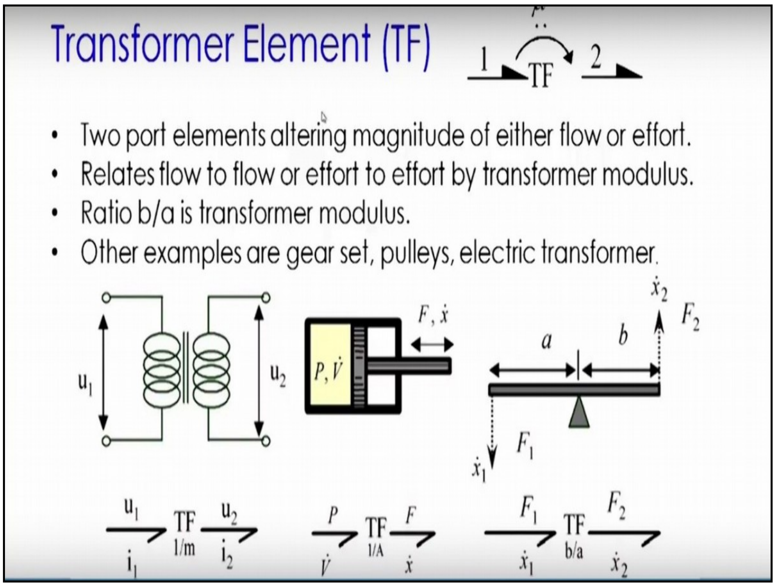
Modelling and Simulation of Dynamic Systems
Dr. Pushparaj Mani Pathak
Department of Mechanical and Industrial Engineering
Indian Institute of Technology - Roorkee

Lecture - 7
Bond Graph Model and Causality

I welcome you all on this course on modeling and simulation of dynamic systems. In this lecture we will be continuing the bond graph elements of my previous lecture. After we complete the description of these bond graph elements, then we will look at the concept of causality, which is a very powerful tool in bond graph modeling and which helps us in deriving the system equation of any dynamic systems in an algorithmic manner?

So, apart from the three elements which we have seen in our previous slides that is energy, storage elements Inc and dissipative element r. In this lecture we will be looking at the two other important elements that are the gyrator and the transform.

(Refer Slide Time: 01:30)



The transformer elements as I said it is two port element and it is represented like this that is, there is a power bond, then we have the TF element and there is a another power bond and we have the TF element and there is another power bond. The arrow basically represents the direction in which the flow relationship is going to be implemented and new represents the modulus transformer.

So, these are the two port elements altering magnitude of either flow or effort and these relates flow to flow or effort to effort with the help this transformer modulus just discussed with you then we can take many examples. Of the transformer elements such as see gear set or pulleys or the electric transformer itself, or the lever fine.

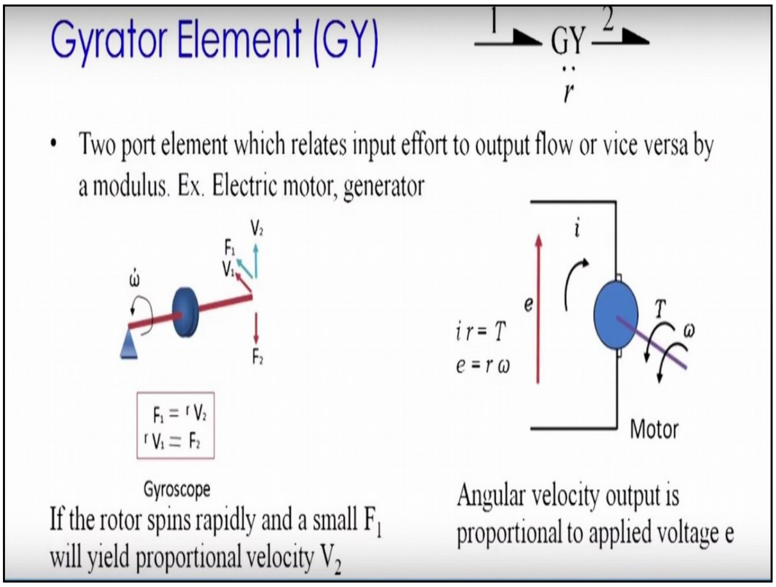
So this figure actually shows you the systematic of the electrical transformer as the input side voltage is u1 and output side voltage is u2, so this is how it is represented that is; this is the input side and this one is the output side and the transformer Modulus is here represented as one by amph.

Likewise, here a piston cylinder arrangement. Here in the piston side we have the pressure p and I volume flow rate at v dot and at the other side. We have the force acting on the piston as f and the velocity x dot. Then this scenario can be modeled using the transformer element and here at the left hand side we have the pressure p, and volume flow rate v dot and the right and side we have the force f and velocity x dot and of course these are related to the transformer modulus 1 by A.

So, you see as I said the velocity relationship it is been from in input side to this output side x dot will be v dot divided by A. That is what it is going to be and if you look at the other side's the f are deregulations. Relationship between will be reverse that will be f divided by A.

Likewise, we can find the transformer modulus for a lever work so here the input side yes in the input side the f 1 and flow level is f 1 and flow level is x 1 dot and at output side f two by flow level is f 2 and flow level x 2 dot and by this seen this figure we can find out the modulus is b by a. So b by a will be modulating the velocity of the effort. Next another very important bond graph element is gyrator which is represented.

(Refer Slide Time: 04:55)



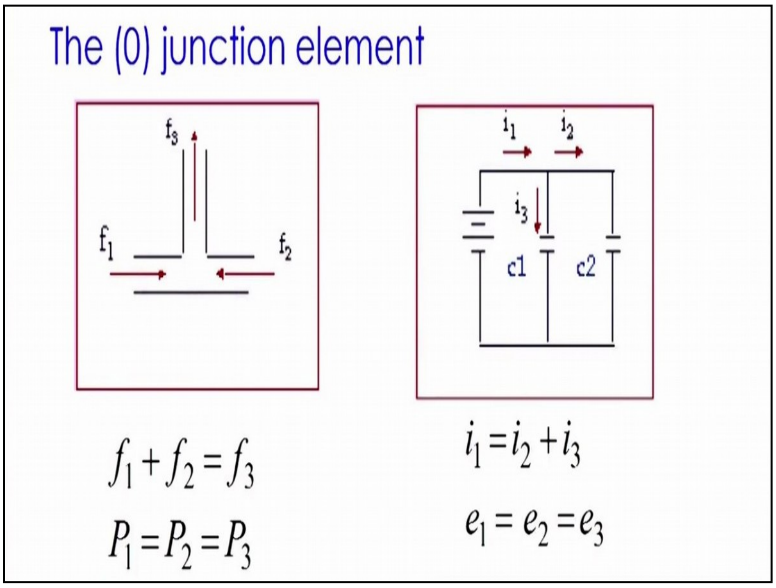
Here like this you have side here input side and output side; so here you have bond and bond to again site is are the two port elements. Because they interest to rest are the system are to side and the modulus of the gyrator is the generated by r. These two port elements relate input effort to output flow or vice versa through a modulus. They are examples like electric motor and generator all the gyroscope pixel which can be model using the gyrator element.

