


Steam and Gas Power Systems
Prof. Ravi Kumar
Department Of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

Module No # 04
Lecture No # 17
Nozzles and Diffusers – Momentum and Continuity Equations

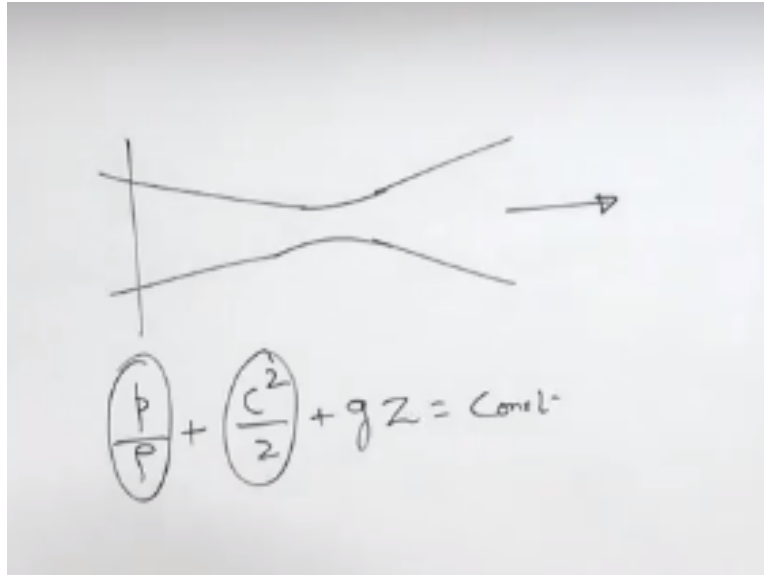
Hello I welcome you all in this course on steam and gas power systems today we will start with the nozzles and diffusers.

(Refer Slide Time: 00:33)

- 
- Type of nozzles and diffusers
 - Equation of continuity sonic velocity and Mach Number
 - Momentum Equation

First of all we will take up type of nozzles and diffusers then equation of continuity sonic velocity and mach number in case of nozzles and diffusers. Then momentum equation now let us start with the nozzles nozzle is a passage of varying cross section.

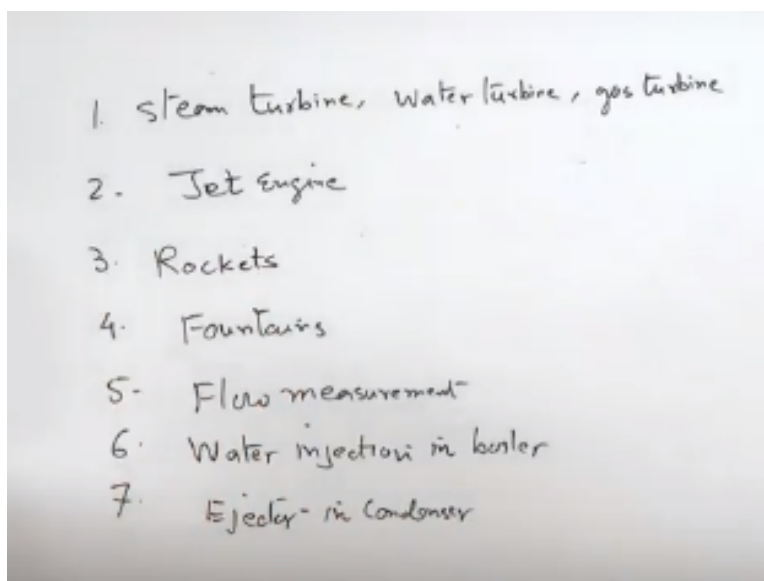
(Refer Slide Time: 00:51)



Nozzle is a passage of varying cross section it may be converging or diverging or the varying cross section may be converging and diverging as well. So it is a passage of varying cross section where pressure energy is converted into kinetic energy all of us know the equation $\frac{p}{\rho} + \frac{C^2}{2} + gZ = \text{constant}$. So in nozzle at the expense of pressure energy kinetic energy is attained.

And this high velocity jet which is emerging from the nozzle is used for variety of the purposes.

