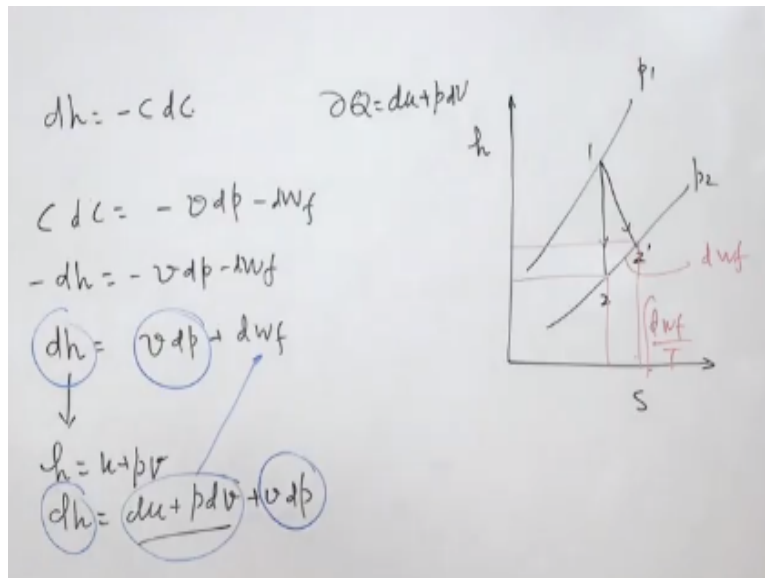


Steam and Gas Power Systems
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Indian Institute of Technology - Roorkee
Module No # 04
Lecture No # 18
Nozzles and Diffusers – Efficiency and Critical Pressure

Hello I welcome you all in this course on steam and gas power systems today we will discuss efficiency and critical pressure for a flow inside the nozzle and diffuser.

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As we know that for the flow through a nozzle the change in enthalpy $DH = -CDC$ change in kinetic energy and from momentum equation we have write that $CDC = -VDP - WF$. V is the specific volume W is the friction loss so if you compare this two equation that we get $-DH = -VDP - WF$ or uses DWF right.

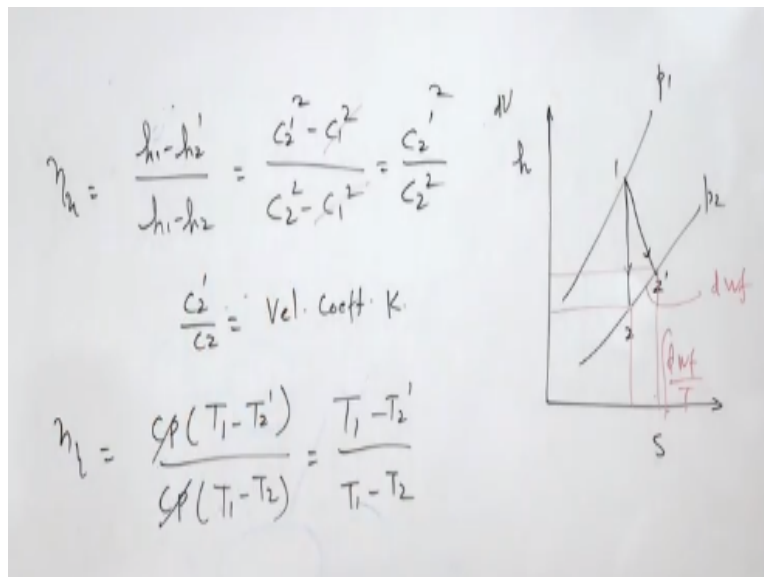
Or $DH = VDP + DWF$ for H we already know that $H = U + PV$ or $DH = DU + PDV$ this is $V + VDP$. Now if we compare this two equations we will find that this VDP is in common and this 2 that this is going to be = this 1 ΔW DW and this is nothing but heat transfer $del Q = DU + VDE$. Now if you want to depict this on enthalpy entropy diagram this is enthalpy specific enthalpy and specific entropy.

There is two specific lines this is P1 and this is P2 expansion is taking place from state 2 to state 2 there is a heat transfer suppose friction is there is a heat transfer. So instead of following this isentropic line vertical line the process will follow this 2 dash. Now at this two dash in this difference this difference in enthalpy is nothing but DWF and there is a rise in enthalpy also right.

