

Steam and Gas Power Systems
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Module No # 04
Lecture No # 20
Problem Solving (Nozzles and diffusers)

Hello I welcome all this in this course on steam and gas power system today we will solve one numerical on flow through the nozzles.

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Numerical

The nozzles of a 100 kW capacity turbine have a throat diameter of 0.5 cm each. The steam flow rate in turbine is 6 kg/kWh. The steam enters the nozzle at 14 bar pressure and 300 °C temperature. The back pressure is 5 kPa. If the flow through nozzles is isentropic, find the number of nozzles and steam consumption. Neglect the velocity of approach.

If the 10% of the isentropic heat drop between throat and exit is wasted, find exit diameter of nozzle and the final condition of steam. Also find overall efficiency of nozzle.

The statement of the numerical is in nozzles of a 100 kilopascal kilo watt capacity turbine it means there are number of nozzles not a single nozzle there are number of nozzles for 100 kilowatt capacity turbine have a throat diameter of 0.5 centimeter each this steam flow rate in turbine is KG per kilowatt hour where steam enters the nozzle at 40 bar pressure and 30 degree centigrade 300 degree centigrade temperature.

The back pressure is 5 kilopascal back pressure means pressure after expansion in the nozzle so that is 5 kilopascal. If the flow through nozzle is isentropic find the number of nozzles and steam consumption neglect the velocity of approach. I mean the velocity of steam which is entering the nozzle. If the 10 % of isentropic heat drop between throat and exit is wasted find exit diameter of nozzle and the final condition of the steam.

Also find overall efficiency of the nozzle so they are number of nozzle in this system and initial condition is 300 degree centigrade temperature and 40 bar.

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600 kPa					
C	v	u	h	s	
158.83	0.31558	2566.8	2756.1	6.7592	
200	0.35212	2639.3	2850.6	6.9683	
250	0.3939	2721.2	2957.6	7.1832	
300	0.43442	2801.4	3062	7.374	
350	0.47427	2881.6	3166.1	7.5481	
400	0.51374	2962.5	3270.8	7.7097	

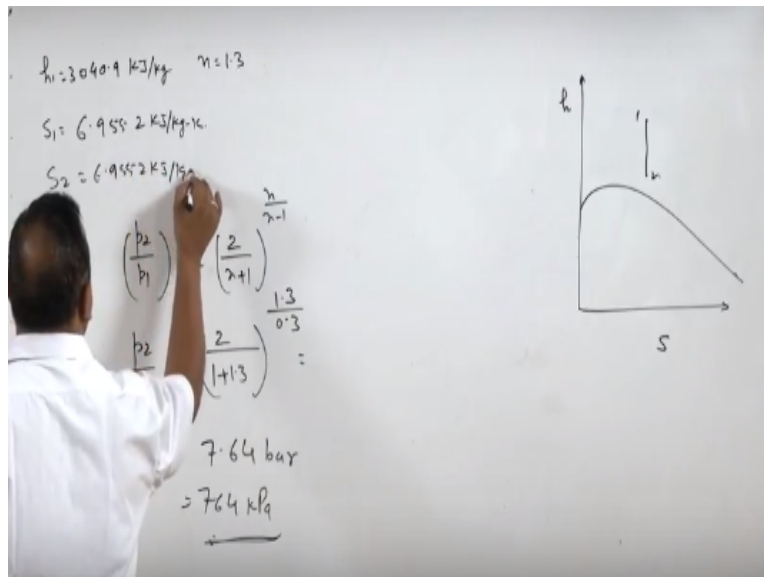
800 kPa					
C	v	u	h	s	
170.41	0.24034	2576.0	2768.3	6.6616	
200	0.26088	2631.0	2839.7	6.8176	
250	0.2932	2715.9	2950.4	7.0401	
300	0.32416	2797.5	3056.9	7.2345	
350	0.35442	2878.6	3162.2	7.4106	
400	0.38428	2960.2	3267.6	7.5734	