(Refer Slide Time: 01:43)



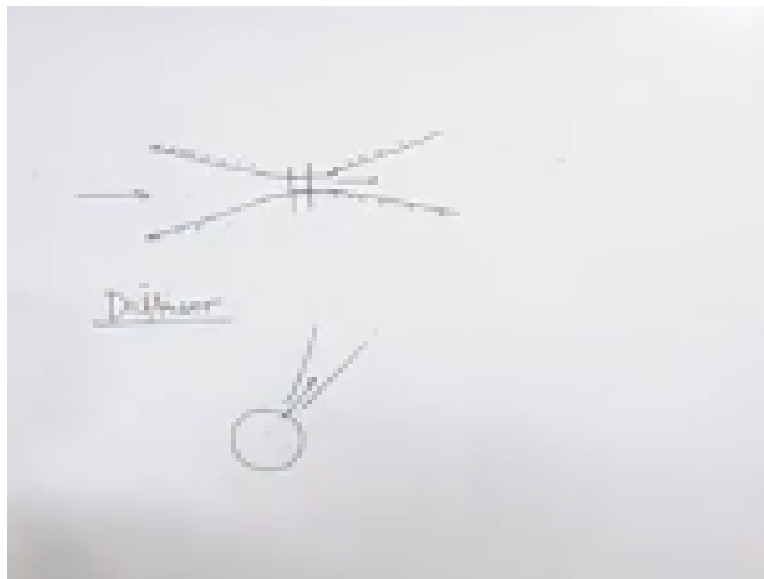
So regarding the uses of the nozzle first of all it is used in steam turbines turbine hydro turbine or water turbine and gas turbines now the purpose of nozzle is to convert pressure energy into the

kinetic energy and mass flow rate remains constant throughout the passage of the nozzle the mass flow rate of the fluid remains constant and this high velocity get emerging from the nozzle hits the turbine blades and causes turbine rotor rotate in a particular direction.

And that is how I grate energy or work produced in a turbine now another use of nozzle is jet engine. In jet engine the purpose of creating thrust the nozzles are used rockets then public use fountains then flow measurement. Nozzles are used for flow measurement in a passage in addition to this water injection in a boiler or further remover of air in condenser steam rejecter in condenser.

There are few examples there are many other used of nozzles which I cannot list here there are three types of nozzles we will go by the classifications of the nozzles or cross section of the nozzles.

(Refer Slide Time: 03:54)



There are three types of nozzles one is convergent nozzle where the cross section of nozzle decrease this is the passage of the nozzle this is wall of the nozzle so flow is taking place in this direction cross section area of the passage keeps on decreasing this is known as convergent type nozzle. Another type of nozzle is diverging nozzle where cross section area of nozzle increases at the same time the velocity of fluid also increases in a particular conditions.

So these type of nozzles are known as diverging nozzles if you connect them then it becomes convergent and divergent nozzles and this is throat area. In throat area there is no change in cross section so nozzle as three part inlet and outlet converging part, diverging part and a throat which in the throat there is no change in the cross section area of the nozzle.

Now another part is diffuser now in diffuser kinetic energy is converted into the pressure energy the function of diffuser is reverse of the function of nozzle it can be axial diffuser like this or it can be radial flow diffuser also.

So in radial flow diffuser the best example is centrifugal pump where kinetic energy of fluid is converted into the pressure energy so that is one of the main application of the diffuser there are many other applications but centrifugal pump the kinetic energy is imparted to the fluid and subsequently this kinetic energy is converted into the pressure energy with the help of it user. Now we will start with the continuity equation let us take continuity equation of a flow through a nozzle.

(Refer Slide Time: 05:48)

$$\dot{m} = \rho A C$$

$$\frac{\partial \dot{m}}{\partial t} = \frac{\partial \rho}{\partial t} A + \rho \frac{\partial A}{\partial t} + \rho A \frac{\partial C}{\partial t} = 0$$

$$\left(\frac{\partial \rho}{\rho}\right) + \frac{\partial A}{A} + \frac{\partial C}{C} = 0$$

$$\frac{\partial \rho}{\rho} + \frac{\partial C}{C} = 0$$

$$\left(\frac{\partial \rho}{\rho}\right) = -\frac{\partial A}{A}$$

So this is $\dot{m} = \rho A V$ is velocity or here we are denoting V by C so C is the velocity and doing partial differential it is going to be $\frac{\partial \dot{m}}{\partial t} = \frac{\partial \rho}{\partial t} A + \rho \frac{\partial A}{\partial t} + \rho A \frac{\partial C}{\partial t}$ right. Now mass flow rate remaining constant throughout the passage so this is 0 now we are leaving only $\frac{\partial \rho}{\rho} + \frac{\partial A}{A} + \frac{\partial C}{C} = 0$.