Let us take example, this is a simple example of the gyroscope here suppose there this which is a state attached soft and it is spinning. About spin p here by mega 1 angular velocity omega 1 and suppose in this case if, I apply a force in f 2 direction it will be short moving in the v 1 direction. This is what is the gyroscope principle is and this f 2 will be given by the gyrator modulo are into v 1. Likewise, if we apply a force in f 1 direction it will be moving v 2 direction and this f 1 will be are times v 2.

Let us take another example of motor which is again two port element; so here in case of motor suppose Ah we supply. Voltages cross the terminal of the motor and see the current troughs are mature am i and others side that is the mechanical side. We have talk t and the speed of the roader as omega then knows that dark generator by the motor is proportional to the current. And so this dark is equal to I cross a constant which is a gyrator modulus in the case are the motor tar constant and motor.

Likewise, we can write it for the reverse case the voltage generator the cross when it is acting is generator. The voltage generator will be equal to r into the angular speed that which speed the soft rotates. Know the 0 junction elements we have been talking about the 0 junction elements. Are basically constant for junction so in case of the 0 junction.

(Refer Slide Time: 07:57)

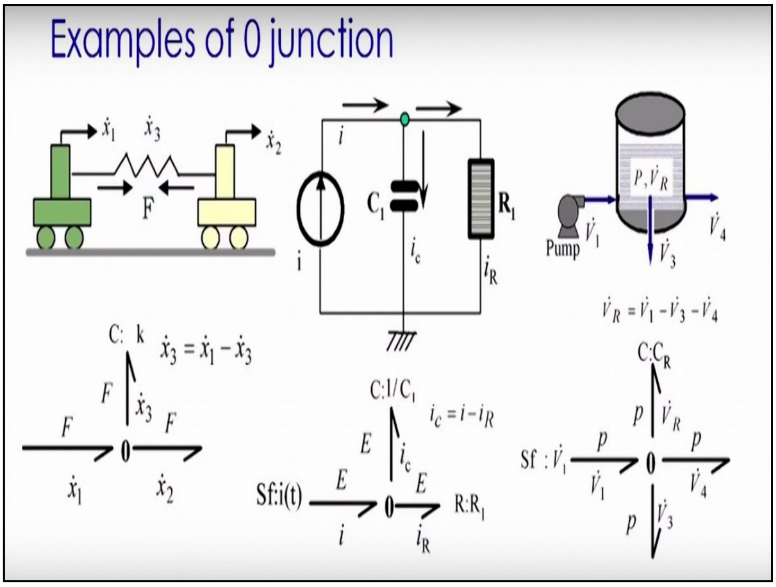


Suppose, this is the scenario of that is the flow of liquid pipes here through fine flow is coming from here f two is coming from here f three and f three is going, then we know that in this example the pressure i is going to be same everywhere. All the three sides so, we write p 1 is equal to and p 2 is equal to and p 3. And in this case f 1 pulse and f 2 is going to be equal to f 3 so this scenario represents is 0 junction is structure which is nothing but is the constant effort junction and here the efforts are the pressures.

So, in case of a 0 junction efforts are equal where is the flows are some as per their direction. Likewise, we can have look at this simple electrical circuit, the parallel circuit here you see that the current I want is geting higher. Cited into i 2 and I 3 so here basically I 1 I equal to I 2 pulse I 3 and Voltages are going to the same across ape a source the shows and these to capacities the e one is equal to e two is equal to and e three and here of the summation of the flows.

This is how the 0 junction elements can be depressant. There are many examples of the 0 junctions here, as I said the 0 junction we also know by the name of the constant effort junction. So suppose if, you have a scenario like this, that there are two cars which are moving velocity x one dot and x two dot then they are connected.

(Refer Slide Time: 09:35)

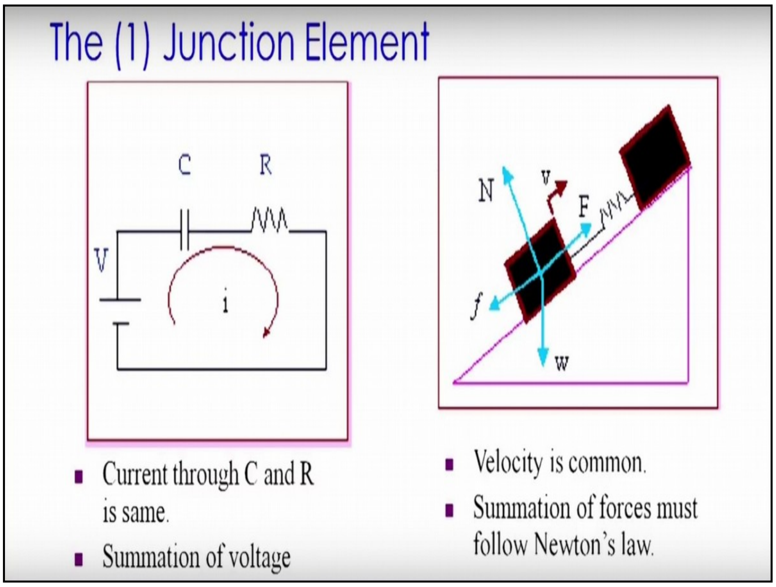


By so here ha the bond bond here one bond here is a bond for the velocity of the first cross x 1 dot you have the velocity of the x 2 dot, and in between we have the spring so they are going to be collected to 0 junction because the effort is going to be same to all these three elements. Otherwise, I can that this spring will be always subjected to the velocity different. Velocity different of the two dots so, from here we can see the x 3 dot is equal to x 1 dot minutes x 2 dot.

Similarly, I can explain you the scenario are a parallel circuit of consisting of capacitor and register. so and the voltage as same across all the capacitor and register here and of course current source, so current source here find. Likewise we can hydrolysis system we can represent here that is pump filling in a dang and flow from the discharge from the dings to discharge. So, in this case again pressure is going to be the same I cross all the elements. So this scenario can be again represented by the 0 junctions.

Next, let us take the example of one junction element know one junction element. Now one Junction elements is the constant four junction so incase of l see circuit we know of that the flow are going to be current is going to be same across all the element.

