Modelling and simulation of Dynamic Systems
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Lecture - 8
Generation of System Equations

I welcome you all, in this lecture on generation of system equations which is a sub module for modeling and simulation of dynamic system a course which you are going through. Now, as I have discussed in my earlier lectures that the greatest advantage of bondgraph modeling is, that it can be used for modeling of systems which are there in the multi energy domains and the system equations, which can be generated in an algorithmic way.

So, let us see now if the system equations are to be generated, the first question is that the system equations are generated in terms of what?

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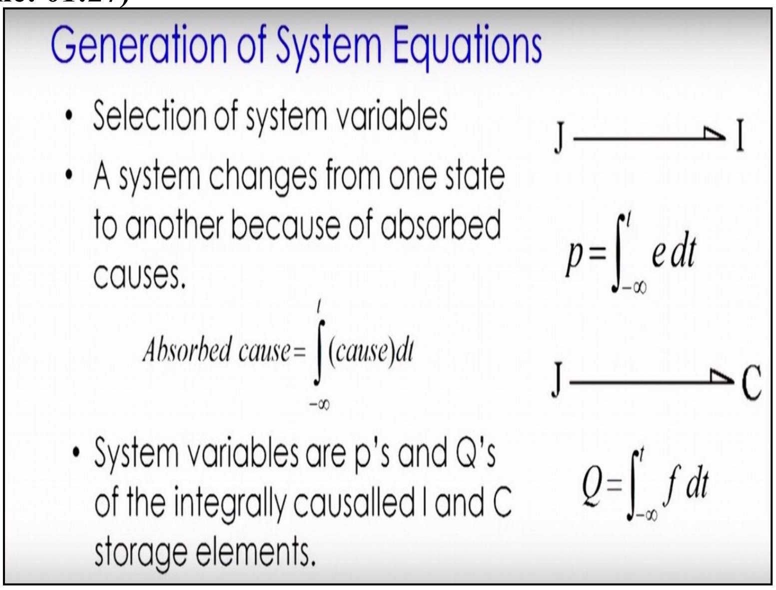

So, the question of selection of system variables arises. Now, you see system changes from one state to another because of the absorbed cause. So, if I define say absorbed cause as integration of cause dt of course integration being carried out from minus infinity then it is the absorbed cause which is responsible for change from one state to another state. So, what we will first do is that, we will identify the system variables or the variables in terms in which we can write the system equations.

So, let us look at say element I. Now if I write the expression for i element then you see that the momentum of integration of the effort. So, here going by this definition of absorbed cause we can see that the absorbed cause is the momentum here for the i element. Likewise, if we see the c element then in case of c element this is our expression which is going to be and here it is the flow which is being integrated in order to get the displacement.

So, it is the q which is the absorbed cause here. So the system variables you can see are p's and q's that are momentum and that are displacements. Of course, if I am talking about a mechanical system. So, the system variables are p's and q's of the integrally caused i and c storage elements. So whatever system equations will be developed during bondgraph modeling this system equation will be in terms of the p's and q's.

Now, as I have been talking to you we get system equation in algorithmic manner. So what could that algorithmic be?

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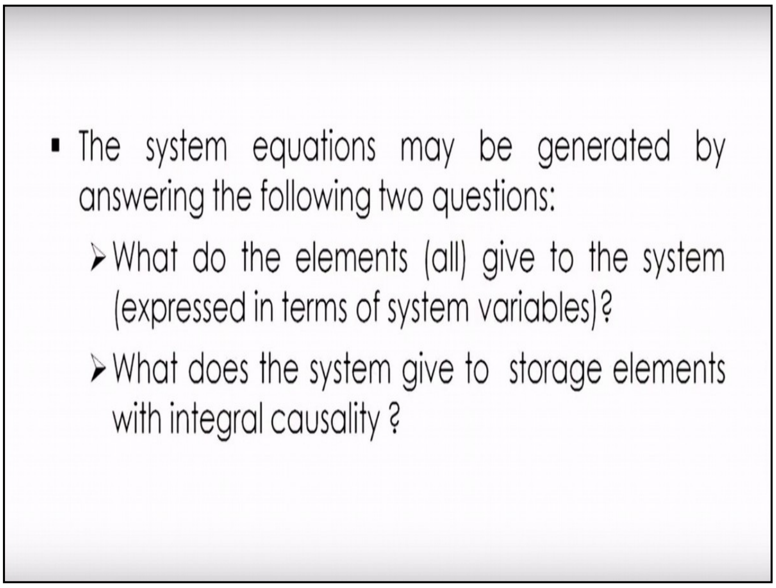

So, here the system equations may be generated by answering the following 2 questions or 2 logic. If, we implement the first one what do the elements give to the system and ofcourse, whatever they are giving to the self system it should be expressed in terms of system variables. So that is the first logic which we are going to use.

