**Principles of Casting Technology**
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**Lecture - 05**
**Solidification**
**Problem solving on solidification**

Welcome to the lecture on, Problem solving on solidification. In this lecture, we will try to solve the problems based on nucleation, finding the critical size, critical radius; also we will try to solve problems based on finding the solidification times. So, let us move to our first question. The first question is, calculate ratio of surface energy term to volume energy term in the nucleation energy equation at critical condition.

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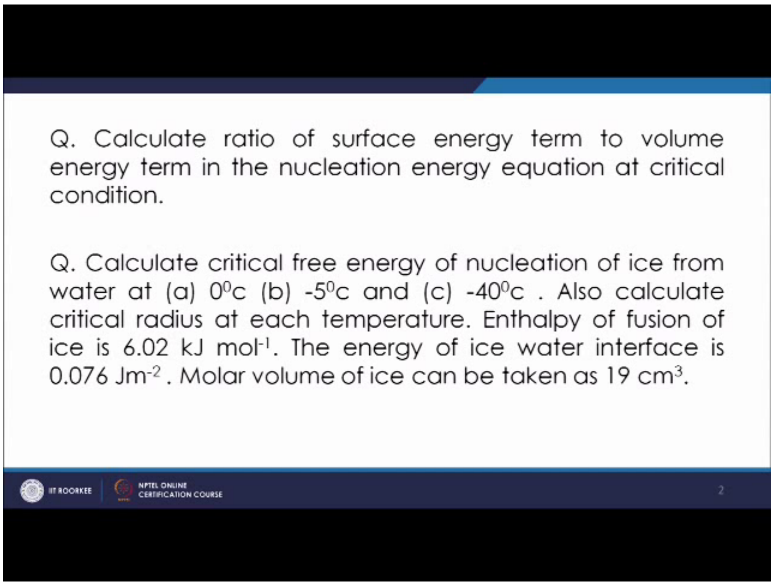


In this case, you have to find the ratio of surface energy term, and you know that surface energy term is nothing but 4 pi r square gamma.

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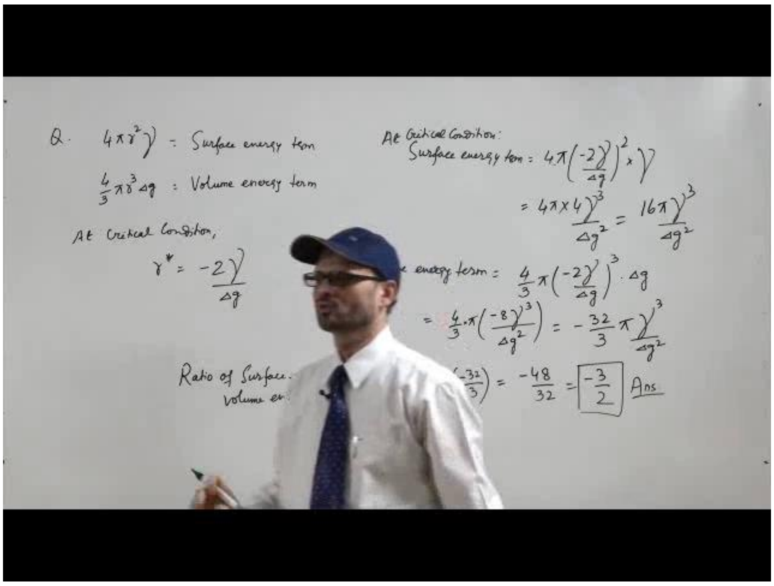


Where, r is radius of the spherical particle which is supposed to be nucleated, and the gamma is surface energy per unit surface area. This is your surface energy term, and the volume energy term is 4 by 3 pi r cube delta g. So, here again the r is the radius of spherical particle which has to be nucleated and delta g is the volume free energy change for unit volume and that is why it is the volume energy term. Now, we are supposed to find the ratio of these two terms, for the condition that there is critical condition. So, at critical condition we know, at critical condition we get critical radius as minus 2 times gamma upon delta g. This is what we have already derived; this is the critical radius which must be achieved at which there was the peak in that expression 4 by 3 pi r cube delta g, plus 4 pi r square gamma for that the peak was achieved at this particular radius.