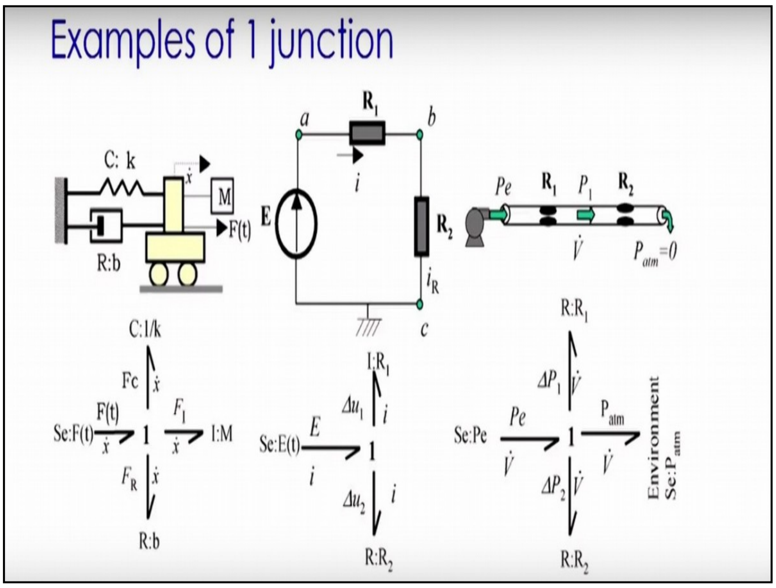
(Refer Slide Time: 11:35)



So, this type of situations is constant flows situations can be model using one junction. And of course when current flows, we are going to have the summation of the voltages across all the elements. Likewise, if we take an example of the mechanical system they are at two block which are connected to the expiring then supposed.

They are supposed if I trying to see this block now, this block here is spring velocity and they are going to the same velocity but, the forces will be subjected to the revolution of the forces in the direction of the motion of the block, in the perpendicular direction of the motion of the block and that summation of the will be this side by the Newton law here again by here s examples one junction.

(Refer Slide Time: 12:36)

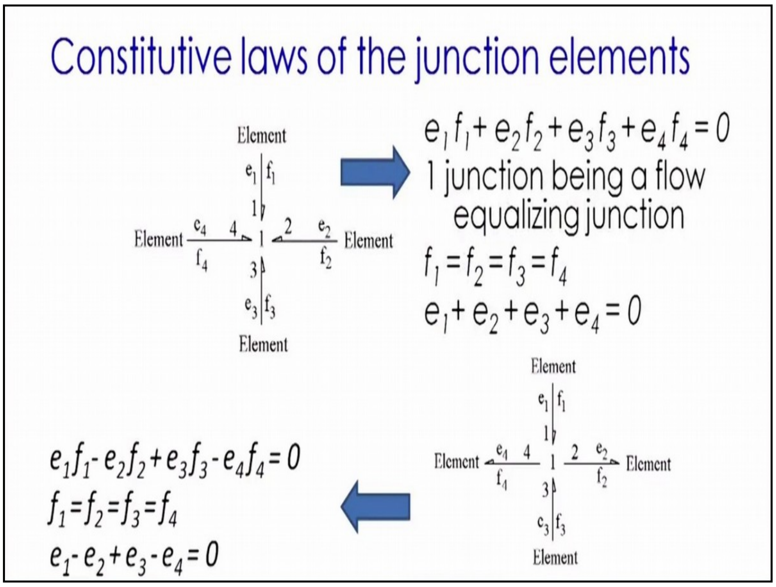


Here they are connected to springs and damper and other end of the spring and damper. It is fact that, the eparchial system here we see the spring damper mass; all are going to the same velocity here. So, this is what we can model as one junction and we can represent as the force and the spring. The mass and the dang are in case of serious circuit again as is the current is going to be same to all the elements.

So, here we are going to have 1 junction or flow through five. Here flow is going same across all the section here the two restrictions. Constrictions on to but, we are going to the same so we are on the one junction as the constant. Now, let us look at the constant lose of the junction element. So aha as are be n talking to you that in one junction we are going to have the same flows through all the elements.

So, naturally the efforts are is going to be same and how they are going to be same that depends on what direction those power bond are. So, supposed in this figure we have by they are elements are here which represent integration of these elements is represented through the bond one, two, three, four and all the power direction of the bonds are to one junction.

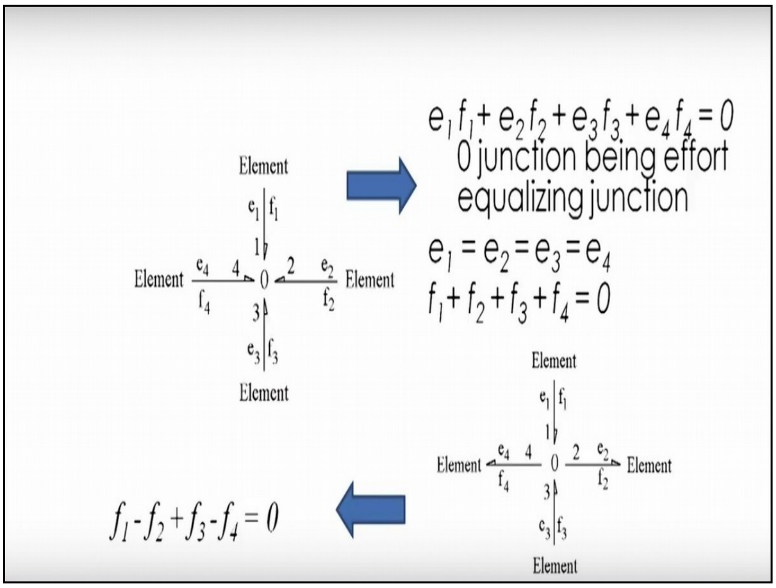
(Refer Slide Time: 14:16)



So, we can simply that power across all the bonds is summation of the power is going to be 0 and one junction being the flowery coiling junction I can and liquid the force and if I substitute this equation we will be having equation constricting the summation of the efforts.

The same concept we can apply to this scenario they are of course, power direction of two of the bond is reverse as this one. so here again I can write a conservation of power is equation, here power in if I take us positive so e 1 f 1 e 2 f 2 is going out so minus e three f 3 again going in so pulse and e 4 f 4 going out so minutes so this is 0 and I put the flow condition f one all the flows is to be 0 equal and so if is substitute is then we get the f are relationship.