While flowing through a nozzle suppose a air or steam is flowing through the nozzle area will vary velocity will vary and density also vary or we can say by virtue of variation and density and the velocity are of the cross section is area of the nozzle will vary. However in the case of liquids in liquids change in density of the pressure is in significant so this can be neglected.

So we can write $\delta A \cdot C + A \cdot \delta C = 0$ or $\frac{\delta C}{C} = - \frac{\delta A}{A}$ right it means at the expanse of pressure when the velocity of fluid is increasing the cross section of the area of the passage will keep on decreasing or it is going to be in this shape even at very high velocity now let us do first law analysis of the flow of fluid through the nozzle.

(Refer Slide Time: 07:37)

$$dh + Cdc + gdz = 0$$

$$(h_1 - h_2) + \frac{c_1^2 - c_2^2}{2} = 0$$

$$\frac{c_2^2 - c_1^2}{2} = h_1 - h_2$$

$$c_2^2 = 2(h_1 - h_2)$$

$$c_2 = \sqrt{2(h_1 - h_2)}$$

$$c_2 = \sqrt{2 \times 1000 (h_1 - h_2)}$$

$$= \sqrt{2000 (h_1 - h_2)}$$

$$= 44.72 \sqrt{h_1 - h_2}$$

Let us take a passage or a of varying cross section the walls are rigid there is no walls on the movement of the muzzle fluid is flowing in this direction this is plain one this is inlet this is plain two at outlet and let us say this is control volume on which our study is focused very close to the wall right. And if we write first law equation for flow that is $\delta P \cdot \rho + C \delta C + G \delta Z = 0$ right or now this is DH.

So DH by $C \delta C + G \delta Z = 0$ or in terms of volume $2 H_1 - H_2 + C_1^2 - C_2^2 = 0$ here G because it is horizontal direction. So there is no change potential energy so this can be

taken as $0 = 0$ or we can write $C_2^2 - C_1^2$ by $2 = H_1 - H_2$. If you want to write in differential form then it is $DH = -C_1 dC_1$.

Now if the C_1 is 0, C_1 is not 0 if you compare the value of $C_2 - C_1$, C_2 is much larger than C_1 or outlet velocity of the nozzle is much larger than the inlet velocity. And it is often neglected because if you consider the inlet velocity it is not going to make much difference. So often it is neglected so we can always find $C_2^2 = 2 H_1 - H_2$ or exit velocity from the nozzle is under root two $H_1 - H_2$.

But we should remember that enthalpies are given in kilo joules so this kilo joules as to be converted into the joules while calculating the exit velocity. So C_2 is going to be under root 2 into 2000 $H_1 - H_2$ and it is going to be under root 2000 $H_1 - H_2$ we have multiplies $H_1 - H_2$ by 1000 just to convert kilo joules per KG to joules per KG and here we get 44.72 under root $H_1 - H_2$.

This is regarding the flow of the fluid suppose there is a gas nozzle the working fluid in a nozzle is gas or air.

(Refer Slide Time: 10:58)

The image contains handwritten mathematical derivations and a schematic diagram of a nozzle. On the left, the equations are:

$$C_2 = \sqrt{2C_p(T_1 - T_2)}$$

$$= \sqrt{2C_p T_1 \left(1 - \frac{T_2}{T_1}\right)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$C_2 = \sqrt{2C_p T_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