We have to put these values of r star, as here and here and then you have to find the ratio. So, having this r star term your surface energy term becomes, so surface energy term becomes at critical condition, surface energy term will be 4 by 3. So, this is 4 only 4 pi r square, in place of r it will be r star so it will be minus 2 times, gamma upon delta g square into gamma. This will be 4 pi into 4 gamma, cube gamma square into gamma cube upon delta g square. Similarly, the volume energy term, this term will be 4 by 3 pi r cube delta g. So, again 4 by 3 pi in place of r we will put r star value it will be minus 2 times gamma upon delta g raise to the power 3 into delta g. It will be 4 by 3 into pi; into minus 8 gammas cube, upon this will be delta g cube into delta g here it will be delta g square. So, it will be minus 32 upon 3 into pi gamma cube upon delta g square, and it has come as 16 pi gamma cube upon delta g square.

So, if you take the ratio, ratio of surface energy term to volume energy term it will be nothing, but the ratio of this term upon this term in that pi gamma cube by delta g square is there as a constant it will be 16 is to minus 32 by 3. So, it will be minus 48 upon 32 and that will be minus 3 by 2. So, this is the answer. What we see is the ratio of the surface energy term to the volume energy term and this particular condition is minus 3 by 2.

Now, we will move to next question. The next question is regarding finding the critical free energy of nucleation of ice from water at the 3 temperatures given; the 3 temperatures are 0 degree C, minus 5 degree C and minus 40 degree C. What as others things been given is the enthalpy of fusion of ice. Also we have to calculate the critical radius at all these temperatures. Enthalpy of fusion of ice is given as 6.02 kilo joule per mole. Energy of ice water interface is 0.076 joule per meter square. Molar volume of ice can be taken as 19 centimeter cube.

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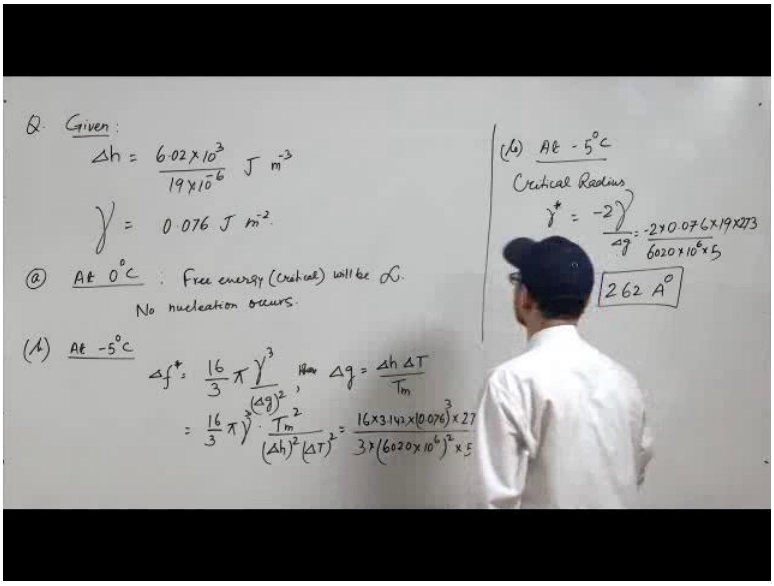


Now, in this question we have few things given. We have been given the temperatures that we will see one by one. We have been given the delta h enthalpy of fusion of ice and this is given as 6.02 kilo joule per mole and the molar volume is 19 centimeter cube. So, basically delta h is taken as the enthalpy of fusion per unit volume, if you take that it will be 6.02 into 10 to the power 3 joule per mole; however, per mole as the volume of 19 centimeter cube. It will be divided by 19 into 10 to the power minus 6 so then it will come in the unit joule per meter cube. This is what we have to convert the unit of delta h. Then we are given the interfacial energy and the interfacial energy is that is gamma and this is given as, that is given as 0.06 joule per meter square, 0.076 joule per meter square.

These are the data which are given we have to find the critical free energy of nucleation of ice from water at 0 degree C minus 5 degree C and minus 40 degree C. Let us take at 0 degree C, for the first case at 0 degree C. Now, in this case we know that 0 degree C is the melting temperature of the ice. So, basically it is nothing but the equilibrium temperature itself the critical free energy required will be infinite and there will be no nucleation. The free energy requirement will be infinite and there is no nucleation and that is why there is no radius. Free energy, critical free energy, will be infinity no nucleation occurs.