(Refer Slide Time: 15:32)



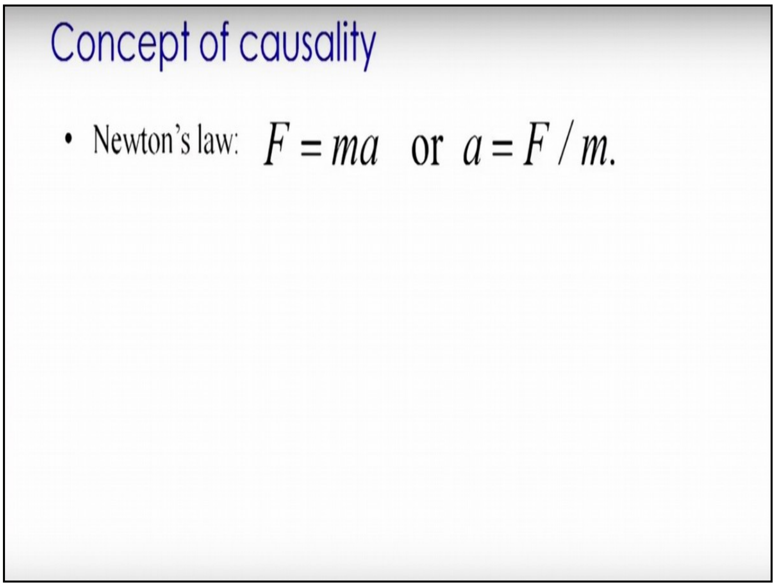
So this is what is a pot one junction next about 0 junction the scenario like this that the constant among these four elements is represented by a 0 junction. So again I can write the power conservation equation. Here and here 0 junction being the effort equalizing junction I can substitute ha the equality of the efforts and if I put is values a then I get the summation of the flows here.

So some of the case of 0 junction we have efforts as equal and flows are some summation of depends as the shine summation a depends are how the power direction on they are the different bonds.

Again, let us take another example and here we can see that the two bonds in two bonds power is in rise where other two bonds power is out so again if, we do thus same process this is what relationship here going to get. So, this is all about the 0 junction another very important concept in ah bond graph modeling is the concept of causality which is nothing but the cause and effect relationship.

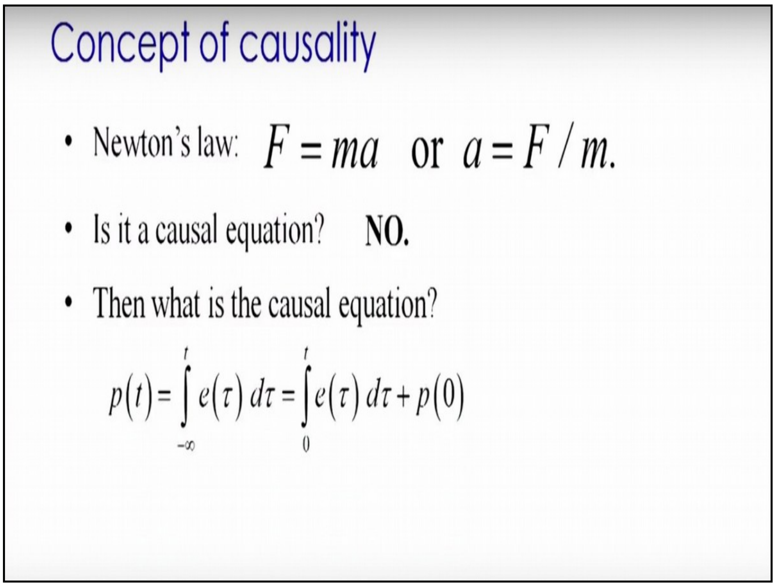
So let us look at what do I mean, what cause and effect to the relationship.

(Refer Slide Time: 16:52)



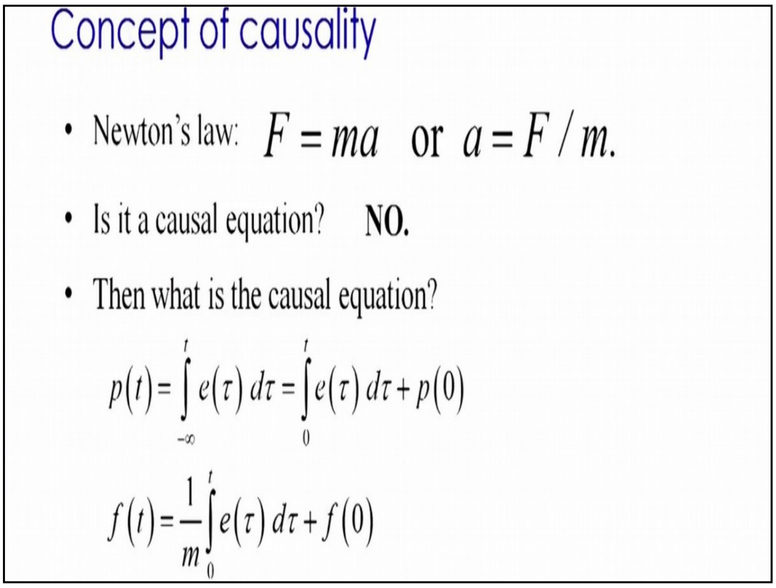
If you look at Newton's law it s that f is equal to m into a, that is force equal to mass into adulation are here if you that adulation is equal to force divided by mass. Now, is this the casual equation. Know why this is not equation because here we don't know what is the cause and what is the consecution.

(Refer Slide Time: 17:23)



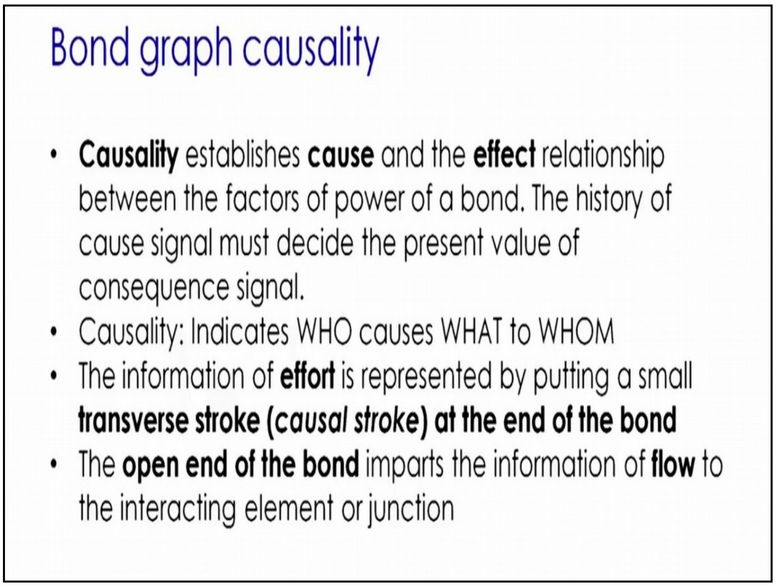
So, if I write this equation to this is that this momentum is equal to integral of efforts. Alright are from live it. With minutes infinite to t or I can write it from 0 as from plus ten initial value. Now in this case we can that the effort is being integrated and momentum is being generated. so effort is the cause and momentum is the consecution, so this type of equation is what is called as the causal equation.