The second logic is that, what the system gives to the storage elements with integral causality. So, if we implement this to logics then we can get the system equations in terms of the system variable. And once we have the system equations then we can use those system equations for the simulation purpose.

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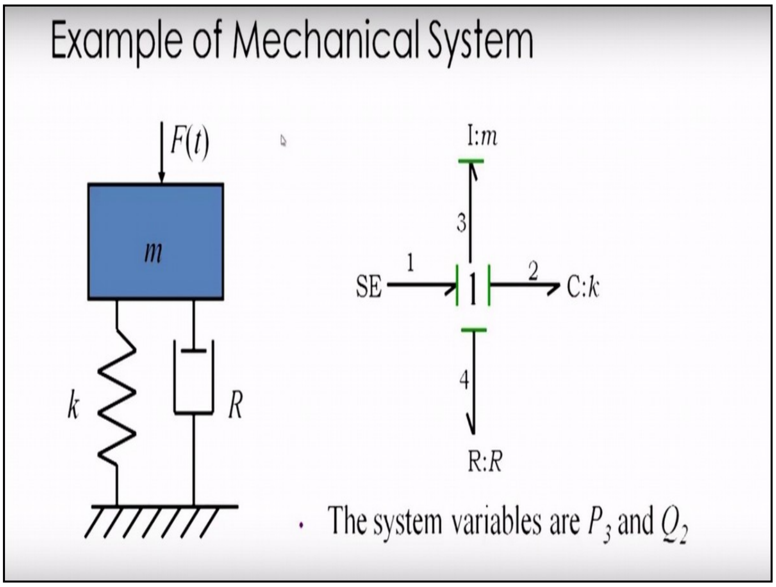

Let us take an example of mechanical system. The figure which you are seeing here is a simple spring mass damper system, which has been subjected to certain force excitation. Now, the question is, first of all we need to understand how we can draw the bondgraph model for this system. So, before I will be talking to you in much more details when I will be discussing the modeling of the mechanical system.

That is I will be talking about how do we draw the bondgraph for any generalized mechanical system. But, to be precise here we can see that spring and damper other end is fixed that is, they are initial so it is implied that the mass spring and damper this end they are all moving with the same velocity. So as we have seen the general procedure first we identify. What all elements constitute this system. So, here as you can see that the elements what we have is, we have a force which is acting on the system.

We have a mass, we have a spring and we have a damper. So these are the four elements which constitute the system. Then what we do we can represent the virtual power pipe lines through which power flows to these elements. All right and after that, we can put the constrains. Now here as i said you can see that all these 3 elements are going to have the same velocity here.

So, plus the force, as well. So, here we can put a constraint of one which is the Constraint flow and of course after this we can put the power direction which is as good as fixing of the coordinate system in our conventional mechanics, and after this we can casual this bondgraph model.

In the last module we have seen how to casual the bondgraph. So, for causing purpose we see that this force, which will be represented by source of effort, here the mass is of course and inertia the compliance here will be represented by k and of course here damping which we represented by r fine. Now coming to the casuality, as I was telling first we casual the sources so this is source of effort it will always be giving effort to the junction.

After causing the sources, we can casual the storage elements so, for i element this is the integral causality. Whereas for the c element, this is the integral causality that is in take Flow returns effort and i element takes effort and returns flow.

Now, this junction is a constant flow junction. So, if we look at the causality of the junctions then we can see that, this flow has to be decided by any one of the bonds and i is already doing that, it means that the r here is going to have the stroke here. So, that is the r element will be giving the effort to the junction. So, this way we can create the bondgraph for this system. Now, after creation of the bondgraph we, have to identify the system variables that are the variables in terms of which we are going to write the system equations.