Then, now we should come to the next temperature that is, minus 5 degree C. We know that the expression for critical free energy is, it is given as 16 by 3 pi gamma cube upon delta g square. Also delta g can be converted or delta g can be taken as where, delta g can be taken as delta h delta T upon T m.

The expression which we have got earlier was 16 by 3 pi gamma cubes upon delta g square, it will be T m square upon delta h square multiplied by delta T square. This is what, is required that is the critical free energy. We will substitute all the values, 16 upon 3 then it is pi we can have 3.142 then gamma, gamma is 0.076 joule per meter square. It will be 0.076 raise to the power 3 into T m square, T m as we know it is 0 degree C that is, 273 Kelvin. So, it will be 273 square divided by 3 into delta h square we know delta h is this. In that case, delta h will be this 10 will go on the upper side, it will be 6020 into 10 raise to the power 6, the square and this 19 square will go at the top, and then delta T square, delta T is 5 here so, it will be 5 square. This has to be basically computed and this value it is coming out to be 2.2 into 10 to the power minus 16 J.

Now, at this temperature this is the critical free energy at minus 5 degree C. We have to find, also the critical radius at this temperature. So, at minus 5 degree C critical radius, so critical radius we know the expression for the critical radius r star will be minus 2 gamma upon delta g and already delta g is taken as delta h delta T upon T m. It will be 2 into gamma, gamma is given as 0.076 upon delta h delta T upon delta m and delta h is given as that. So, it will be 6020 upon 10 to the power 6 by 19. That is delta h delta T is 5 and then T m that 273 will go here. And when we calculate this value it is coming out to be 262 Angstrom.

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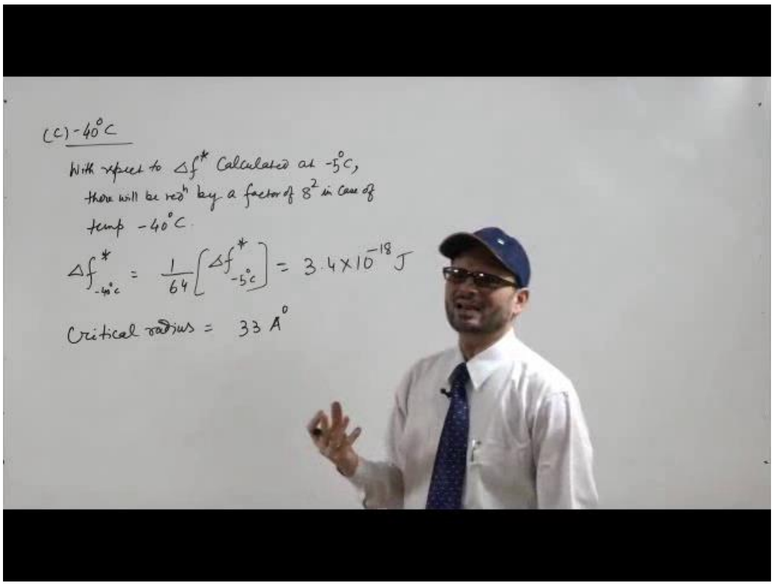


Next is the same thing we have to calculate at minus 40 degree C. What we see is in the same expression, now your delta T is 40. Whereas, in the earlier case it was 5 basically here it will be nothing, but the critical free energy will be reduced by a factor of 8 square. With respect to delta f star calculated at minus 5 degree C, there will be reduction by a factor of, by a factor of 8 square in case of temperature minus 40 degree C.

So, delta f star at minus 40 degrees C will be the 1 by 64 times delta f star at minus 5 degree C, and that comes out to be 3.4 into 10 to the power minus 18 joule. What we see, in the same expression 16 by 3 pi gamma cube and delta and upon delta g square. So, in delta g again it will be delta h delta T upon delta m. So, delta everything is constant only delta t is changing, delta T is in the denominator and it is increasing by 8. That is why this value is decreasing by 64 and that is why we are getting this. Similarly you can also find the critical radius; critical radius will be reducing by factor 8 that will be 33 Angstrom. This is how; you calculate the critical free energy as well as the critical radius for all these problems.

Now, the next question what we are going to discuss is, to calculate the under cooling required for liquid to crystal transformation in tin.