(Refer Slide Time: 17:58)



I can write it in terms of our power verb variables of bond graph that is, the effort and flows I can write flows as one by into integration of efforts. So this equation is what is called as the causality equation. So here effort a mask as flow and this is an integral form of equations.

(Refer Slide Time: 18:23)

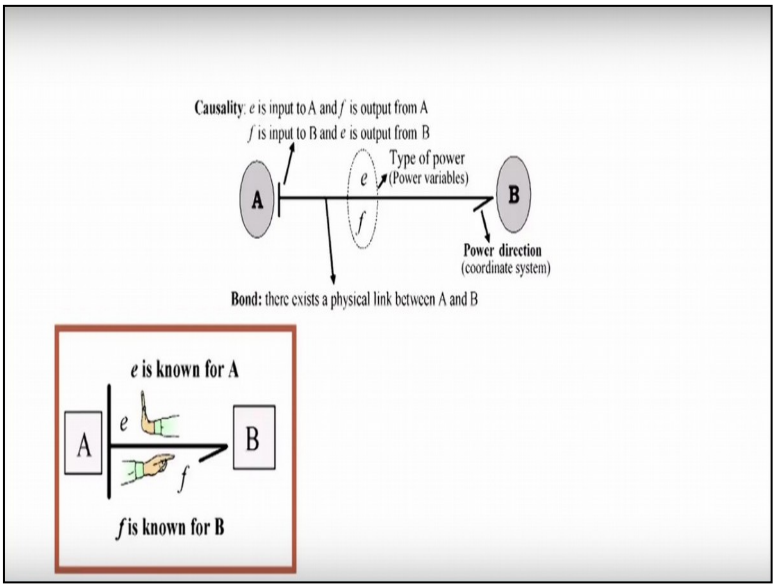


Now this causality stabilizes cause and effect relationship between the factors of a power of the bond. The history of cause signal must decide the present value of the consequence signal. This lashing the indicate who cause what to home, so in the bond graph modeling to repressed causality the what is then the information of the effort are reprehended by putting a small transverse stroke causal stroke at the end of the bond graph.

So the open end of the bond imparts the information of flow. To the interacting junction of the element. So, there this is a that junction element at junction a and system element at junction b here and this is powered direction. As I the power direction means it is in fiction of coordinate system in the bond graph modeling.

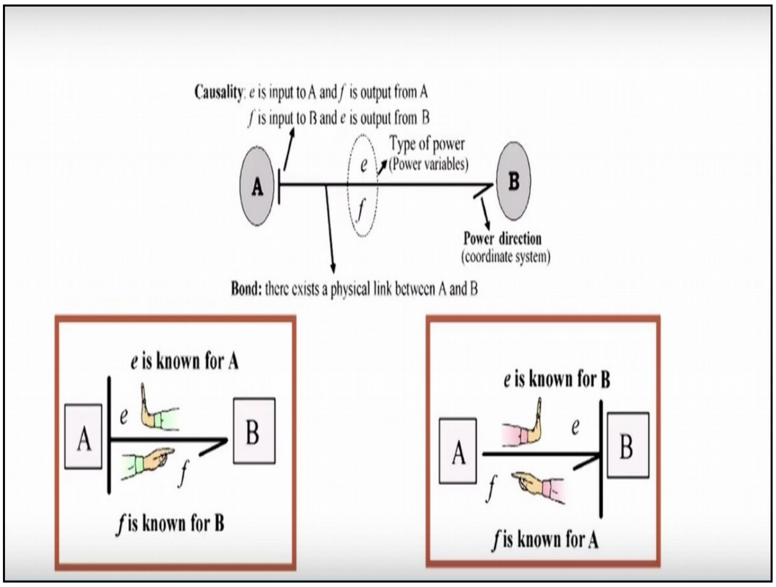
Now here f are and flow they represent are power variables fine and here stork here basically means that Effort information. Is being known for a here and here there is know is stroke it means that the flowing formations for is being known for b.

(Refer Slide Time: 19:15)



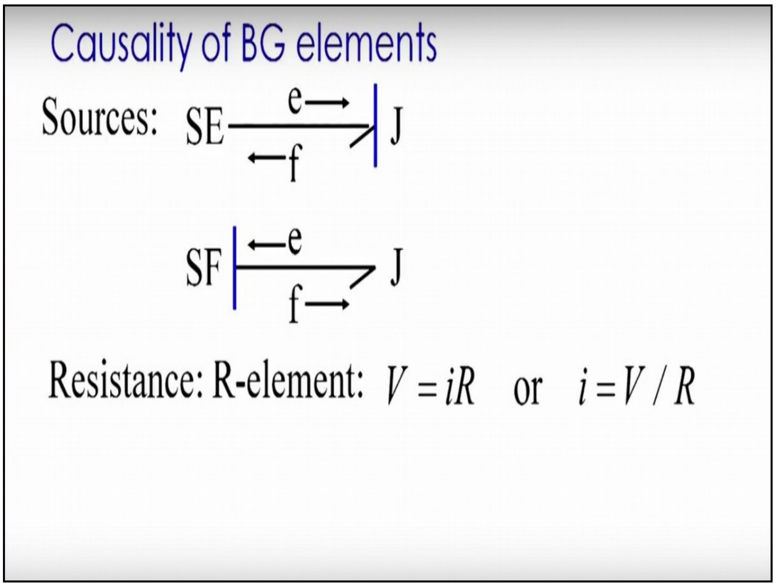
So here the causality means is input two a two f output are of form a are here I can that the x it input to be are e is output form b., so we can have the same thing is being the explained the by another way here now you can see that some element of junction a and some element of junction b that, if input is stroke here means known as for a and known stroke at it means that known for b.