So, here if we see for i element the variable will be momentum of that so that is p3 and for the c element the system variable will be q2. So this way, we have the displacement of the spring and the momentum of the mass these are going to be 2 system variables in this case. Now after this, we go by the same logic the logic which I just discussed in my previous slide.

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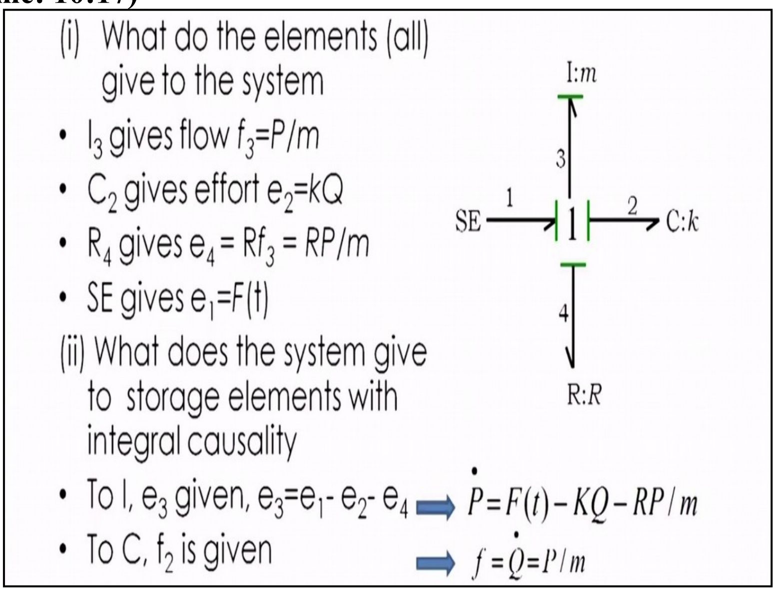

The first logic is what the elements give to the system. What do all elements give to the system? So here, let us look at this the i 3 i here i element 3 means that it is the bond number 3. So i3 is giving to the system a flow. Here you see that there is a noise stroke that's it means that it is giving if flow and that flow f3. We can write as p by m that is momentum by mass. So we will be getting that as the velocity. So i f3 is p by m then the next element is c element that is c2 it is giving an effort here.

So, that effort I can write e2 as k into q where k is the spring stuffiness constant, and the damper r4 is giving the effort here. So that effort e4, I can write as r into f3 and you see its a constant flow junction. So whatever effort is there; it is the same as that of f3. So, here directly it is written as f3. So, these are f3 then I can substitute for this f3, so I can write this as are p by m and as seen f3 will be giving an effort and in the case it is the force see e1 equal to ft fine.

Then we go for the implementation of the second logic, and the second logic is what does the system give to the storage element with integral causality? So, first of all we need to identify which are those storage elements who have the integral causality? So, here it is the i and the c; these are the two storage element that has the integral causality. So, you see that the system I so giving this i element an effort you can see here the stroke is there it means that, it is giving an effort e3.

Now, these e3 I can write as e1 minus E2 minus e4 of course this relation comes from the basic concept of conservation of power as we have seen earlier and since it is, all the bonds are connected to a constant flow junction.

So, flows are going to be the same, so here basically what we will be having rather that it will be e1 is equal to e2 plus e3 plus e4 and from here I am interested in finding out e3 and so this will be e1 minus e2 minus e4. So, this is what is going to be, now this e3 i can write as p derivative that is the derivative of the momentum. E1, I have already seen it is ft here e2 I have seen it is kq and e4, I have seen that it r p by m. All right, so this is my first equation fine. Similarly, I can find out that, what is the system giving to the storage elements c.

So, if you look, the system is giving to the storage elements c a flow and that flow is f2 and again this f2 will be same as what is the derivative of the displacement that is, q dot and this q dot we have already found out that is going to be equal to p by m because, this is flow and this element is connected to constant flow junction. So, this flow here is going to be the same as this one so this q dot is p by m. So, we have these two equations for this system.

Okay these equations are p dot is equal to ft minus kq minus r p by m and q dot is equal to p by m. So here, we can see that these 2 equations are returned in terms of system variables that is momentum and displacement, I can write these equations in state space. From that, i can write here p dot q dot is equal to a matrices of the state space formulation and then will have pq here plus the input vector here.