764 kPa					
C	v	u	h	s	
168.33	0.25388	2574.3	2766.1	6.679168	
200	0.27730	2632.5	2841.7	6.844726	
250	0.31133	2716.9	2951.7	7.065858	
300	0.34401	2798.2	3057.8	7.25961	
350	0.37599	2879.1	3162.9	7.43535	
400	0.40758	2960.6	3268.2	7.597934	

1400 kPa					
C	v	u	h	s	
195.04	0.14078	2591.8	2788.8	6.4675	
200	0.14303	2602.7	2803	6.4975	
250	0.16356	2698.9	2927.9	6.7488	
300	0.18232	2785.7	3040.9	6.9552	
350	0.20029	2869.7	3150.1	7.1379	

So if we look at 40 bar that is 400 and 1400 kilopascal the saturation temperature is 195.04 centigrade it means then steam is superheated while entering the nozzle.

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So on a enthalpy entropy diagram steam is superheated while entering the nozzle right and since the temperature is a 300 degree centigrade directly from table fourteen bar pressure that is 1400 kilopascal pressure and 300 degree centigrade the enthalpy is 3040.9 kilo joules per KG. So H1

is 3040.9 kilo joules per PG entropy will be requiring entropy is 6.9552 kilo joules per KG Kelvin right.

Now pressure ratio P_2 by $P_1 = 2$ by $N + 1$ N upon $N - 1$ this will not give the exit pressure. This will give the pressure at the throat right so P_2 by for this superheated vapor $N = 1$ by 3. So just by putting the value $N = 1$ by 3 so just by putting the value 1 by 3, 2 by 1 + 1 by 3 raise to power 1 by 3 divide by 0.3 we are getting and this is P_2 by P_1 and then P_2 is in this case we are getting 7.64 bar or 764 kilopascal that is the pressure at the throat right.

And temperature now this 764 kilopascal let us look at entropy 764 I have taken the properties of because 764 kilopascal will not find from the steam table. So what I have done I have taken properties at 600 kilopascal properties at 800 kilopascal and linear interpolation I have calculated the properties at 764 kilo pascal.

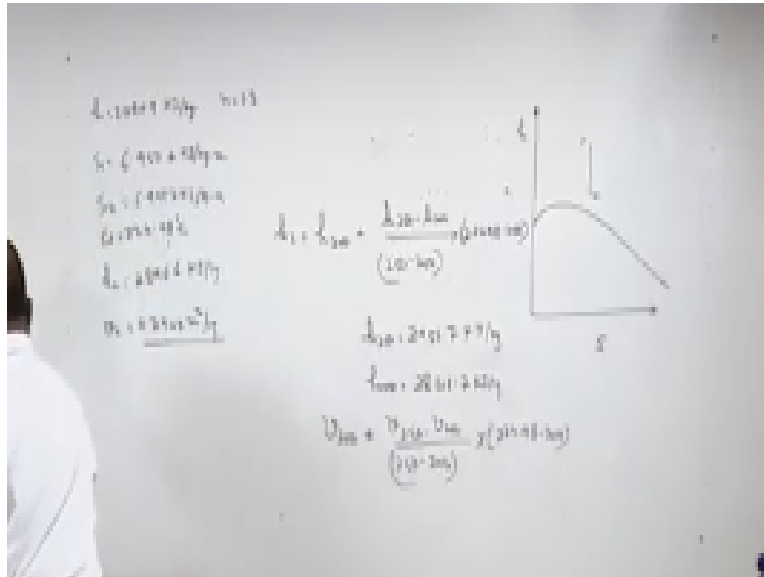
So if you have steam table with you this exercise shall also be done by you in order to find the properties at 6764 kilopascal. At 764 kilopascal if the vapor is saturated entropy is 6.679 here it is 6.955 it means the vapor is superheated because when there is a isentropic expansion from state 1 to state 2 right in this case the entropy is going to remain constant.