(Refer Slide Time: 20:38)



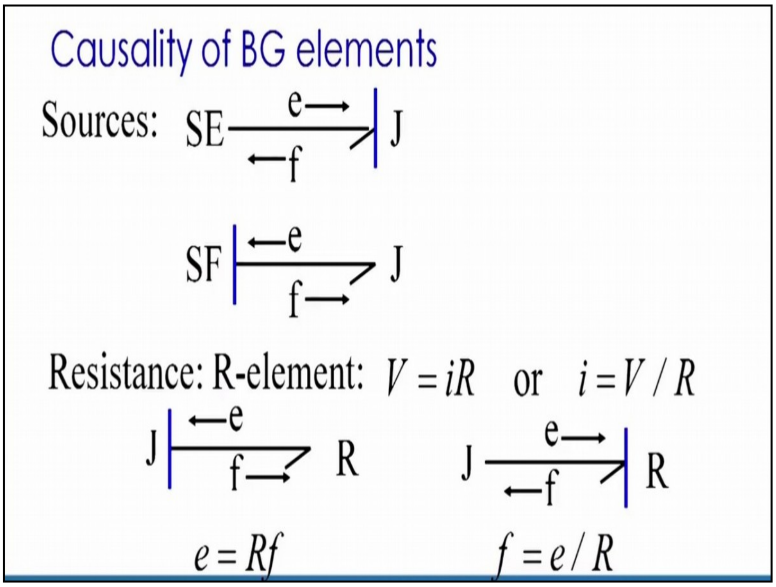
We can have another situation like this that is, stroke input being near the b this means that is e is known for b and f is known for a. So, with this concept we can go for bond graph modeling sorry, the causality of the bond graph element. So, sources as you know that, the source is sources of an effort will always be importing effort to a junction so, here will of stroke; here there will not be stroke. Then we have the sources of flow always be importing flow to the junction.

(Refer Slide Time: 21:10)



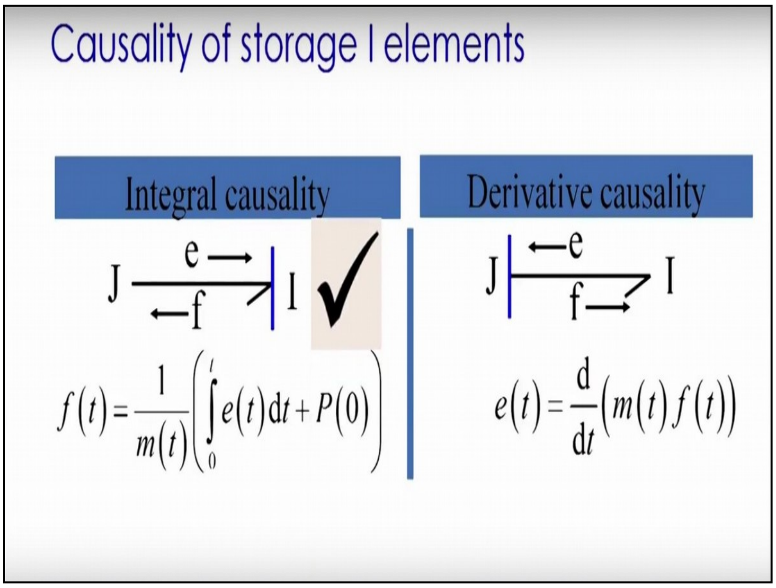
so, here there will not be stroke so, here you are going to have the stroke, and for r element you see the relationship. From the ohms law is v equate I r or I can write it I equal to v by r so for the r the element causality. Could be either way that is, here it receive flow or it returns f for as for this law.

(Refer Slide Time: 21:48)



R it could be otherwise here that it resist for and effort flow as per this law if, look at the causality of the store as element I, element so for I element this is going to the equation which we can derive from the Newton's law in the integral form.

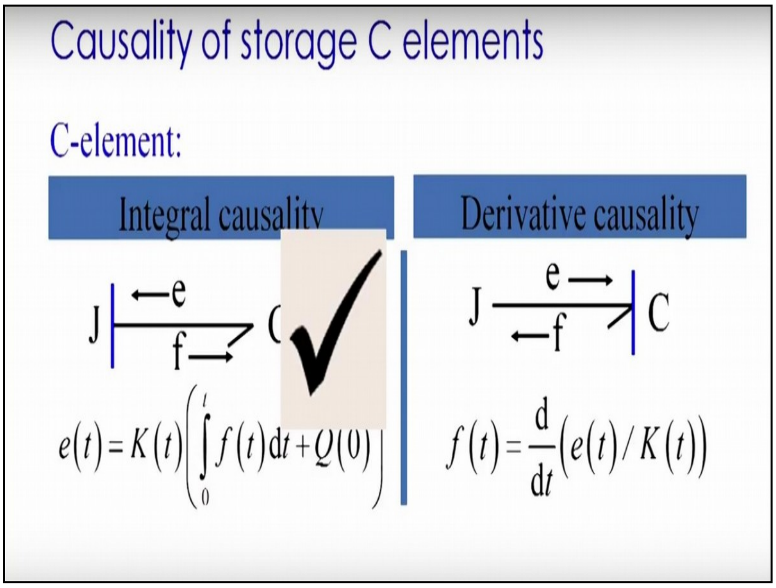
(Refer Slide Time: 22:54)



So, in this equation we see that the effort is being integrate and flow is being given and so the I element will always be receiving effort; and it will return running in the flow so this disgusts the casualty will be here. Similarly, if this integral causality is not, they are then the other form of causality.

That is I receiving flow and determine effort is what we call casualty it is the derivatives causality and derivatives causality is going to be the equation, which is going to be applicable. and since all the numerical algorithms are creating in nature be prefer the integral causality so that able to similar system.

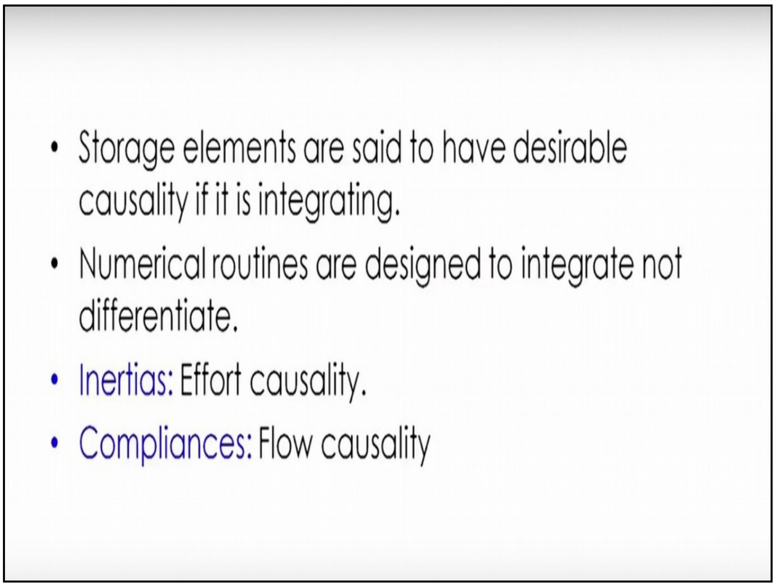
(Refer Slide Time: 23:11)



Next let us see the causality of storage c element now, for the c element in the integral causality is like this that is, they flow and they return the effort. Why because for the c element this is going to the expressions which is being applicable that is the flow being integrated and is being multiple with some constant it is going to return the effort. The c element see is flow and it returns the effort, and likewise if we have causality otherwise that is seeing effort and the returns.