And the rise in enthalpy will be DWF by T so this is the rise in entropy due to friction and this is the loss in enthalpy drop. When there is a loss in enthalpy drop definitely the velocity of vapor or air coming out of the nozzle will be less in comparison to that the case when expansion is taking place from 1 to 2 here efficiency of the nozzle comes into the picture. Efficiency of the nozzle is 100 % when the expansion is isentropic expansion.

This total enthalpy drop is converted into the kinetic energy but here in this case what is happening only part of this is converted into the kinetic energy and this part is going in terms of increasing entropy of the fluid.

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So the efficiency of the nozzle is going to be H1 - H2 dash divide by H1 - H2 right or we can write C2 dash square - C1 square divided by C2 square - C1 square. Always while doing the analysis of the nozzle we have neglected this so we can take always take as C2 dash square - divided by C2 square and this is this C2 dash by C2 is nothing but it is velocity co efficient.

We can say velocity coefficient K now in case of gas nozzles efficiency can also be expressed as $\frac{C_p T_1 - T_2}{C_p T_1 - T_2}$ dash divided by $\frac{C_p T_1 - T_2}{C_p T_1 - T_2}$. And there is C_p and C_p will be cancelled out so efficiency will be $\frac{T_1 - T_2}{T_1 - T_2}$ dash divided by $\frac{T_1 - T_2}{T_1 - T_2}$ right. There is another term in the nozzles is coefficient of discharge C_D .

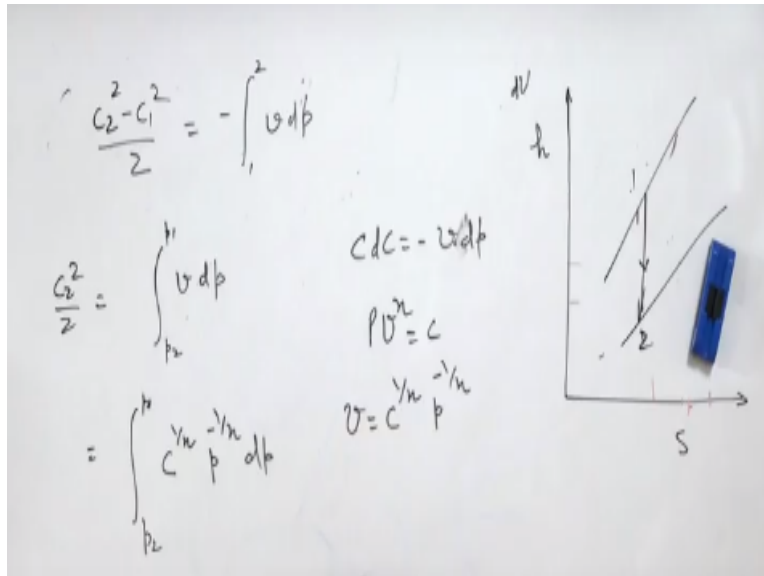
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The image shows a handwritten equation for Thrust. The word "Thrust" is written at the top and underlined. Below it, the equation is written as $(1 + F/A) (V_j - V_a) \times V_a$. To the right of the equation, there is a velocity vector V_a represented by a horizontal line with a circle at the end, pointing to the right.

So coefficient of discharge is actual ratio of actual flow divide by the ideal flow for all passages coefficient of discharge any passages actual flow divided by ideal flow. Now let us take case of diffuser is pressure arranging is increased at the cost of kinetic energy. So one two this is two dash so efficiency of the diffuser will be $\frac{H_2 - H_1}{H_2 - H_1}$ dash divided by $\frac{H_2 - H_1}{H_2 - H_1}$ right.

Now we will derive and expansion for the mass of discharge for the nozzle how much discharge is taking place through the nozzle we do not have expression yet for this.

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So as we know for the flow of the nozzles $C_2^2 - C_1^2$ square by 2 = - 2 to VDP. This we have derive from $VDC = -VDP$ right C_1 we can always neglect so C_2^2 square by 2 = integral P_2 to P_1 VDP . We always know that for polytropic this is polytropic process not his one expansion through nozzle.

State one to state two this a polytropic process so we can always say PV raise to power $N =$ constant right or $V = C$ raise to power 1 by N and P raise to power -1 by N . Now putting this value V here we will be getting P_2 to P_1 C raise to power 1 by N P raise to power 1 by N DP .