On the right, the equations are:

$$C_2 = \sqrt{2 \times 1000 (h_1 - h_2)}$$

$$= \sqrt{2000 (h_1 - h_2)}$$

$$= 44.72 \sqrt{h_1 - h_2}$$

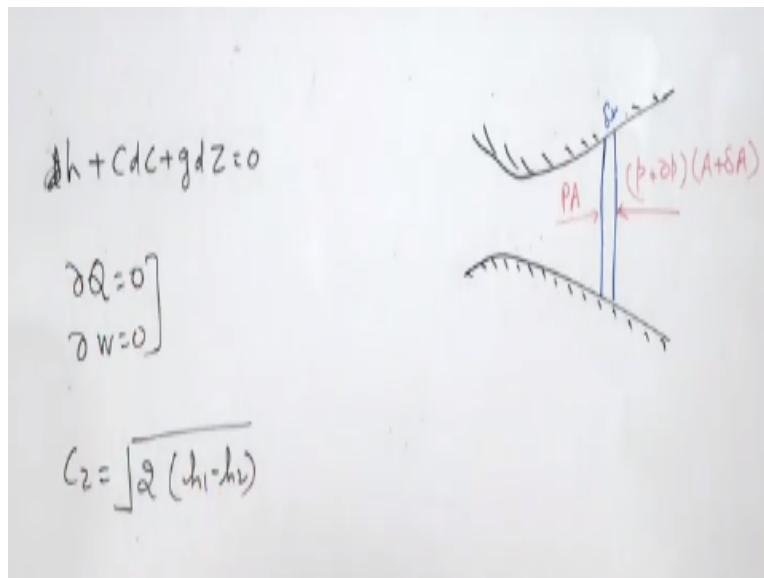
The diagram shows a nozzle with an inlet section labeled '1' and an outlet section labeled '2'. An arrow indicates the flow direction from section 1 to section 2. The nozzle walls are shown as converging lines.

In that case C_2 also be taken as under root $2C_p T_1 - T_2$ this is the change in enthalpy of the base or if you want to further simplify then two $C_p P_1^{1/\gamma} (1 - T_2/T_1)$ here you can use the ideal gas T_2

by $T_1 = P_2$ by P_1 raised power $\gamma - 1$ over γ . So C_2 is going to be = under root $2CP$
 $P_1^{1-\gamma} - P_2$ by P_1 raised power $\gamma - 1$ over γ . Because pressure drop in nozzle is often
 expressed in terms of ratio of down stream pressure and up stream pressure right.

So this can be expression for the velocity of air or gas coming out of the nozzle now after this we
 will come to the momentum of equation for the flow inside the nozzle. Let us take a generalize
 case were there is a convergent divergent nozzle.

(Refer Slide Time: 12:18)



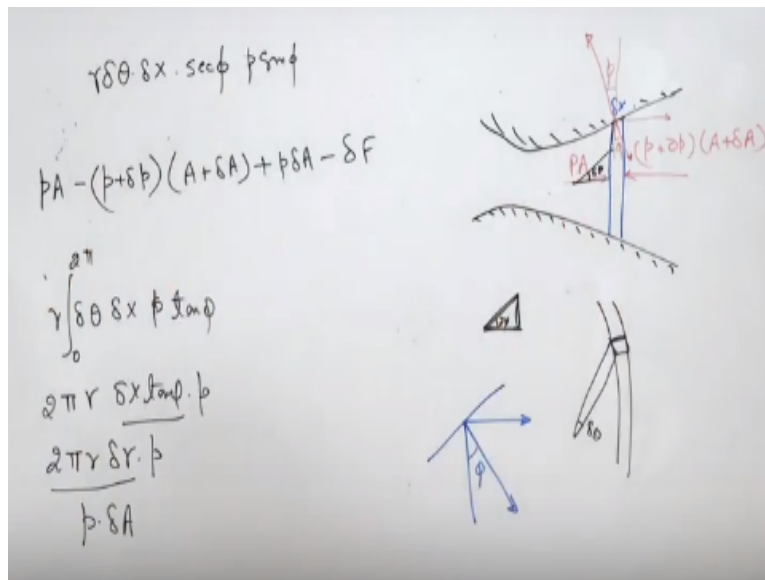
And let us take a thin section out of this of distance δx now if you do the force balance on
 the section in this direction the force is pressure into area. Whatever pressure is exist on existing
 on this side multiplied by the area. Now existed on this from this side is $P + \Delta P A + \delta A$
 whatever it is because normally in the x mean in this direction there is a fallen pressure but to
 have a generalize equation we have taken $P + \Delta P$ existed on this direction right.

