

Steam and Gas Power Systems
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Module No # 04
Lecture No # 19
Nozzles and Diffusers and Supersaturated Flow

Hi I welcome you all in this course on steam and gas power system today we will develop a general relationship between different operating parameter in a nozzle and supersaturated flow shall also to be discussed in this like lecture.

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- Maximum discharge of steam
 - General relationship between area velocity
 - Supersaturated Flow

Now to begin with we will start with the maximum discharge of the steam then we develop general relationship between area velocity and then we discuss supersaturated flow.

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$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

$$\frac{m}{A_2} = \sqrt{2 \cdot \frac{n}{n-1} \cdot \frac{p_1}{v_1} \left(\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}} \right)}$$

$$\frac{m}{A_2} = \sqrt{2 \cdot \frac{n}{n-1} \cdot \frac{p_1}{v_1} \left(\left(\frac{2}{n+1}\right)^{\frac{2}{n-1} \cdot \frac{2}{n}} - \left(\frac{2}{n+1}\right)^{\frac{n}{n-1} \cdot \frac{n+1}{n}} \right)}$$

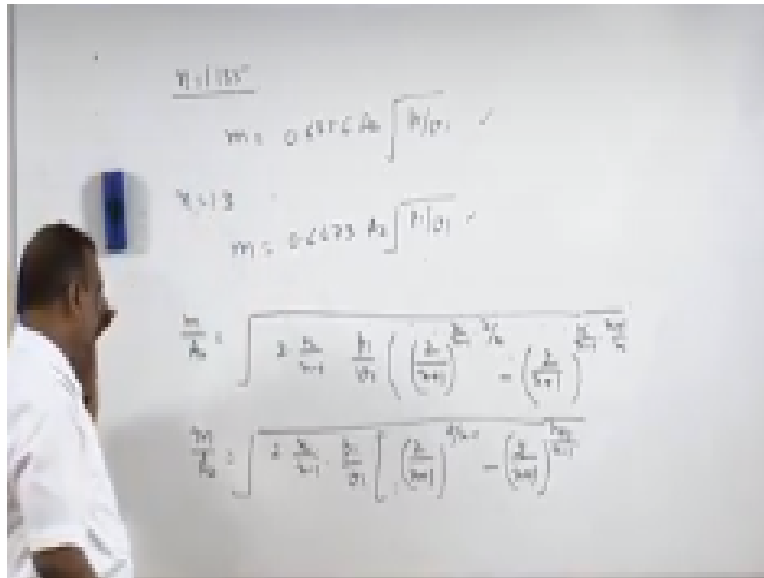
$$\frac{m}{A_2} = \sqrt{2 \cdot \frac{n}{n-1} \cdot \frac{p_1}{v_1} \left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}} \right]}$$

Now for the flow through a nozzle we already know the pressure ratio P_2 by $P_1 = 2$ by $N + 1$ raise to power N upon $N - 1$. For this pressure ratio the discharge is maximum we know the discharge equation M by A_2 that = $2 N$ over $N - 1 P_1 V_1 P_2$ by P_1 raise to power 2 by $N - P_2$ by P_1 raise to power $M + 1$ by N . Now this is the equation for the mass flow rate through the nozzle this is for pressure ratio when the discharge of the nozzle is maximum.

Now putting this P_2 by P_1 here we will getting M by $A_2 =$ under root $2 N$ over $N - 1 P_1$ by V_1 . Now P_2 by P_1 is 2 b $N + one$ raise to power M over $N + 1$ multiplied by 2 by N . And this 1 is - P_2 by P_1 raise to power sorry 2 by $N + 1$ raise to power N over $N - 1$ multiplied by $N + 1$ by N . Now this this N will be carried out this N will be cancelled out and will be getting M by A_2 as $2 N$ over $N - 1 P_1$ by $V_1 2$ by $M + 1$ raise to power $N - 1 - 2$ by $M + 1$ raise to power $M + 1$ divided by $M - 1$.

Now we will further simplify because once we know the value of N we can find the mass flow rate for the cross section area A_2 . Cross section area A so for dry and saturated steam when dry saturated steam enters the nozzle and its gets expanded.