Fine now, here you see that the coefficient of p are minus r by m and coefficient of q are minus k and here this is ft and for the second row q dot is coefficient of p is 1 by m and here this is zero and of course, in this there is no input so it is zero. So, here we can see that we get this equation in the state space form.

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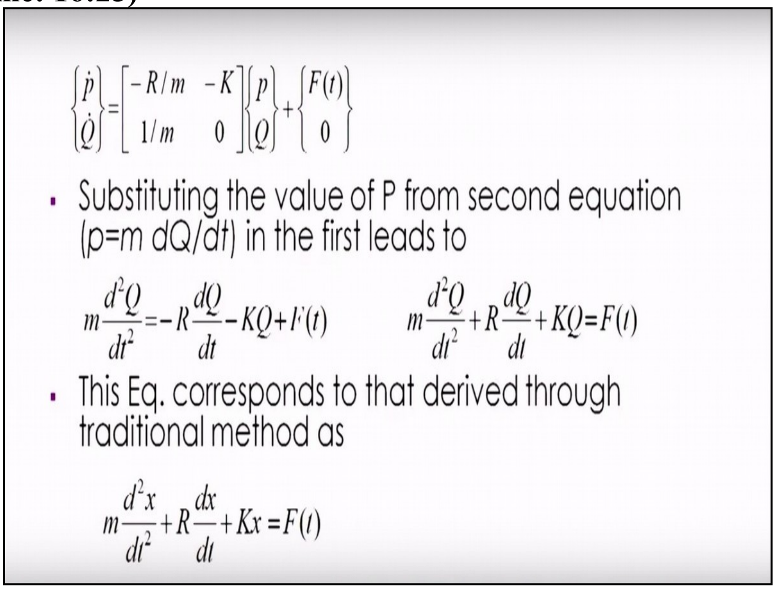

Now, you can see that, we are more conversant with the equation in this form in terms of displacement. So that is what we can get when we substitute for p in the first equation so, if we substitute for p in the first equation then, we can get this expression or finally it is this equation that is m d square q by dt squared plus r dq by dt plus kq equal ft, or as I said that is very similar to what we are more conversant with our vibration course that is m x double dot plus ax dot plus kx equal to ft okay, so this is how the systems equation is derived using these two logic's in the bondgraph.

So, we can see that the derivations of the system equation we have algorithmzed okay. Next, let us take an example of an electrical system. In the previous case, we have seen example of mechanical system. Now let as see an example of an electrical system.

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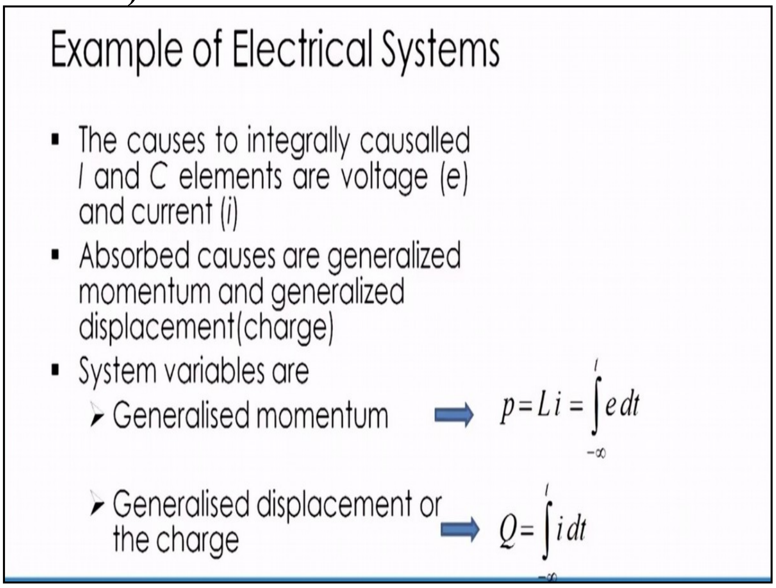

Now in case of electrical system the causes to integrally causal i and c elements are the voltage and current. Fine, so voltage current is the causes. So, the absorbed causes are the generalized momentum and the generalized displacement that is charge to the equations in electrical system will be written in terms of generalized momentum and the charge so the system variables.

Will be generalized momentum p which is integral of idt, which can be written as l i were l is the inductions and i is the current and the generalized displacement. Or the charge q can be written as integration of i dt. Now, let us take an example of this circuit.