So S_2 as to be 6.9552 kilo joules per KG kelvin now if you look at the superheated property of steam at 764 kilo pascal. So in that case 9.955 so this entropy will lie between this 200 degree centigrade and 250 degree centigrade right. For 200 degree centigrade or will write 200 = how much 6.866726 kilo joules per KG kelvin.

And entropy at 200 and 50 is 7.065818 kilo joules per KG kelvin. And our entropy is lying between these two so we will do the linear interpolation of temperature in order to find the temperature of the paper it is going to be $200 + \frac{S_2 - S_2 100}{S_2 50 - S_2 100}$ multiplied by 250 - 200.

Now these values S2 will take from here S2 100 or entropy at 200 here and entropy at 250 from here and while this linear equation we are getting temperature as to 224.98 degree centigrade. So T2 is 224.98 degree centigrade.

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In similar fashion we can find other properties also other properties means suppose I want to have a enthalpy H2. So enthalpy at 2 is again enthalpy are 200 + enthalpy at 200 enthalpy at 250 - enthalpy at 200 divided by 250 - 200 this is temperature multiplied by 224.98 - 200.

So this is linear again linear interpolation of the enthalpy at 250 can be taken from here at 2 it is 952 to 951.7 kilo joules per KG. And enthalpy at 200 = enthalpy at 200 = 2632 enthalpy at 200 yes it is 2841.7 kilo joules per KG.

This we have taken from the interpolated table so now putting this values as 200 here and H 2 50 and 200 here we will get the value of H2S 2896.6 kilo joules per KG. In similar fashion we can find the specific volume at 2 also and that is 0.2943 meter cube per KG.

Again specific volume again is specific volume at 200 + specific volume at 250 - specific volume at 200 divided by 250 - 200 multiplied by 224.98 - 200. So this is linear interpolation and this will give the value as S 0.2943 meter cube per KG right. So in such case when the steam

remains the superheated zone all the values are calculated by taking cross interpolation of the values.

Now entropy is remaining constant it is further expanded to 5 kilopascal this is state 3 P_3 P_s is 5 kilopascal. If you look at the statement at of the problem the back pressure is 5 kilopascal right at 5 kilopascal the entropy is 6.9552. So $S_3 = S_1$ and saturated vapor when it is at 5 kilo pascal the entropy is 8.3938.

It is the vapor is no longer superheated it is wet vapor having certain quality so $S_3 = S_1$ and S_1 is 6.9552 this is $S_1 = S_f + X S_g$. So S_f is 0.4762 + X multiplied by 7.9176 now from here so this is 6.9552 from here we will get the value of X and the value of X is X_3 is in this case 0.8183.

Now once we have value of X_3 we can find the value of H_3 also H_3 is $H_f + X_3 H_{fg}$ right. Now H_f is the pressure of the liquid at 5 kilopascal and it is going to be 137.75 multiplied by 0.8183 that is the quality of vapor multiplied by this is + multiplied by latent heat of vaporization that is 4222.95 and this will give the value of H_3 S 2120.4 kilo joules per KG.

And will be volume of the vapor at 3 will be X_3 and V_{X_3} and this will give because X_3 is already we know 0.8183.