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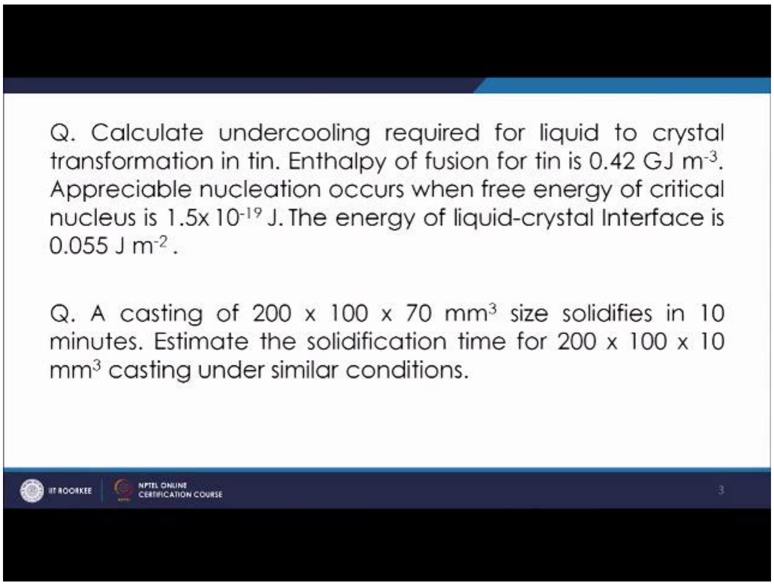


Enthalpy of fusion for tin is given as 0.42 giga joule per meter cube. Appreciable nucleation occurs when free energy of critical nucleus is 1.5 into 10 raise to the power minus 19 joule. The energy of liquid-crystal interface is 0.055 joule per meter square. In this case, in this question we have to find the under cooling that is delta T.

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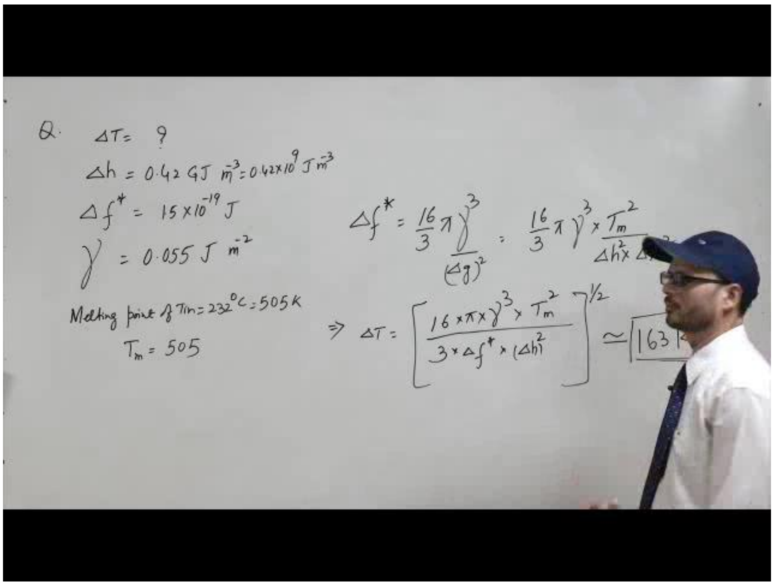

What has been given is enthalpy of fusion for tin that is delta h; delta h is given as 0.42 giga joule, per meter cube. So, here you have been given the enthalpy of fusion in the proper unit that is per unit volume.

Now, free energy of critical nucleus is given, delta f star is given as 1.5 into 10 raise to the power minus 19 joule, and the energy of liquid crystal interface that is surface energy, this is given as 0.055 joule per meter square. We had been give this data and we have to find, the under cooling. Now, for that we need to know the melting temperature of the tin, melting temperature of tin is melting point of tin is 232 degree C that is 505 Kelvin. Now, we have to simply put in the expression, the expression which we have is delta f star as we have calculated is 16 upon 3 pi gamma cube upon delta g square and for that, and delta g we know it is nothing, but delta h delta T upon T m, it will be 16 by 3 pi gamma cube into, that T 1 will m will go up and delta h into delta T both square. This will come, from here we will get the expression delta T will be, so delta T will come and this delta f will go and then we have to find the square root.

So, it will be 16 upon 3, 3 into f delta f star into delta h square and in the numerator you have pi gamma cube into T m square, and on this you will have the square root. This way what we see is you have been given all this values, you have to know the actual formula for calculating this and once you solve this you will get this value as approximately 163 Kelvin. If I do the computation calculation for putting all these values, all the values are given you have gamma as 0.055, you will have delta h as 0.42 rises to the power, it will be nothing, but 0.42 into 10 raise to the power 9 joule per meter cube. So this value is to be taken here T m is given as, so this is T m is given as 505. We have T m also, delta f star is given as 1.15 10 to the power rise to the power minus 19. So, ultimately the value which comes out to be 163 Kelvin, this is the answer.