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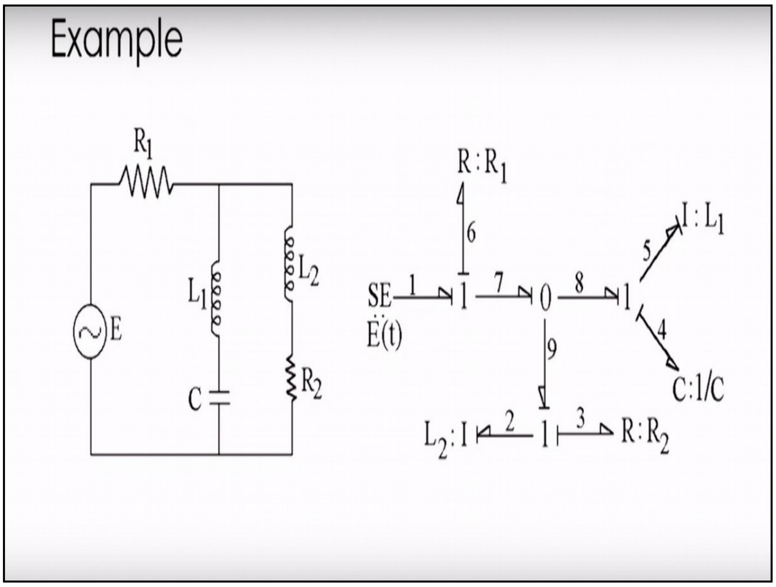

Now the thing is that, first we need to draw the bond graph model for this circuit, then we need to casual it and after casualing it using those two logics which I just told you, we need to derive the system equations and that is going to be the aim of this topic find so to derive the bondgraph of this system. You can see here that this system composed of a voltage source a resistance here, Inductor and capacitor r in series and of course these to r in the parallel so, to derive the bondgraph model.

This system we need to use the concept of masking which is there in the bond graph and I quickly explain you how can we draw the bond graph for such a system, so we have a resistance. Here an inductor then there is a capacitor and again an inductor and there is a resister. So, the voltage source e we have resister r1 and inductor l1 capacitor c and inductance l2 and the resistance r2.

Fine, now you see here voltage source and this resistance r1 they are in series. So, suppose I for the moment, I just mask this portion then what happens. I have this source resistance and source something in this series I do not know what this something is so, what I will do I will put a cs element see and c resistance r equal to r1 they are in series plus something is there that I am not aware of. Now here in this something you can see that there are two things in the indictors or capacitor.

They are in series and again this inductor and resister they are in series so, if I take this as one junction because, that parallel means they have the same voltage and the same effort. So the zero junctions here I put for these two, okay now you see here in side of this the inductor and capacitor are in series so constant current so here, I put 1 junction and I just put i equal to l1 and here i put see c equal to 1 by c.

Likewise, here indictor and resistor they are in series so i can put 1junction here and then i can put say i equal l2 and r equal to r2 so this way, I can draw the bondgraph model for this system. So in fact this is the same as this one just a little portion has been put here at the bottom at this position i will be here at the top fine and then of course, we can casual this so counseling as our concept.

What we did first, we casual sources a of port it will be giving for a tl casual the integral casuals i and c elements. So i casuals here i casuals c here and i casual i here line this like and after that we do the casualing, for the rest of the bond graph. So that, the whole system is integrally casuality. Now, here you see that this is giving flow; this is first so junction is determine so i can put effort here, so effort is here you have flow.

Fine all right and here efforts so flow this is one junction so, this as to ohh bringing give effort here has to be there, so the flow here is fixed flow so and this is zero junction so, this has to be determined by someone so that, information has to come from here okay, now flow s coming from here. So, this way we can casual a bondgraph. Now, let us see how do we derive the system equation?

