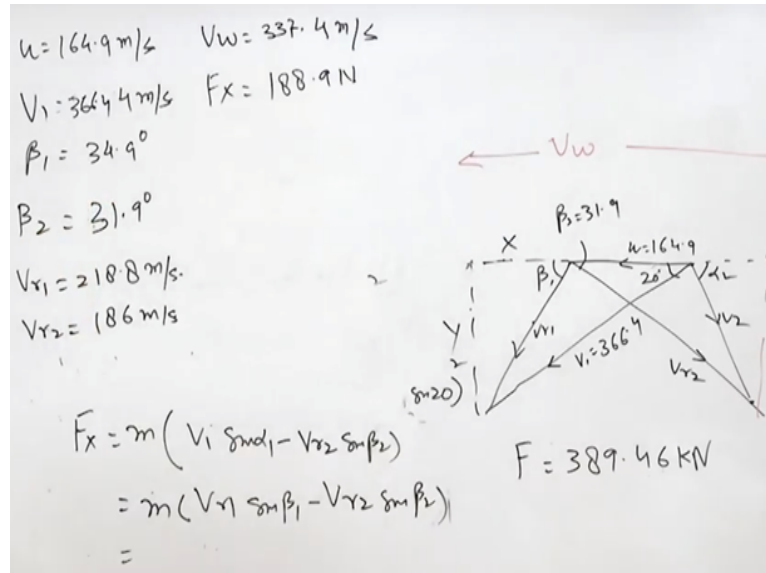
$$= \frac{7 \times 337.4 \text{ kN}}{1000}$$

$$= 389.46 \text{ kN}$$

Force is on the thrust on the blade is force is equal to mass flow rate multiplied by VW. Mass flow rate is 7, multiplied by 337.4 this will give, this will be in newtons right. If you divide this by 1000, it will become kilo newtons and this is 337.46 kilo newtons.

So force, here we can write so first answer is, tangential thrust on the blade, $F = 389.46$ kilo newton. Now second one is axial thrust on the blades. Now axial thrust on the blades is perpendicular to this it means this Y- this Y.

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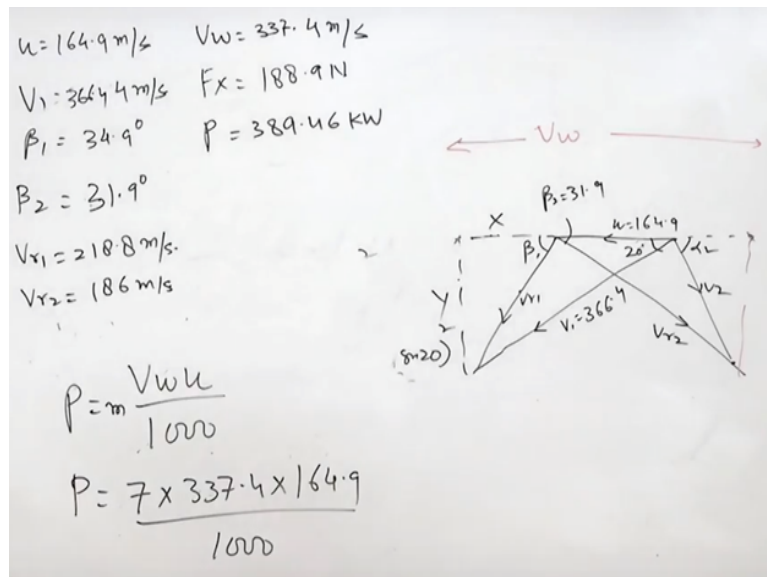


Again, in order to find axial thrust on the blade we can take axial thrust on the blade, mass flow rate and $V_1 \sin \alpha_1$, this is this 1 - we can take $V_{r2} \sin \beta_2$, or we can take similar fraction. We can take $V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2$ sorry yes $V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2$ can also be taken because this is $V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2$ right.