Now this is A this is the nozzle valve and pressure of the fluid is outside pressure it is greater
 than the pressure outside this nozzle. This is valve so for whatever analysis we have done we
 have considered flow through nozzle is adiabatic flow. If you remember the equation is this
 equation $DH + CDC + GDZ = 0$. Here also we have considered that the flow through nozzle is
 adiabatic and because valves are rigid that is not work.

So $\Delta Q = 0$ and ΔW is also 0 considering this only we derived the equation $C_2 = \sqrt{2H_1 - H_2}$ right. Now here when we are doing the force analysis force is also being exhausted on the wall of the nozzle. So force is also being exhausted on the wall of the nozzle virtue of pressure of fluid inside this nozzle and reaction will work in this direction.

Reaction will work on this element in this direction if you take from the vertical this is ϕ and this is nozzle diverging angle is also ϕ .

(Refer Slide Time: 15:12)



Because nozzle is a diverging passage this is a diverging passage and this angle is ϕ . So when we take pressure from normal to for so pressure is normal to this surface so this angle is going to be ϕ right. Now second thing is this is a ring of this vector ring around the periphery of the nozzle of thickness δx .

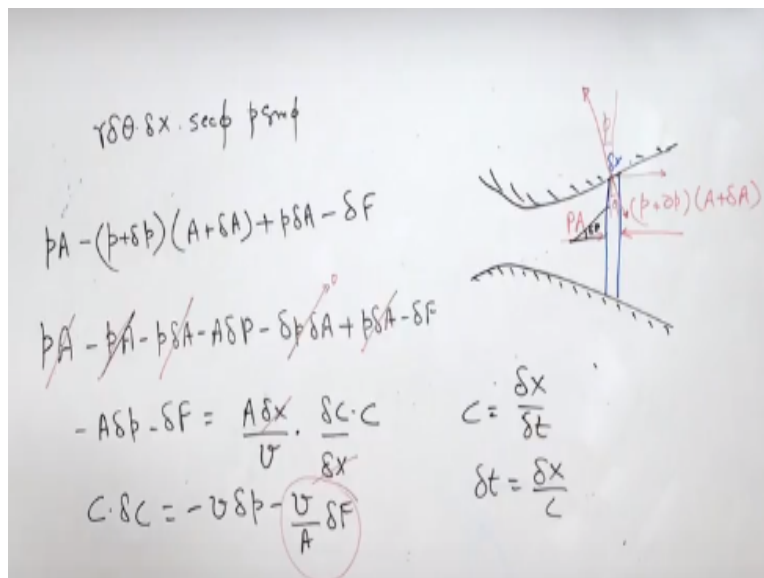
If it is the ring of δx if you take a small element on the ring which is exerting $\delta \theta$ at the center small element on the ring which is exerting $\delta \theta$ on the center right on the entire ring suppose it is a part of the ring is exerting $\delta \theta$ on the center if I want to area of this this this element it is going to be $r \delta \theta$ multiplied by δx right. Now this area this then again now we have found this area now projection in this area in this direction.

So project of this area in this direction and this is $\delta x \sin \theta$ this going to be $\sec \theta$ now force is exerted on this force is exerted on this and reaction of this force in this direction and component of force this direction is going to be this is $\delta x \cos \theta$. Component of this force in this direction is going to be I will explain this diagram it is something like this force is working in this direction reaction force this is $\delta x \sin \theta$ and component of force in this direction.

So component of this force in this direction is going to be $\sin \theta$ $P \sin \theta$ and this is $P \sin \theta$. So this will give you the force working in this direction in this passage due to pressure now the net force is net force is PA this pressure into area right - $P + \delta P$ $A + \delta A$ this force. Not this force $R \delta x \sin \theta$ and $\sin \theta$ is $\tan \theta$ $P \tan \theta$ right.