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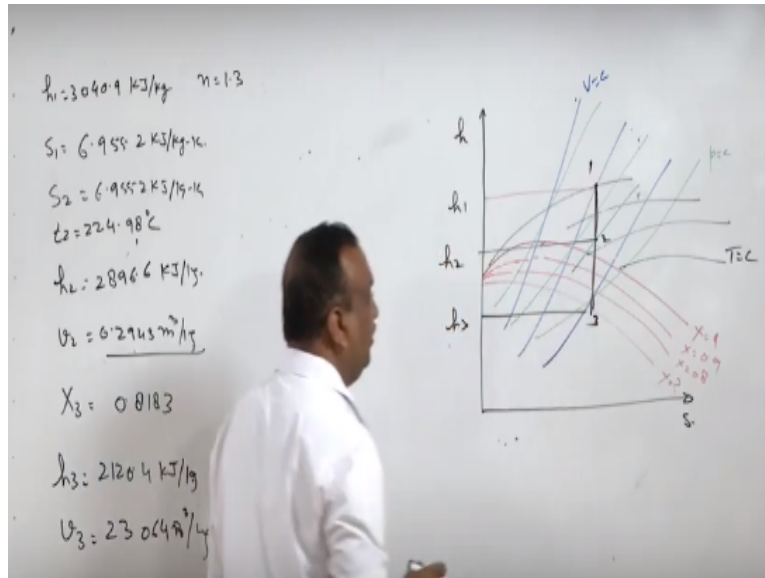
kPa	deg.C	vf	vfg	vg	uf	ufg	ug	hf	hfg	hg	sf	sfg	sg
5	32.874	0.001005	28.18399	28.185	137.74	2282.06	2419.8	137.75	2422.95	2560.7	0.4762	7.9176	8.3938
750	167.75	0.001311	0.254399	0.25551	708.4	1865.6	2574	709.24	2056.36	2765.6	2.0195	4.6643	6.6836
775	169.09	0.001313	0.246577	0.24769	714.26	1860.74	2575	715.12	2051.88	2767	2.0327	4.6397	6.6724

And V3 three we can take from here it is 28.185 meter cube per KG. So multiply this two will be getting volume of vapor and that is 23.064 meter cube per KG. Now we have all the properties right these properties we have derived from steam table right and the steam table also at a particular pressure that we calculated the pressure 764 by this equation for optimum pressure ratio.

And from seven sixty four for all the properties at this pressure we have taken properties at 600 kilopascal we have taken properties of superheated steam at 800 kilopascal. These properties at these two pressures of a steam we have calculated the properties of superheated steam at 764 kilopascal.

Again once we have found the temperature of the steam that is 224.98 degree centigrade we have done interpolation and we have found the value of different properties of the steam at all 1, 2 3 all at all the states 1 and 2 and 3. So this is through steam table if we use Mollier diagram the job becomes very simple when we use the mollier diagram.

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In a mollier diagram or mollier chart it is a chart between enthalpy and entropy it is also known as enthalpy entropy diagram. In mollier diagram there is a line when vapor is saturated this is known as saturation line $X = \text{right}$. In mollier diagram there are lines which are inclined these lines are known as constant pressure lines.

There are certain lines which are constant volume lines volume = constant right and these lines quality wise also these lines quality wise also this is the line for $X = \text{one}$ and you will find another line $X = 0.9$. Then you find another line $X = 0.8$ and so on. Now in this diagram suppose you want to take the value suppose you have initially you have pressure and temperature right.

So there are having constant temperature lines also and the mollier diagram having constant temperature lines also like this temperature = constant. So now I have constant temperature line I have constant pressure line immediately I can look out suppose I know the temperature and I know the pressure I can immediately locate the point right. So state one can be easily located on the mollier chart.

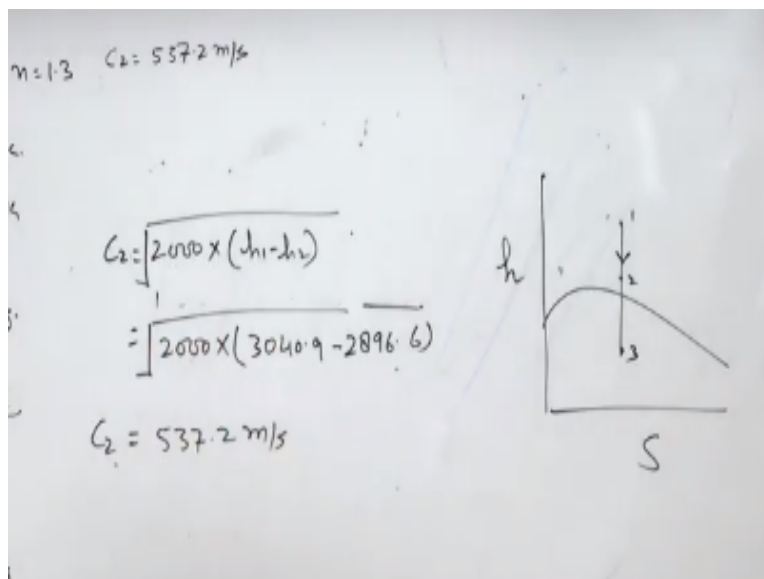
When state one is located you can directly take enthalpy you do not have to do the interpretation in the steam table immediately you can take the value of $H1$ right. Process is isentropic when the process is isentropic it means entropy is remaining constant it is going to be vertical line three