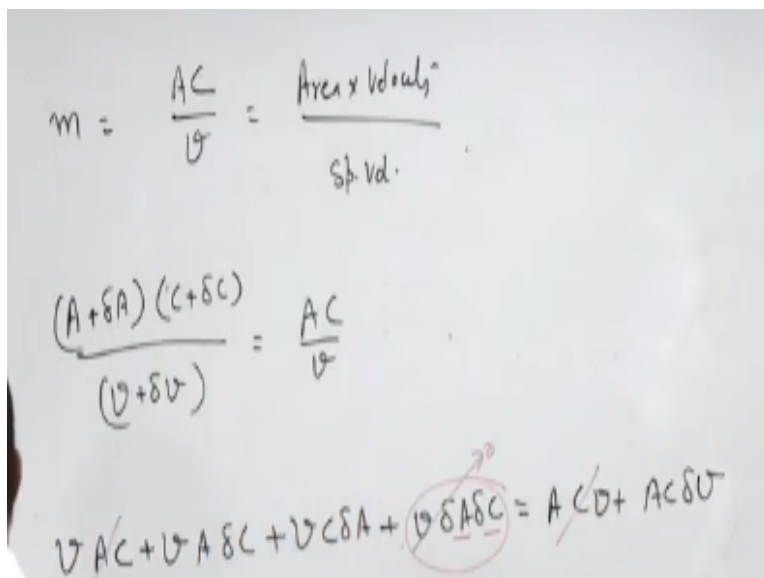
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The value of N is 1.135 now if you put the value of N = 1.135 here we get mass flow rate as 0.6356 A2 under root P1 by V1. Mass flow rate when N = 1.3 when steam is superheated when entering the nozzle then N = mass flow rate = 0.6673 A2 P1 by V1.

We are getting this expansion by putting this value N in this ratio so this is the maximum mass flow rate while flowing through a nozzle initial condition are P1 and V1 we keep on putting the value of different P1 and V1 will be getting a different maximum mass flow rate through the nozzle. Now we develop a general relationship among the velocity and pressure and density now we will be developing a general relationship between the parameters.

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This is a relationship between area and pressure mass flow rate through the passage is AC by V mass flow rate is area into velocity divided by specific volume right. Okay now $A + \Delta A$ $C + \Delta C$ ΔV divided by $\Delta V = KC$ by V. Now we can write $VA \Delta A + V$ sorry $VAC + VA \Delta C + VC \Delta A + V \Delta A \Delta C = ACV + AC \Delta V$ right.

Just simply multiply with this and this with this one now here we can we cancel this cancelling this by this $AC \Delta V$ ok and this can be taken as 0 because ΔA is final decimal again ΔC also decimal this is will be turning to 0. So the final expansion is $VA \Delta C + VC \Delta A = AC \Delta V$ divided by VAC divided by VAC divided by VAC it will give us the expansion.

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The image shows handwritten mathematical derivations on a whiteboard. At the top, the equation $\frac{dA}{A} = \frac{1}{n} \frac{dp}{p} \left(\frac{a^2}{c^2} - 1 \right)$ is written. Below it, the speed of sound a is defined as $a = \sqrt{np\omega}$ and $a = \sqrt{\gamma RT}$. To the right, the Mach number is defined as $M = \frac{c}{a}$. At the bottom, the equation $\frac{dA}{A} = \frac{v dp}{c^2} - \frac{1}{n} \frac{dp}{p}$ is written, followed by its rearranged form $\frac{dA}{A} = \frac{1}{n} \frac{dp}{p} \left(\frac{np\omega}{c^2} - 1 \right)$.