We can take either of these so $V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2$ now we here we have the value of V_{r1} with us. We have value of V_{r2} , $\sin \beta_1 \sin \beta_2$ values are also with us right because mass flow rate is also given, and this will give the axial thrust as, if you are putting the values and solving this we will be getting the axial thrust as 188.9 newtons right.

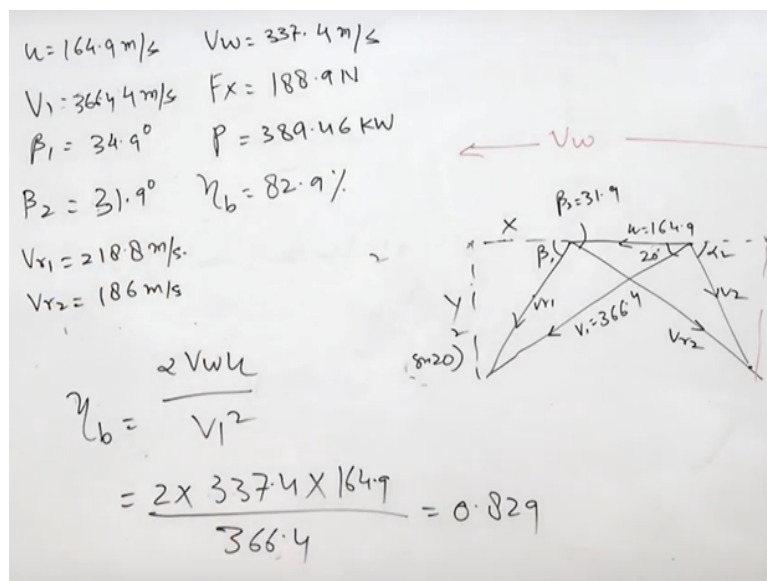
This is the axial thrust means when the shaft is rotating, then force working perpendicular to, sorry parallel to the axis of the shaft that is known as axial thrust. So axial thrust is 188.9 newtons now after axial thrust, the second is power developed in the blade.

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So power is VWU by 1000 this is the force multiplied by the velocity so and multiplied by mass flow rate. So VW is given so power is mass flow rate is seven, VW is 337.4 multiplied by U , peripheral velocity, we have calculated. 164.9 divided by 1000 okay and this gives the power as 389.4 kilo watt right so we are getting power also.

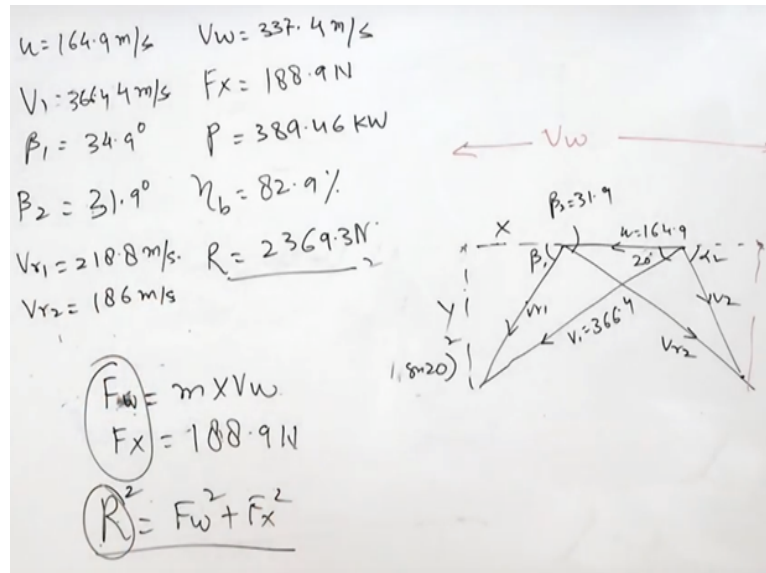
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Now blade efficiency is $2VWU$ by V_1 square it is the blade or diagram efficiency. Now VW is with us so 2 into 337.4 into U , 164.9, divided by V_1 , we have it is given here 366.9, sorry 366.4 and this will give the blade or diagram efficiency 0.829 if you multiply this by 100, this is 82.9 %.