The next question is related to find the solidification time. This question tells that there is a casting of size 200 mm, length 100 mm width and 70 mm thickness; this solidifies in 10 minutes. Estimate the solidification time for a casting with 200 mm length, 100 mm width and 10 mm thickness under similar conditions.

In this case, what we see that the thickness has varied and then we have to find how the solidification time will varied and what will be the solidification time for this particular casting. So, what we have so for a studied is that the total solidification time for a casting is proportional to V by surface area square.

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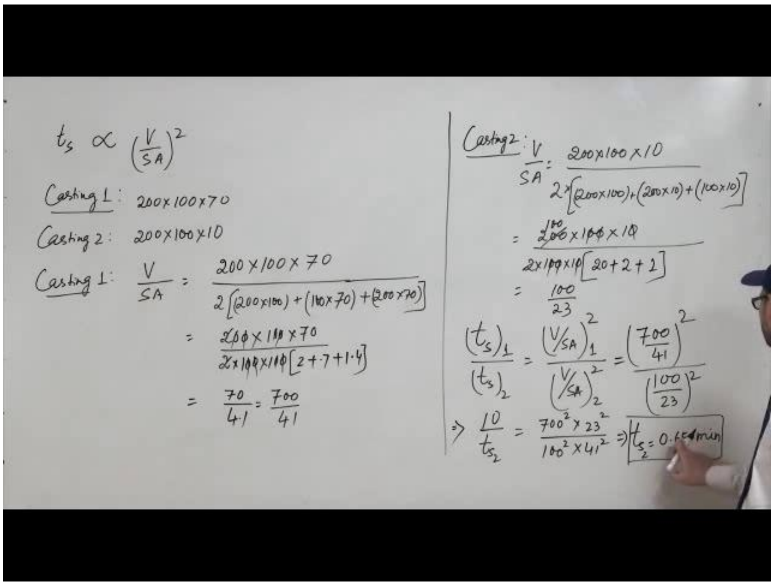

We have to calculate, the ratio of their volume upon surface area. If you take this as casting 1, casting 1 which has the dimension 200 by 100 by 70 and casting 2, have the dimension 200 by 100 by 10.

For casting 1, volume upon surface area you have to calculate. Volume upon surface area will be, volume will be 200 multiplied by 100 multiplied by 70 that are the volume of the casting 1 and surface area will be 2 times, 200 into 100 plus 100 into 70 plus 200 into 70. So, 2 into l b plus b h plus l h that is the total surface area of the casting. If you look at that, we will have 200 multiplied by 100 multiplied by 70 upon 2 times. If we take 100 times 100 out, this will be 2 plus this will be 0.7 plus this will be 2 into 0.7 so, 1.4. We can cancel few zeros, this 2 will also cancel you will have 70 upon this will be 4.1. This is how you get the volume upon surface area for casting 1.

Now, we have to find the V by SA for casting 2. So, for casting 2 V by SA will be 200 times 100 times 10, upon 2 into 200 times 100 plus 200 times 10 plus 100 times 10. We can take it as 200 upon 100 into 10, upon 2 into, we can take 100 into 10 out. If you take 100 into 10 out, it will be 20 plus 2 plus 1 and we can cut all these. So, this will be 100 you will have 100 upon 23 and this will be basically, you can have it as 700 upon 41. This is coming as 700 upon 41 and this is coming as 100 by 23.

Now, the ratio of the time t s of first casting, upon t s of second casting will be V by SA a square of first casting upon V by SA a square of second casting. It is nothing, but V by SA for first is 700 by 41 square divided by this is 100 by 23 square, and this is given as t s 1 is given as 10 minutes. It means 10 by t s for 2 will be equal to 700 square into 23 square upon 100 square into 41 square.

So, this way you have to calculate this and you can find t s 2 directly from there. From here you directly get t s 2 as 0.65 minutes, you can do that computations and get these results by doing the calculations basically it will be 49 and then we will do all these calculations and this will come as 0.65 minutes. This is how you compute the ratio of solidification time for 2 castings, under similar conditions.

Thank you very much.