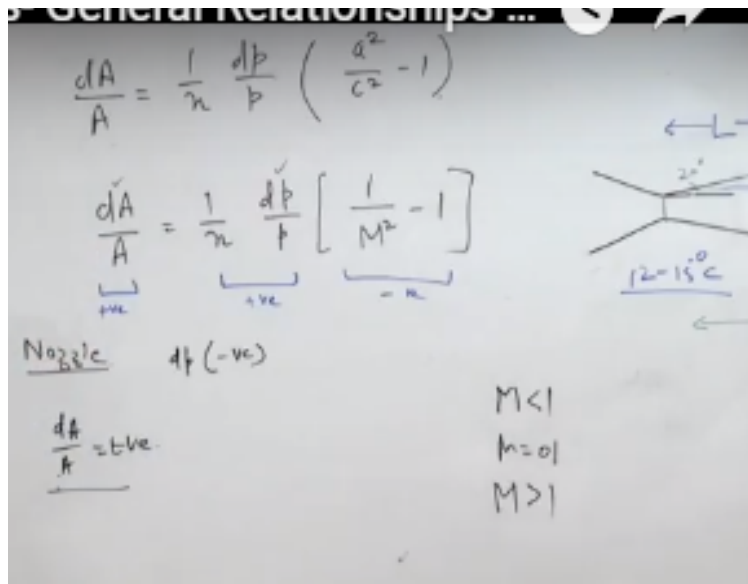
ΔC by $C + \Delta A$ by $A - \Delta V$ by $V = C$ because in this expression this VA will be cancelled here VAC will be cancelled here AC will be cancelled here ok. And if ΔC ΔA ΔV are tending to 0 then we can always write $\Delta C - C + \Delta A$ by $A - \Delta V$ by $V = 0$. Now all of us know that PV raise to power $N =$ sorry constant not zero constant.

PV raise to power $N =$ constant or we can write in differential form PT by $P + N \Delta V$ by $V = 0$ or ΔV by $V = - \Delta P$ sorry one by $N \Delta P$ by P that is one thing second is we already know that $\Delta C = -VDP$ from momentum equation. So ΔC by C is going to be ΔC by $C - DVP - C$ square when we take this $\Delta C = -VDP$ by C and ΔC by $C = -VDP$ by C square.

Now + DA by A - DV by V = -V are putting this then it is going to be + 1 by N DP by P = 0. We will take DA by A on left hand side = VDP by C square - 1 by N DP by P right. Now here DA by A can be taken as 1 by N PD by P inside the bracket because this we are taking out. So inside the bracket will remain NPV by C square - 1 now DA by A = 1 by N DP by P.

Now NPV if you remember the sonic velocity of steam is going to be NPV for gas it is going to be gamma RT. So this is nothing but sonic velocity and this is going to be A square by C square - one. There is a term which is known as there is a dimensional term which is known as number MAC number and MAC number is actual velocity divided by sonic velocity.

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So this expression further written as DA by A = 1 by N DP by P 1 by M square - 1. Now this is the dromatic equation now this equation as two differential terms which is which are DA and DP. Let us start with the flow through the nozzle flow through nozzle fine there is a flow through nozzle this DP will be negative because in the direction of the flow there is a fall in pressure due to this fall in pressure the kinetic energy is attain by the fluid.

So in the direction of the flow there is a drop in the pressure so here DP is going to be negative now DP is negative or we can say this DP is negative 1 by M square - 1 the velocity of steam is less than the sonic velocity or A is sorry this C is less than A or M is less than 1 when M is less than 1 this expression is positive right and DP is negative so DA by A is negative this expression

I will write again when M is less than 1 this expression is positive because this is greater than 1 by M is greater than 1.

1 by M square greater than 1 and this expression we have already assumed negative so DA by A is going to be negative it means up to the sonic velocity or less than for velocity less than sonic velocity if you want to draw the shape of the nozzle. The shape of the nozzle is going to be like this now second case $N = 1$. When $N = 1$ DA by A this expression is zero one by M square is 1 and this expression is 0.

When this expression is 0, DA by A is also 0 it means there is no change in velocity sorry there is no change in cross section area for a sonic. Third one is well M is less than 1, M is 0 we have already discussed M is less than 1 sorry $M = M$ is less than 1 we have discussed now the flow is supersonic. M is greater than 1 when M is greater than 1 this expression is negative when the M is greater than 1 then M is greater than 1 this one by M square is less than one and this going to be negative.