If you integrate this sorry integrate this 0 to 2π this will become $2\pi \delta x \tan \theta$ multiplied by P right. Then $\delta x \tan \theta$ $\delta x \tan \theta$ is δR so it is going to be $2\pi R \delta R$ multiplied by P and $2\pi R \delta R$ is nothing but δA . And this force is also working in this direction so we will also add $P \delta K$ now the third will consider the force of friction which is opposing the movement of the flow of in this direction. So that is going to be let us say minus we do not know this force that if is going to be in this direction.

(Refer Slide Time: 19:22)



So that is the net force working on this element right now this net force we will simplify this equation $PA - P \delta A - A \delta P - \delta P \delta A + P \delta A - \delta F$ right. Now here we can

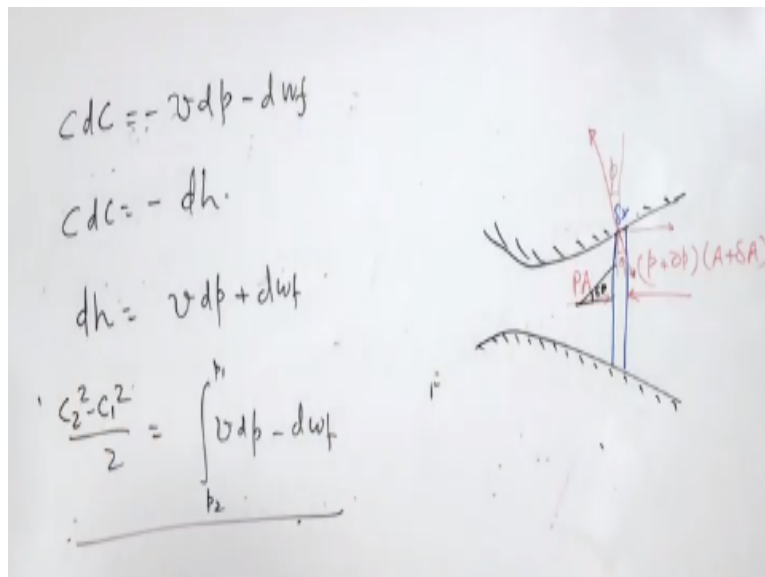
cancel out some terms some of the terms that is PA this will be cancelled out P delta A P delta L is cancelled out now this we consider to be 0 because delta P is very infinite decimal this is also infinite decimal.

So there product will turn to be 0 so we are neglecting this term so net force we are getting in the direction of the flow is - A delta P - delta now this force is mass into acceleration mass of the fluid in this and acceleration of this. So mass of the fluid is going to be A delta X by specific volume this is volume of this element divided by this specific volume multiplied by acceleration.

Acceleration is delta C by delta T okay here there is a unknown term delta T so delta T C is del X by del T. So del T is always del X by C so we will replace this del T by del X by C now here we can again cancel out this right and further simplification will give us the equation C del C is = - V del P - V by A del F. Just multiply by V divide by A we can further simplify this equation like this right.

Now again we have this term work in the friction per KG of fluid right work in the friction per because here we are considering friction. So work in the friction per KG of the fluid work is F in force in velocity right.

(Refer Slide Time: 22:30)



So friction force is ΔF velocity C DY per KG per Unit pass in order to find mass of the fluid mass of the fluid we will divide it by area velocity divided by specific volume. So this is work and this is mass of the fluid which is flowing we can further simplify it has $C \Delta F$ this C and C will be canceled out right. So we will be getting $V \Delta F$ by A this is friction per unit mass flow rate and this means this is friction per unit mass flow rate and this is denoted by NS this is denoted by WF .

So we can further write this equation as $C \Delta C = -V \Delta P - \Delta WF$ if the change is tending to be zero then this equation can be written as $CDC = -VDP - DWF$ right. If you remember that equation $CDC = -DH$ energy equation right in that case DH is going to be $= VDP + DWF$ right or we can write $C_2^2 - C_1^2$ by $2 = \int_{P_2}^{P_1}$ am not writing 1 to 2 am writing 2 to 1.

So it is $VDP - DWF$ right so this is the momentum equation for the flow of fluid inside the nozzle if the friction is in ideal when the friction is zero we can always consider this equation as the moment in this equation for the flow inside a nozzle this is all for today thank you very much.