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$$\frac{C_2^2}{2} = \frac{\left[C^{1/n} p^{1-1/n} \right]_{p_2}^{p_1}}{1 - \frac{1}{n}}$$

$$C_2^2 = \frac{2n}{n-1} \left[p_1^{1/n} v_1^{1-1/n} p_1 - p_2^{1/n} v_2^{1-1/n} p_2 \right]$$

$$C_2^2 = \frac{2n}{n-1} \left[p_1 v_1 - p_2 v_2 \right]$$

$$C_2 = \sqrt{\frac{2n}{n-1} (p_1 v_1 - p_2 v_2)}$$

Now we can easily integrate this equation and we will be getting C_2^2 square by 2 = C raise to power one by N P raise to power 1 - 1 by N divided by 1 - 1 by N from P_2 to P_1 right. Now C_2^2 square by 2 or we will say that C_2^2 square is $2N$ over $N - 1 + C$ raise to power 1 by N.

T raise to power one by N = P raise to power 1 by N and V so here we can write P_1 raise to power 1 by N, V_1 P_1 raise to power 1 by N - again C we can always write P_2 raise to power 1 by N V_2 P_2 raise to power 1 - 1 by N right. If we further simply this then C_2^2 square = $2N$ over $N - 1$ this is $P_1 V_1 - P_2 V_2$ right.

We have velocity terms for the velocity 2 is under root $2N$ upon $N - 1$ $P_1 V_1 - 2V_2$ now we have find the mass flow rate.

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So mass flow rate is C_2 at the exit A_2 divided by V_2 right now in order to achieve this what we can do we can take $C_2 =$ under root $2N$ upon $N - 1$ we can take out $P_1 V_1$ So we will be getting $1 - P_2$ by P_1 multiplied by V_2 by V_1 fine now V_2 by $B_1 = P_1$ by P_2 power 1 by N or we can write P_2 by $P_1 = P_2$ by P_1 raise to power - 1 by N.

Now putting thus value here P_2 by P_1 here we will get C_2 is equal to under root $2N$ upon $N - 1$ $P_1 V_1$ $1 - P_2$ by P_1 raise to power 1 - 1 by N. Now C_2 multiplied by N ok so C_2 multiplied by A_2 by V_2 will give us the mass flow rate so this is going to be = A_2 now V_2 is nothing but $V_2 =$

$V_1 P_1$ by P_2 by P_1 raise to power -1 by N . P_2 we can take from here P_2 by P_1 raise to power -1 by N multiplied by P_1 .

And if this goes inside then $2N$ upon $N - 1$ $P_1 V_1$ because this V_1 will get squared so this is P_1 by V_1 and this will take inside the bracket. So this is will become P_2 by P_1 raise to power 2 by N - P_2 by P_1 this 2 by $N + 1 - 1$ by N . Am repeating we have taken from here we have taken out $P_1 V_1$ we get the velocity C_2 . Now C_2 velocity at the exit arial cross section at the exit divide by specific volume at the exit.

So velocity of the exit will get from here we have further simplified this equation we will we taken P_1 out and this expression is modified by this expression then C_2 is multiplied by A_2 divided by V_2 and V_2 we have taken P_2 by P_1 raise to power -1 by N multiplied by V_1 this expression is taken inside so we are getting P_1 by V_1 and this expression.

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The image shows handwritten mathematical work and a graph. The equations are:

$$\frac{m}{A_2} = \sqrt{\frac{2n}{n-1} \frac{p_1}{\rho_1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2n}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]}$$

$$y = \left(\frac{p_2}{p_1} \right)^{\frac{2n}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}}$$

$$y = y^{\frac{2n}{n}} - y^{\frac{n+1}{n}}$$

$$y' = \frac{2}{n} y^{\frac{2n}{n}-1} - \frac{n+1}{n} y^{\frac{n+1}{n}-1} = 0$$

$$\frac{2}{n} y^{\frac{2n}{n}-1} = \frac{n+1}{n} y^{\frac{1}{n}}$$

The graph on the right shows a coordinate system with a vertical axis labeled $\frac{dV}{dh}$ and a horizontal axis labeled S . Two curves are plotted: a straight line labeled '1' and a curve labeled '2'. A vertical line segment connects the two curves, and a horizontal line segment is drawn from the intersection of the two curves to the horizontal axis.