And the last one is resultant thrust on the blades. Now resultant thrust, we have wheel component. We have axial component ok and they are perpendicular to each other. So net thrust will be, now we have the value of VW we can calculate the value of FX.

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We have already we can calculate the force wheel force, that is mass into VW and FX is mass flow rate, this is already calculated so this is already calculated. It is 188.9 newtons and FW and FX, the net resultant force is FW square + FX square now this will give the resultant force acting on the blades.

After this we will take up another numerical which states that, steam flows from the nozzles of a single row impulse turbine with a velocity 400 meters per second and 15 degree nozzle inlet angle. Steam at 5 kg per second comes out of the equiangular moving. Equiangular means, inlet blade angle = outlet blade angle, so beta 1 = beta 2.

With an absolute velocity of 80 meters per second so absolute velocity at the exit is also given with the nozzle outlet angle of 60 degree. So in this case, inlet and outlet information's are giving given. Find the power developed and the loss due to friction.

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$$\begin{aligned}
 V_1 &= 400 \text{ m/s} \\
 \alpha_1 &= 15^\circ \\
 \dot{m} &= 5 \text{ kg/s} \\
 V_2 &= 80 \text{ m/s} \\
 \beta_1 &= \beta_2 \\
 \alpha_2 &= 60^\circ \\
 u &= 131 \text{ m/s}
 \end{aligned}$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} = \frac{400 \sin 20}{400 \cos 20 - u}$$

$$\tan \beta_2 = \frac{V_2 \sin \alpha_2}{V_2 \cos \alpha_2 + u} = \frac{80 \sin 60}{80 \cos 60 + u}$$

So here in this case V_1 is given V_1 is 400 meters per second and α_1 is also given is 15 degree. Steam at 5 kg per second, so mass flow rate is also given 5 kgs per second comes out of the equiangular moving blades with an absolute velocity of 80 meters per second. So $V_2 = 80$ and equiangular, so $\beta_1 = \beta_2$. Nozzle outlet angle is 60 degrees so α_2 is also given 60 degree.

Find the power developed and loss due to friction here U is not given in this case. So analytically we will have to find the U . So first of all we will draw the tentative velocity diagram, and that is V_1 , VR_1 , U this is β_1 , this is β_2 , α_1 , α_2 , VR_2 and V_2 right.

Now here $\tan \beta_1$ is, we have the value of V_1 ok, but we do not have value of so we have the value of α_1 also. So it is $V_1 \sin \alpha_1$ divided by this Y divided by X that will give $\tan \beta_1$. So X is $V_1 \cos \alpha_1 - U$ now V_1 is given 4000 meters per second. $\sin \alpha_1$ is 20, sorry yes this is \cos not \sin and then $4000 \cos 20 - U$, U is not known to us. Now for $\tan \beta_2 = V_2 \sin \alpha_2$, this is again this is suppose Y_1 this is X_2 and Y_2 .

So $\tan \beta_2$ is $V_2 \sin \alpha_2$ or $VR_2 \sin \beta_2 - U$ either of these you can take but since we have the value of V_2 80 meters per second and we have the value of α_2 also. So it is easy to find the value of Y_2 and then X_2 is $V_2 \cos \alpha_2 + U$ right. We have all the values, we have the value of V_2 is $80 \sin 60$ divided by $80 \cos 60 + U$.

Now this 1 = this 1, because we have said that we have equiangular blades. And out of the, if we make this equal to this, the only unknown is U. And from here we can get the value of U. And the value of U is coming out as 131 meters per second right. Once we have calculated the value of u, then other values will automatically come out.