right now P2 is known to me now P2 is 764. So immediately I will see the 764 pressure line at when this pressure line constant pressure line.

When this is cutting this vertical line this is state 2 so state 1, state 2 so state 2 S2 can be taken from here right. And state 2 we can find everything specific volume and temperature further expansion is taking place we are getting attaining state 3. When we are attaining state 3 immediately we can take quality of vapor from here where this vertical line is cutting the constant entropy line sorry constant quality line and at the same diagram we can find the value of H3 and specific volume at everything.

So such type of issues I will problems related with nozzles the use of mollier diagram is quite convenient but the problem in the volume in mollier diagram is we do not get accurate value. The accuracy is a little problem when we solve the numericals that is why in order to have high accuracy we go for interpretation in steam table but for all practical purpose purposes this mollier diagram is sufficient to find the properties at different states 1, 2 and 3.

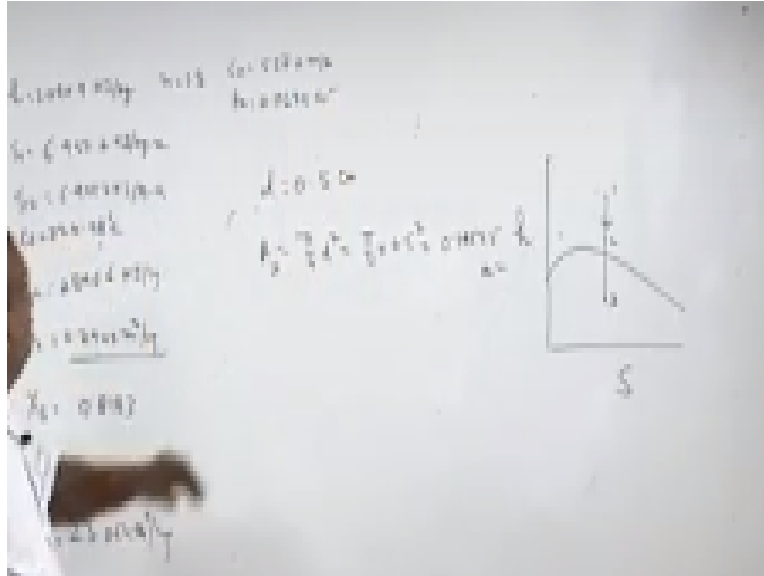
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Now velocity state 2 C2 is going to be 2000 multiplied by H1 - H2 now I will rub this off H1 - H2 and under root. And this = under root 2000 multiplied by H1 is 3040.9 and H2 is 2896 and this will give the value of C2S 537.2 meter per second.