If you further simply this you will get $M = A_2$ under root $2N$ upon $N - 1$ P_1 by V_1 P_2 by P_1 raise to power 1 by N - P_2 by P_1 raise to power $N + 1$ by N raise to power 1 by N is 1 by N . 1 by $N + 1$ by N now this is the mass flow rate we can take here also N by A_2 mass flow rate per unit area is this right. Now here I want to have maximum this is about the discharge of the nozzle.

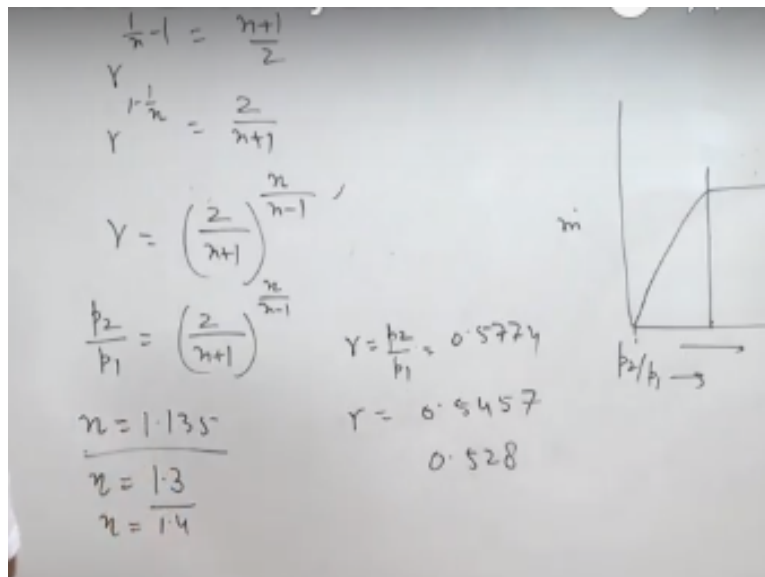
Now I want to have maximum discharge in order to have maximum discharge I should differentiate this equation this is normal practice so n then this term these terms are constants inlet pressure inlet specific volume nozzle is constant right and this constant right. We want to find for what pressure ratio the discharge is maximum when discharge this pressure is 1 discharge is 0.

Obviously when the leading side and the trading side if the pressure is same P_2 and P_1 is same there is no flow but when we start decreasing the P_2 the flow the fluid id start flowing into the nozzle it does not mean that we if this we make this expression zero the flow become infinite. Normally what happens after attaining the certain value the flow becomes constant irrespective of the value of this pressure ratio that is known as choking of the nozzle.

So first of all we will differentiate this in fact we will take a function $Y = P_2$ by P_1 raise to power 2 rest are constant. So $- P_2$ by P_1 raise to power $n + 1$ by n . So we will differentiate Y this we can taken as for the sake of convenience $R Y = R_2$ by $n - R n + 1$ by $n Y$ dash is 2 by $n R$ raise to 2 by $n - 1 - n + 1$ by $n R$ raise to power $n + 1$ by $n + 1$ and this is = 0.

So 2 by n or raise to power tow by $n - 1 = n + 1$ by $n R$ 1 by n .

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Now if you further simply this R raise to power 1 by N - 1 = N + 1 by 2. Or R 1 - 1 by N 2 by N + 1 or we can write this ratio R is 2 by N + 1 raise to power N by N - 1. So we have to maintain this pressure ratio P2 b P1 in order to have maximum flow through nozzle. If the pressure is less than this it will not increase so if we want to show on a graph it is flow is going to give something like this is P2 by P1 this is N this is decreasing in this direction right and this is mass flow rate.

So first of all it will increase and then it will become stagnant and this is known as critical pressure ratio. Now for steam saturated steam saturated steam is in that case the value of N = 1.135. If the value of N = 1.135 here in that case the R is going to be R = P2 by P1 is going to be 0.5 this is for saturated steam is getting expanded in a nozzle.

Suppose getting expanded in a nozzle suppose steam is superheated than N = 1.3. In that case R is going to be = 0.5457 simply just putting the value of N = 1.3 here we getting this expansion. Suppose it is a gas nozzle so N = gamma suppose it is air 1.4. In that case it is going to be 0.528 so for any value of N or we can find the pressure ratio for which the flow is maximum during flow through a nozzle.