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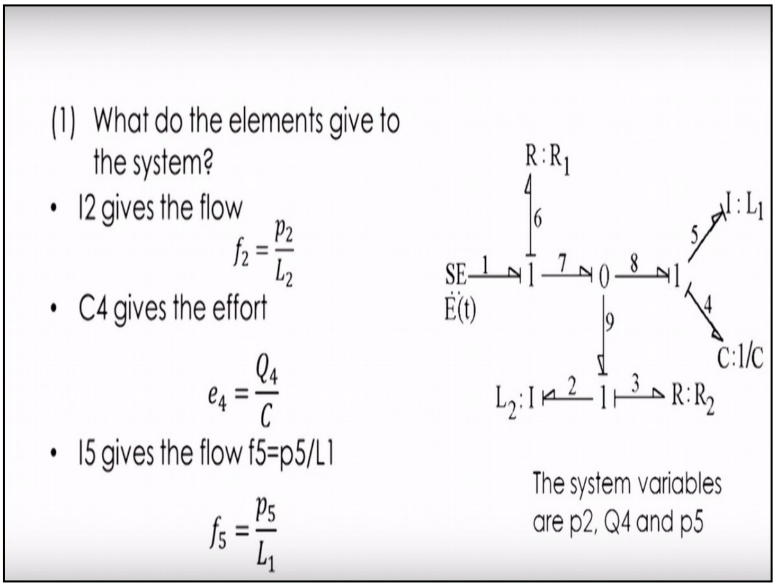

Okay, so again we go by same logic what do the elements give to the system, the first logic okay and here for book-keeping purpose we number the bondgraphs, okay, we number the different elements of the bondgraph so this number is orbitterly you can have your own numbering scheme for this so, first what do the elements give to the system.

So, let us first try to see what all elements given to the system so you see this is i 2 and this is giving flow so I can write f 2 p2 by l2 okay so this is i2 then we have the c four this is giving effort here and this effort I can write as r voltage I can write q four by c okay and the i five here this is giving flow and I can right is f five as p five upon l 1 then the r 3.

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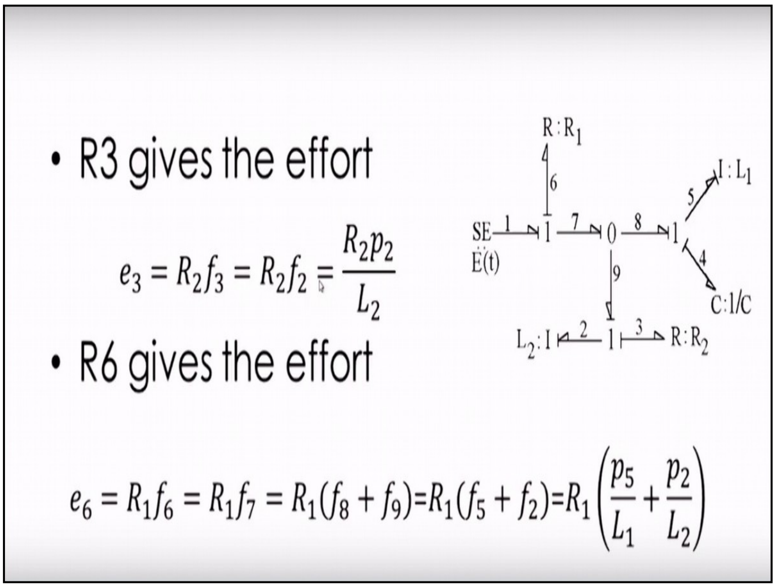

Here we can see the r 3 its giving an effort so this effort I write e 3 as r 2or r 2 f 3 or this is r 2 f 3 is same as f 2 so i put f 2 and what is f2 I can put as p2 by l 2 upjust dirive in the previous slide so, this way i can write e3 likewise my r six its giving the effort six so this is six what r one f six now what is f six as same as f seven here what is f seven this is f eight plus f nine so a f eight plus f nine and a what is effort is determined form this is one five and f nine determine from f 2 so f five plus f 2 and then I can substitute values of five and f 2 as p five by l one + p 2 by l 2.