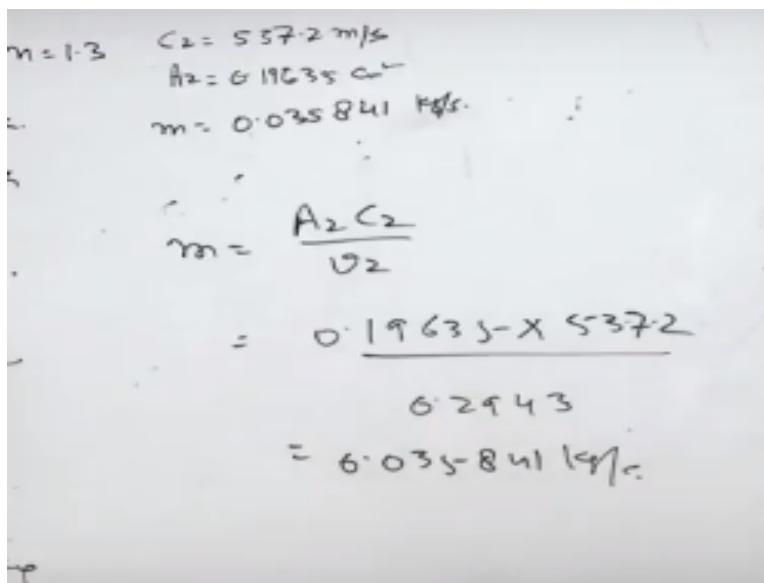
So we can write somewhere here $C_2 = 537.2$ meters per second this is the velocity of a steam at state 2 right.

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Now throat diagram we have already assumed it is 0.5 so D is 0.5 centimeter is it so or it is different .5 centimeter area we can easily find PIE by $4 D \text{ square} = \text{pie by } 4 \text{ into } 0.5 \text{ square} = 0.19635 \text{ centimeter square}$. So A_2 is 0.09635 centimeter square now at state 2 velocity is known area is known and specific volume is known.

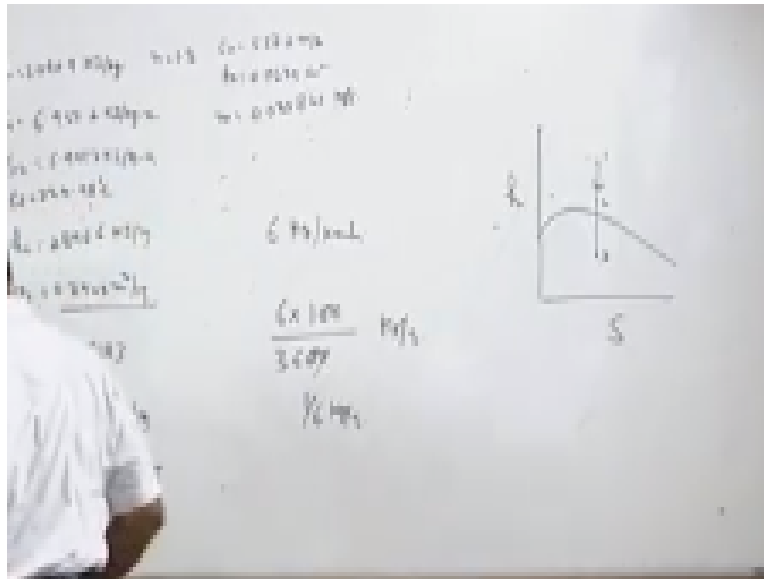
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We can easily find the mass flow rate per nozzle $M = A_2 C_2$ by $V_2 = 0.19635$ that is area velocity is 530 seven by 2 divided by specific volume this 0.2943 and this will give the mass flow rate as 0.035841 KG per second.

So mass flow rate is 0.035841 KG per second and mass flow rate is going to remain constant from inlet to exit.

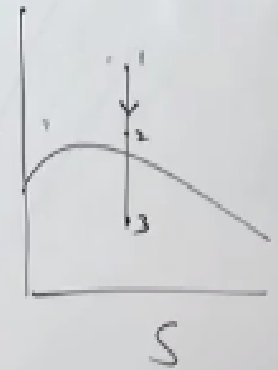
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This is mass flow rate per nozzle and a steam consumption is in this case is how much 6 KG per kilowatt hour 6 KG per kilowatt hour this is steam consumption rate it means 6 into 100 because plant capacity is 100 right and per hour it is 3600 that will give KG per second this much of steam is required. So if we cancel it out it is 1 by 6 KG per second this is the total mass flow rate of steam.