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The image shows four equations for the mass flow rate C_2 and a graph. The equations are:

$$C_2 = \sqrt{2 \frac{n}{n-1} (p_1 v_1 - p_2 v_2)}$$

$$C_2 = \sqrt{2 \frac{n}{n-1} p_2 v_2 \left(\frac{p_1}{p_2} \frac{v_1}{v_2} - 1 \right)}$$

$$C_2 = \sqrt{2 \frac{n}{n-1} p_2 v_2 \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n-1}} - 1 \right]} \quad \frac{v_1}{v_2} = \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}}$$

$$C_2 = \sqrt{2 \frac{n}{n-1} p_2 v_2 \left[\left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} - 1 \right]} \quad \frac{p_2}{p_1} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}}$$

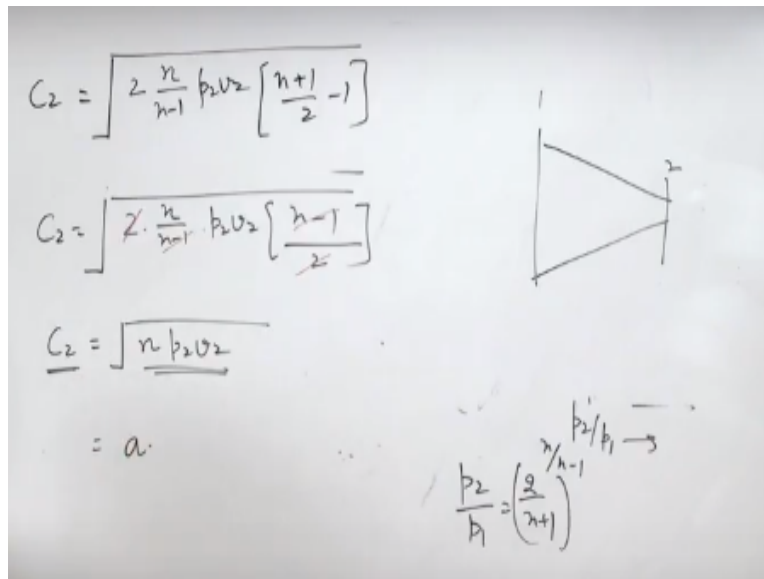
The graph shows mass flow rate m on the vertical axis and pressure ratio p_2/p_1 on the horizontal axis. The curve starts at the origin, rises to a peak, and then levels off to a horizontal line, indicating that the mass flow rate reaches a maximum and remains constant for pressure ratios below the critical value.

Now let us go back to the same equation right so we take P2 V2 common C2 here 2N over N - 1 P2V2 P1 by P2 V by V2 - 1 right. Now V1 by V2 = P2 by P1 raise to power 1 by N so we can

write $C_2 = \sqrt{2 \frac{n}{n-1} p_2 v_2^2 \left[\frac{n+1}{2} - 1 \right]}$ because this will reverse then it will be -1 and -1 .

Now again $C_2 = \sqrt{2 \frac{n}{n-1} p_2 v_2^2 \left[\frac{n+1}{2} - 1 \right]}$ here for the choking condition p_2 by $p_1 = \frac{2}{n+1}$ raise to power $\frac{n}{n-1}$ we have already driven this. So two multiplied by $\frac{n}{n-1}$ p_2 by $p_1 = \frac{2}{n+1}$ $\frac{n}{n-1}$ and this is $1 - \frac{n}{n-1}$ right.

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We can easily simplify this equation to find the value of C_2 at the throat $C_2 = \sqrt{2 \frac{n}{n-1} p_2 v_2^2 \left[\frac{n+1}{2} - 1 \right]}$ here this $1 - \frac{n}{n-1}$, $\frac{n}{n-1}$. So and this n and n will be cancelled out this is $\frac{n}{n-1}$ this is $1 - \frac{n}{n-1}$ so this expression is going to be $\frac{n+1}{2} - 1$. Now C_2 is under root $1 - \frac{n}{n-1}$ over $\frac{n}{n-1}$ $p_2 v_2^2$ divide by $2 \frac{n}{n-1}$ now this $\frac{n}{n-1}$ we will get cancel with this $\frac{n}{n-1}$ this 2 will get will get cancel with this 2 .

And the final expression is going to be $C_2 = \sqrt{n p_2 v_2^2}$ right so in a nozzle if a flow is adiabatic friction less ideal flow. This is one this is throat of the nozzle two right the velocity is going to be $\sqrt{n p_2 v_2^2}$ and this is nothing but sonic velocity of the fluid $\sqrt{n p_2 v_2^2}$ I going to be sonic velocity of the fluid at this particular condition that is all for today thank you very much