When this is negative this is negative DA by A is going to be positive it means when the flow is supersonic flow the passage has to be diverging. DA by A is positive is positive so the three cases we have discussed when velocity supersonic velocity is sonic and velocity is supersonic. When the steam velocity is sub sonic the passage is converging when steam velocity is sonic there is no change in cross section area when the velocity is supersonic the passage will become diverging and this is a very interesting situation.

It means in this area the velocity will increase in cross section area will also increase but normally what happens when we deal with the fluids. When we reduce the cross section area the velocity increases but in supersonic zone if we want to increase the velocity zone we have increase the cross section area and in most of the nozzles most of the nozzles you will observe that these two lines are not equal.

The converging length is shorter much shorter than the diverging part of the nozzle it is something like this reason being in the diverging part if we take this angle there is a limiting

value for this angle and that limiting value is 20 degree right. If this angle is more than this separation of flow separation flow because the fluid is diverging this is a diverging passage.

So separation of flow will take place from the wall and this is reduce the efficiency of the nozzle because this will incur losses in the flow of the fluid and this will reduce efficiency of the nozzle. So normally this angle because 20 is the limiting value 12 to 15 degree the value of diverging value is taken and since we know this time it was pro diameter we know the exit diameter that is how the length of the diverging section is decided.

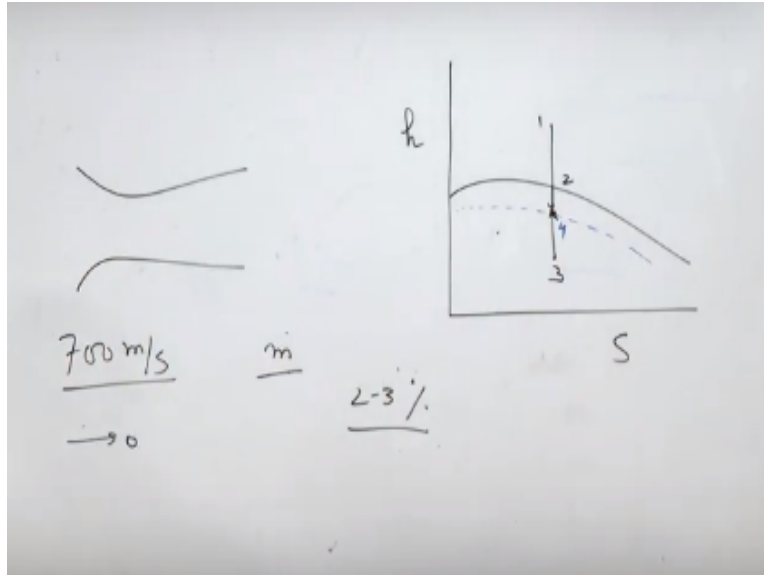
This angle is known to us this diameter is known to us this diameter we have already calculated while designing the nozzle and this will decide the length of the nozzle length of the diverging part of the nozzle. Now let us take the case of diffusers now in diffusers the kinetic energy is converted into pressure energy right. So velocity is reduced in diffusers the velocity is reduced and the pressure is increased.

So in diffusers this DP is positive right this DP is positive let us take case of diffusers this diffuser there is a pressure raise so DP is positive N is greater than one entry is supersonic in diffuser it is reverse on the nozzle entry is supersonic M is greater than 1 right. This is going to be negative right when this is greater than 1 at normal positive this is positive this negative just a minute.

When M is greater than 1 then this is negative right DP by P is positive so DA by A is going to be same process we will adopt here DA by A is going to be negative right. When M is N then there is no change in the throat area. When M is less than 1 subsonic right when M is less than 1 that case this is positive this is positive and this is going to be positive right. So this is how we decide the shape of nozzles and diffusers.

There is a phenomena during the flow inside the nozzle which is known as super saturated flow of the fluid.

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In supersaturated flow of fluid what happens suppose if I depict expansion in a nozzle on an enthalpy entropy diagram let us say inlet is superheated steam is superheated right and this is state 3 this is exit state in fact the condensation should start after this point this is saturation point. So during expansion the moment steam two is attained the condensation should start but this does not happen.