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$$\frac{M}{m} = \frac{1/6}{0.035841}$$

$$= \underline{4.65} \quad \underline{4 \text{ or } 5}$$


$$= 5 \times 0.03581$$

$$= \underline{6.45 \text{ kg/s}}$$

Per nozzle mass flow rate is this much so number of nozzles is going to be total mass flow rate divide by mass flow rate per nozzle total mass flow rate is 1 by 6 KG per second here is 0.035841 and that is going to be = 5.645 number of nozzle is 4.65 but we cannot have 4.65 nozzle. It has to be increase either 4 or 5 then we will have to take five nozzles.

And five nozzles if we are taking then for five nozzles we will have to connect the mass flow rate. So actual mass flow rate will come around we can do about two things either we increase the mass flow rate same type of nozzle or we change the diameter of the nozzle right + changing diameter nozzle is difficult so we can do we can increase the mass flow rate of the steam and that can be = 5 into 0.03581 and that = actual mass.

While designing the nozzle we change the diameter of the nozzle keeping the same mass flow rate. Now next part of this numerical is if 10% of the isentropic heat drop between throat and exit is wasted. Find the exit diameter of the nozzle now from two to three 10 % of the heat wasted right.

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$h_1 = 3040.9 \text{ kJ/kg}$ $\eta = 1.3$ $c_2 = 557.2 \text{ m/s}$
 $h_2 = 6116.35 \text{ cal}$
 $S_1 = 6.9552 \text{ kJ/kg}\cdot\text{K}$ $m = 0.035841 \text{ kg/s}$
 $S_2 = 6.9552 \text{ kJ/kg}\cdot\text{K}$ $h_3' = 2198.02 \text{ kJ/kg}$
 $t_2 = 224.98^\circ\text{C}$
 $h_2 = 2896.6 \text{ kJ/kg}$
 $v_2 = 0.2943 \text{ m}^3/\text{kg}$
 $X_3 = 0.8183$
 $h_3 = 2120.4 \text{ kJ/kg}$
 $v_3 = 230.64 \text{ m}^3/\text{kg}$

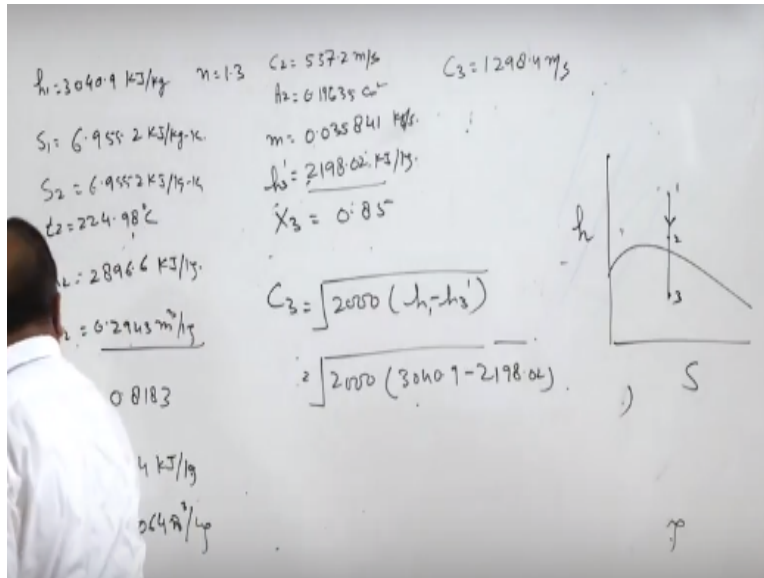
$\Delta h = (h_1 - h_2) + 0.9(h_2 - h_3)$
 $= (3040.9 - 2896.6) + 0.9(2896.6 - 2120.4)$
 $= 842.88 \text{ kJ/kg}$
 $h_1 - h_3' = 3040.9 - 842.88 = 2198.02 \text{ kJ/kg}$

So for the 10 % wasted of the heat it means the total heat drop in the process is Δh is $H_1 - H_2$ right + 90 % of $H_2 - H_3$ right. This will give now we can write $H_1 - H_2$ that means 040.9 and H_2 is 2896.6 and 90 % of remaining enthalpy drop that is again $H_2 - H_3$ and 20.4.