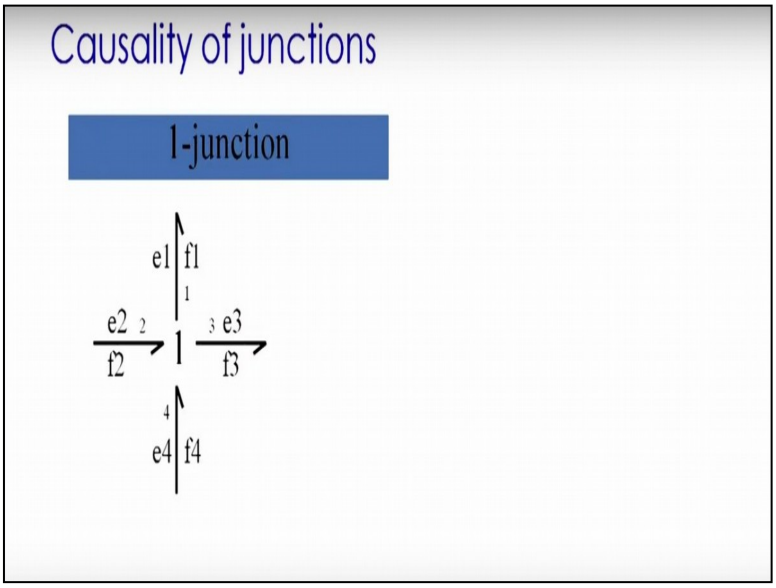
The flow call type of causality as derivative causality and is equation which is applicable here is this one and as I set we are always interested in integral form of causality. So is storage elements are, to have desirable causality if it is integrating and numerical routines are designed to integrate not to differentiate, that is why prefer the integral causality.

(Refer Slide Time: 24:02)



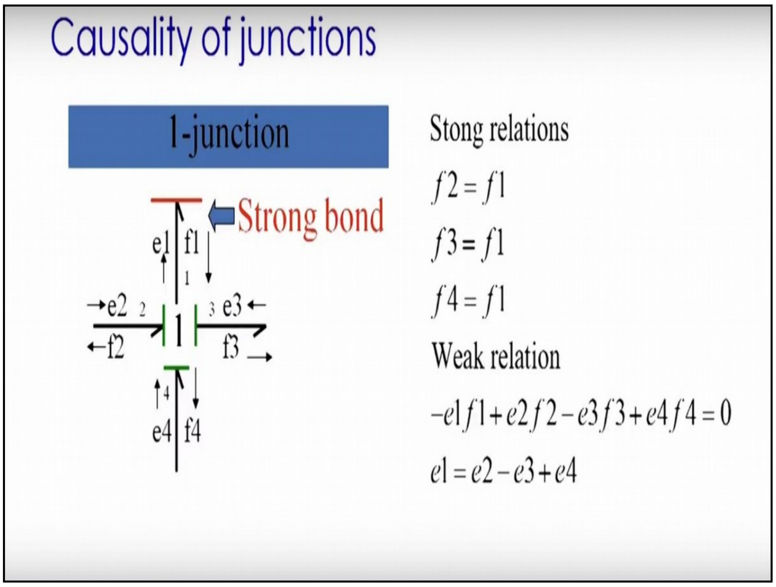
And inertia are always effort causality, because they take effort and determent flow where is the complaint is the flow causality, because they take flows and effort.

(Refer Slide Time: 24:26)



Causality of junctions know if, I talk about one junction then forever one junction is constant the flow junction so, the flow has to be decided by any one of the bonds which is part of the junction. so in flow, information should come from any one of the bonds and rest of the bond should have the effort Information.

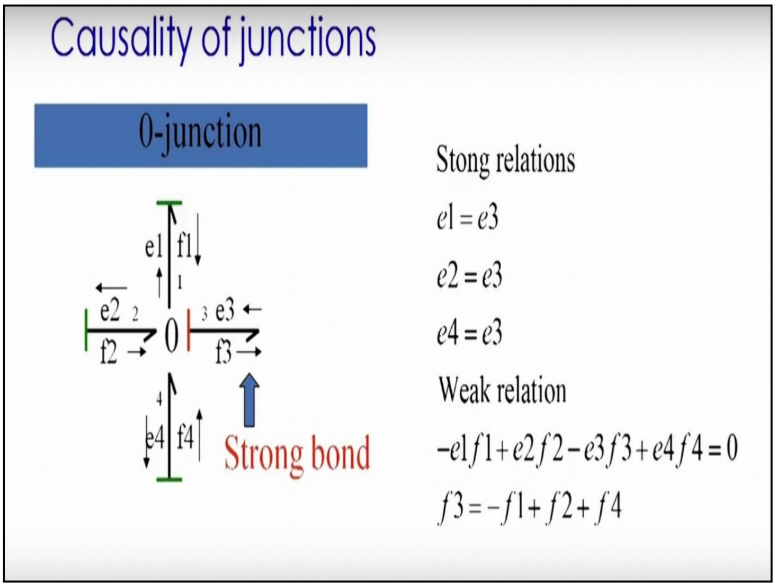
(Refer Slide Time: 24:44)



So, in case of these this is how the causality structure is going to be so here is bond will bring in the flow what we call it as the strong bond. So, likewise this one junction has strong relationship we can write, as we have seen in the previous slides. Equality and flows is the close and weak relationship the one that is relationship among the efforts, and which comes out from the junction of power principle.

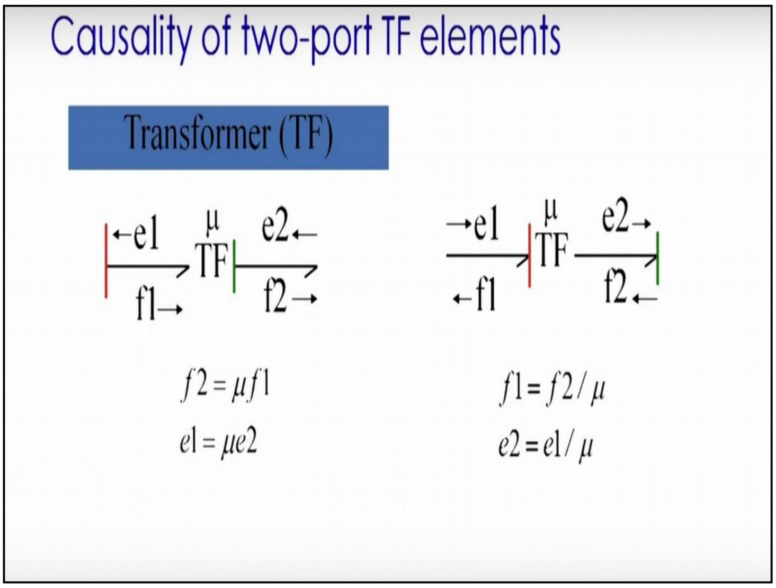
Then let us look at causality at the 0 junctions. The 0 junctions are again a constant effort junction and so the effort information should come from any one of the bonds and they should have the flow information.