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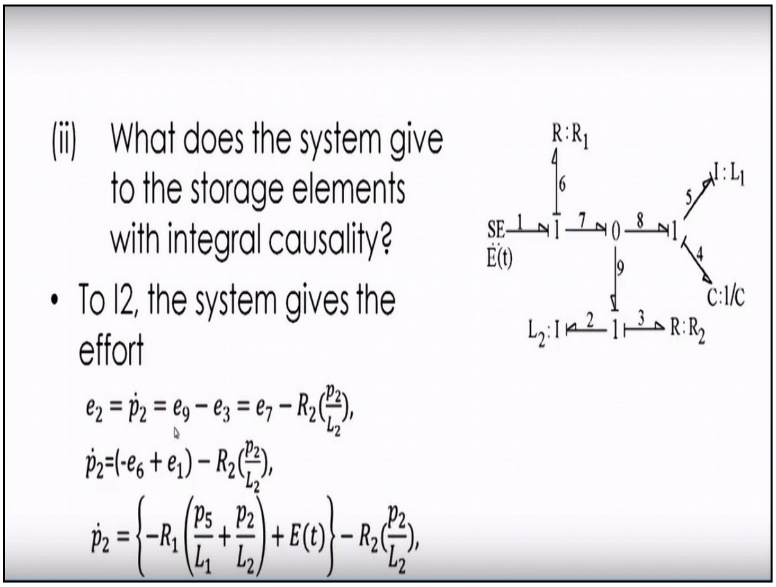

Then, we go for the second logic that is what does the system give to the storage element with integral causality? so you see here the i 2 to i2 the system is giving effort okay so, this e2 I right as p2 dot and is p2 dot as e nine minus e3 and e nine, e as same as e seven here because, this is constant effort junction so e seven and e3 we already found out which is are to be 2 by l2.

So that is they are the now further these p2 dot I can write e seven, where is by e seven this e seven is e one minus e six, so e one - e six I put and then the put the value of e one and e six here then I get this equation so e one and my what e one as six this one, sorry, e one yes et and e six this value so, this way I can find out the p to dot.

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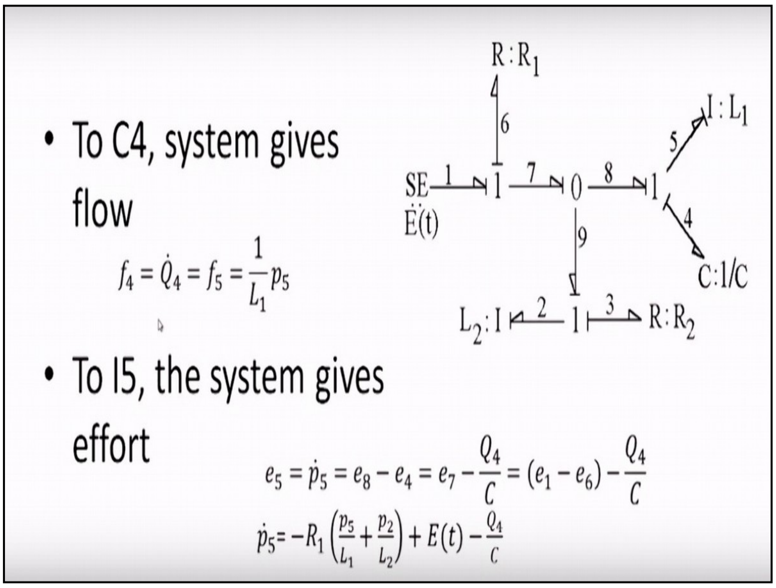

Next, c four the system gives flow that is f four, which is q four dot and this I can write as p five by l one. Okay and to I five the system gives effort that is, e five r p five dot and this e five is e eight minus e four and e eight is same as e seven and e four i am already found q four by c, c and what is by e seven that e one minus e six this is the I substitute the value of e one and e six here and by this way I get the p five dot so i can write the is state space equation in term s of system variable p2 q four and p five as this one.

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- Thus the state space equations can be written as

$$\begin{Bmatrix}\dot{p}_2\\ \dot{Q}_4\\ \dot{p}_5\end{Bmatrix} = \begin{bmatrix} -\left(\frac{R_1}{L_2}+\frac{R_2}{L_2}\right) & 0 & -\frac{R_1}{L_1}\\ 0 & 0 & \frac{1}{L_1}\\ -\frac{R_1}{L_2} & -\frac{1}{C} & -\frac{R_1}{L_1}\end{bmatrix}\begin{Bmatrix}p_2\\ Q_4\\ p_5\end{Bmatrix} + \begin{bmatrix}1\\ 0\\ 1\end{bmatrix}E(t)$$

So here you can see that the input vector, this is vector corresponding to the system matrices and this is the matrices of the state space format a, from this the system variable derivative okay so this variable we can derive the system equations and these are the references our book is and there is the book by professor Mukerji, which you can go through for further understanding of this topic. Thank you