And this will give the enthalpy drop as 842.88 kilo joules per KG and this is $H_1 - H_3$ dash. So H_3 dash is going to be = 3040.9 - 842.88 and that is going to be = 2198.02 so H_3 dash = 2198.02 kilo joules per KG right. So find the diameter of the nozzle and the final condition ok.

Now we have H_3 dash that is the enthalpy of the fluid leaving the nozzle once we have value of H_3 dash we can find the quality of the vapor because this pressure is 5 kilopascal.

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So 5 kilopascal the enthalpy is $2198.02 = h_3'$. h_3' is 137.5 kilo joules per KG and latent heat is $2560.72, 422.95 \times 2422.95$ from here we will get the value of X_3 and that is going to be $= 0.85$.

Once we have power at state 3 we can find the velocity C_3 is under root $2000(h_1 - h_3)$ right. and this C_3 we have getting C_3 we are getting 1298.4 meter per second all we are getting just we are putting the value of h_1 that is 3040.9 and h_3 it is 2198.02 that is how we are getting the velocity.

Now we have to find the area of exit velocity now velocity is with us is specific volume with us and mass flow rate is constant right.

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$C_2 = 557.2 \text{ m/s}$
 $A_2 = 0.19635 \text{ m}^2$
 $m = 0.035841 \text{ kg/s}$
 $h_0' = \frac{2198.02 \text{ kJ/kg}}$
 $X_3 = 0.85$
 $d_3 = 2.9 \text{ cm}$

$C_3 = 1298.4 \text{ m/s}$

$A_3 = \frac{m C_3'}{C_3}$
 $= \frac{0.035841 \times 23.96}{1298.4}$

$A = 6.61 \times 10^{-4} \text{ m}^2 = \frac{\pi}{4} d^2$

So area A_3 is $M V_3$ dash or V_3 , V_3 dash and this is V_3 dash and C_3 dash. Now V_3 dash is X multiplied by V_3 is how much 28 on 8528.1 right 5 multiplied by the quality will give V_3 dash. So now here mass flow rate is 0.035841 into 23.96 divide by 1298.4 and this will give the area as 6.61 into 10 to power - 4 meter square.

And this area is $\frac{\pi}{4} D^2$ and from here we can find the value of D and the exit area or D_3 is 2.9 centimeter. Simply we will just multiply this by 4 divide by π we will take the under root will find the area and meters multiply by 100 we will get the area sorry will get the diameter in meter and the multiplied by 100 will get the diameter and centimeter right.

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$C_2 = 557.2 \text{ m/s}$
 $A_2 = 0.19635 \text{ m}^2$
 $m = 0.035841 \text{ kg/s}$
 $h_0' = \frac{2198.02 \text{ kJ/kg}}$
 $X_3 = 0.85$
 $d_3 = 2.9 \text{ cm}$

$C_3 = 1298.4 \text{ m/s}$

$\eta = \frac{h_1 - h_3'}{h_1 - h_3} = \frac{3040.9 - 2198.02}{3040.9 - 2120.4}$

$= 0.9157 \approx 91.6\%$

Now the last thing remaining is efficiency of the nozzle as I stated earlier the efficiency of the nozzle is this is 3 dash efficiency of the nozzle is $H1 - H3$ dash divided by $H1 - H3$. So we will take the value of $H1$ and $H3$ three dash $H3$ 043.9 - $H3$ dash is 2198.02 divide by 3040.9 and $H3$ is 2120.4. And this case the efficiency as 0.9157 or 91.6 per second and that is all for today thank you very much