There is a lack and there is a lack condensation starts after further expansion up to state 3, 4 let us say 4. Then it happens this happens because the condensation of process which cannot occur spontaneously right and the time spent by the fluid inside the nozzle because the velocities are of order of let us say 700 meter per second. So such a high velocity the fluid is flowing at the high velocity the time spent by the fluid inside the nozzle is almost tending to 0 right.

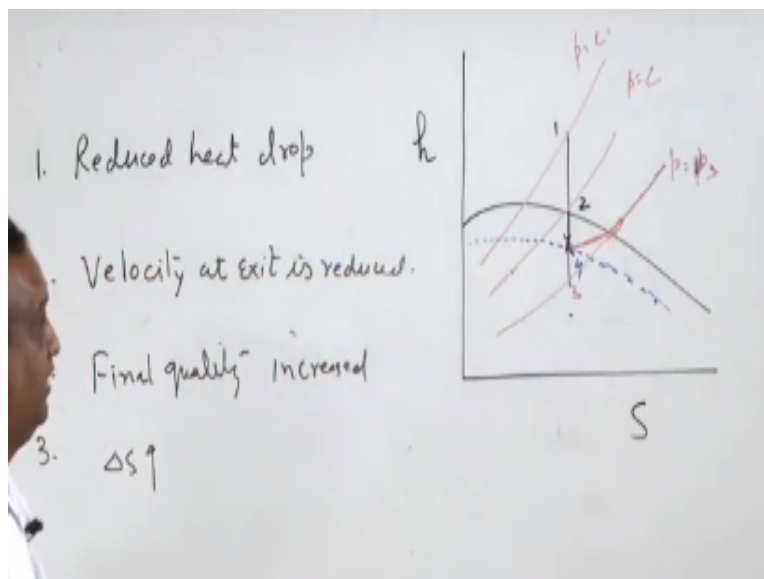
Very less time is spent inside the nozzle and during that time if this state arises the condensation of the fluid cannot take spontaneously because for condensation some nucleation site is required right. After certain value I mean initially condensation does not occur but after this state 1 because the state is continuously changing when the state comes when condensation takes place without any nucleation site that happens when the density of fluid is almost eight times the density of the saturated vapor.

So while expanding from this point to this point condensation will initiate at this point condensation this lag is known as super saturated flow of the fluid. The fluid is suppose to do with some condensation was suppose to take place inside the fluid but it is not taking place condensation is initiated it at this point right.

Second this is the vapor as in the molecules are have random energy, random kinetic energy and to contain because when the fluid attain this state the inter molecular attraction is sufficient to confine the molecule inside the droplet. And the droplet formation starts due to this supersaturated flow there are very interesting phenomena there is increase in the mass velocity.

Normally there is a 2 to 5 % friction losses in during the flow is had a nozzle but due to this phenomena super saturated flow there is increase in the mass flow rate inside the nozzle. So normally it is compensated the flow the losses due to friction are compensated by increase in mass flow rate in the nozzle. If there is no friction loss in that case increase 2 to 3 % increase in the mass flow rate so the nozzle will be observed.

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Now in addition to this they are certain affects of super saturated flow of fluid in the nozzle that is number one is reduced heat drop the heat drop is reduced and this reduces the velocity as at exit is reduced. So heat drop is reduced with corresponding velocity is reduced and number three

final quality of the vapor is increased and entropy is also increased right entropy is also increased.

That is the effect of super saturated flow through the nozzles and this as bearing on the discharge of the nozzles. Now this line when where the condensation is initiated this fiction line is known as Wilson line right. And if you draw the constant pressure line on the graph this is these are the constant pressure line right this is constant pressure line and this is the constant pressure line for this is three.

So this is the constant pressure line = P_3 now if we extend the curve of this to in this direction this one. So this will follow if the process will follow this this this curve right and here the steam or the vapor will be dry and saturated instead of state 2 the vapor will be dry and saturated in state 2 at pressure P_3 . This is all for today thank you very much.