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Handwritten calculations showing the derivation of wheel velocity (Vw) and power (P):

$$V_1 = 400 \text{ m/s}$$

$$\alpha_1 = 15^\circ$$

$$\dot{m} = 5 \text{ kg/s}$$

$$V_2 = 80 \text{ m/s}$$

$$\beta_1 = \beta_2$$

$$\alpha_2 = 60^\circ$$

$$u = 131 \text{ m/s}$$

$$V_w = 426.4 \text{ m/s}$$

$$V_w = V_1 \cos \alpha_1 + V_2 \cos \alpha_2$$

$$= 400 \cos \alpha_1 + 80 \cos 60$$

$$P = 279.3 \text{ kW}$$

$$P = \dot{m} \frac{V_w u}{1000}$$

$$= \frac{5 \times 426.4 \times 131}{1000}$$

For example wheel velocity VW wheel velocity is going to be 4000 Cos alpha 1 + this 80 Cos alpha 2. Alpha 2 is also known to us, so it is going to be 4000 we can write sorry. We can write here V1 and V2. So it is 4000 Cos alpha 1 + 80 Cos 60 right. This will give the wheel velocity and this is going to be = VW. We will write here VW = 426.4 meters per second right now power.

Power is mass flow rate VW U divided by 1000. VW is with us, U is also with us mass flow rate is given here 5 kg per second. So 5 kg per second multiplied by 426.4 multiplied by 131 divided by 1000. And this will give power as 279.3 kilo watts.

Now the second part, find power developed and loss due to friction. Now loss due to friction is change in relative velocity. It can be found thru change in relative velocity.

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$V_1 = 400 \text{ m/s}$ $V_w = V_1 \cos \alpha_1 + V_2 \cos \alpha_2$
 $\alpha_1 = 15^\circ$ $= 400 \cos 15^\circ + 80 \cos 60^\circ$
 $\dot{m} = 5 \text{ kg/s}$ $P = 279.3 \text{ kW}$
 $V_2 = 80 \text{ m/s}$
 $\beta_1 = \beta_2$ $P_f = \frac{m(V_1^2 - V_2^2)}{2 \times 1000}$
 $\alpha_2 = 60^\circ$
 131 m/s
 426.4 m/s

$V_{r1}^2 = V_1^2 + u^2 - 2uV_1 \cos \alpha_1$
 $V_{r2}^2 = ?$

QF or power friction sorry power friction, mass flow rate $V_{r1}^2 - V_{r2}^2$ divided by 2 into 1000 right. Now we have the value of V_1 we do not have the value of V_{r1} .

But V_{r1} we can easily calculate, $V_{r1}^2 = V_1^2 + u^2 - 2uV_1 \cos \alpha_1$. This will give us the value of V_{r1} now V_{r2} is also not with us similarly we can find the value of V_{r2} right. And then once we have the value of V_{r1} and V_{r2} , or this is one way.

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$V_1 = 400 \text{ m/s}$ $V_w = V_1 \cos \alpha_1 + V_2 \cos \alpha_2$
 $\alpha_1 = 15^\circ$ $= 400 \cos 15^\circ + 80 \cos 60^\circ$
 $\dot{m} = 5 \text{ kg/s}$ $P = 279.3 \text{ kW}$
 $V_2 = 80 \text{ m/s}$
 $\beta_1 = \beta_2$ $P_f = \frac{m(V_1^2 - V_2^2)}{2 \times 1000}$
 $\alpha_2 = 60^\circ$
 $u = 131 \text{ m/s}$
 $V_w = 426.4 \text{ m/s}$ $= \frac{5 \cdot (275^2 - 184^2)}{2 \times 1000}$
 $= 103.96 \text{ kW}$

This is one way another way is $V_{r1}^2 = V_1^2 \sin^2 \alpha_1 + V_1^2 \cos^2 \alpha_1 - u^2$ it is more of the same thing right. So this from here we can get the V_{r1} and V_{r2} as well. Once we have the value of V_{r1} and V_{r2} , we will be putting here.

Then power will become 5 times 275 square - 184.184 square divided by 2 into 1000. And the power, friction power is going to be 103.96 kilo Watt. So this is how the friction power loss due friction is calculated. That is all for today and from the next class we will start with the impulse reaction turbines.