(Refer Slide Time: 25:37)



So, in that case this is what the causality is going to be, this bond brings the information and rest bonds have the flow information. And this bond we call it as the strong bond. So, again the relationship is strong relationship; will be the equality of the effort and the weak relationship will be relationship among the flow variables where weak will be coming from the conservation of power equation.

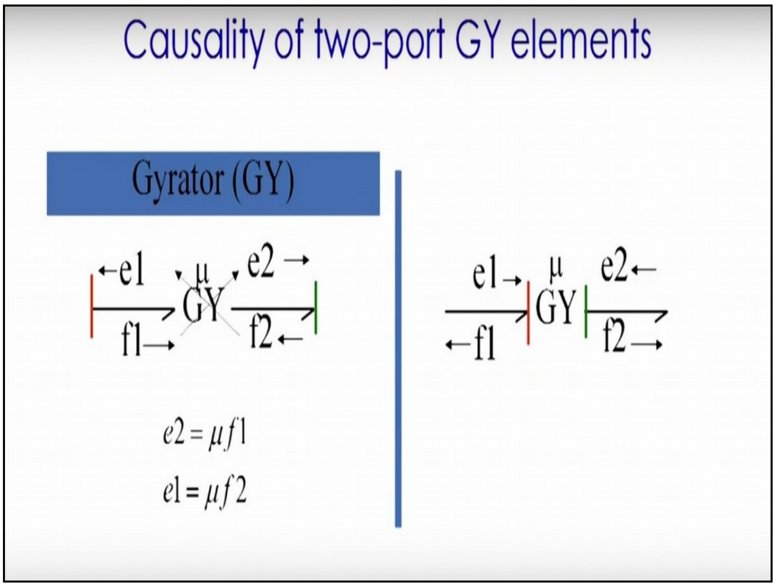
(Refer Slide Time: 26:05)



Then, let us look at the causality of two port to and gy elements so, for two port elements are they manipulate either flow are effort so they will be bringing effort in and they will be sending; sorry, they will be bringing flow in and the flow will be out. so the relationship here will be f two is equal to mean time of f one so if flow information is here the effort information has to be here.

And the transformer element can have the other type of causal which is structure as well that is bring in effort and then they give out the effort. In this case this is going to be the relationship which will be applicable.

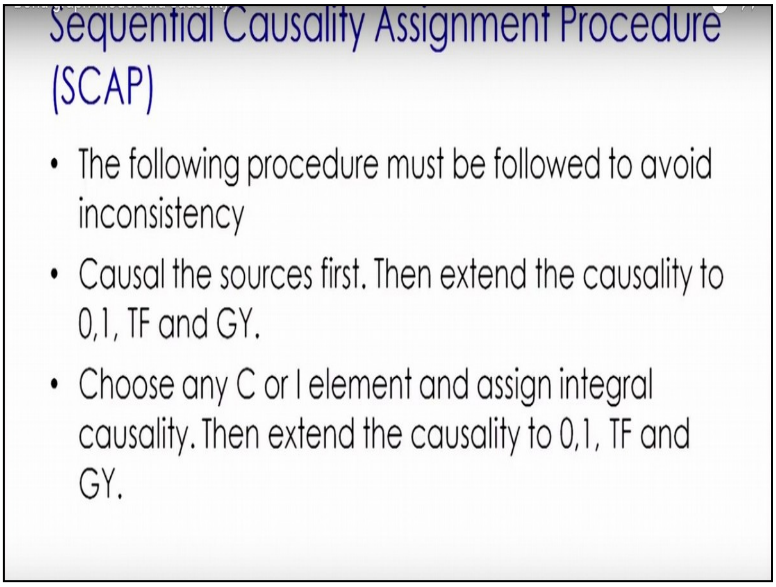
(Refer Slide Time: 27:03)



Next, let us see the causality of the two port g y element now, go element as I its greater element it manipulates efforts and flows both. so, here in this case the applicable relationship is more times of that of f 1 so in this case bring in flow and they return the effort so this is what is going to be the relationship in this case and if they have the causality otherwise then here you can see that bringing effort and they return the flow.

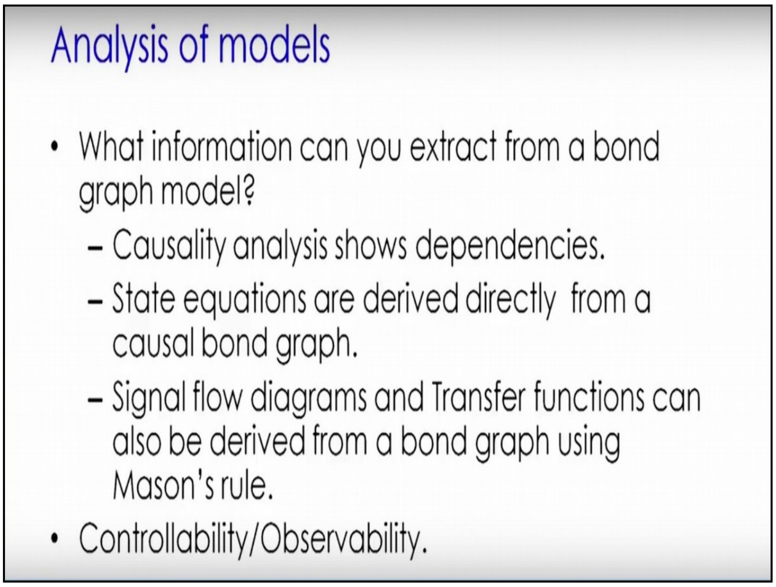
So, in this case this is going to be the relationship, which will be applicable now how is this casual assignment to be in an actual bond graph model so that, there is no confusion. and there is no earns inconsistence in the system modeling.

(Refer Slide Time: 28:05)



So there is certain procedure we need to be the flow that is, first of all we casual all the sources. and then extend this causality to the junction elements 0 one and two port element to and GY after castling sources we go for the counseling of the integrally casual c and I elements, and again extend the causality top 0 1 to and g y and after this r element is casual and the causality of the r element is given in such a way that here the whole system model and is integrally casual so this is what is done.

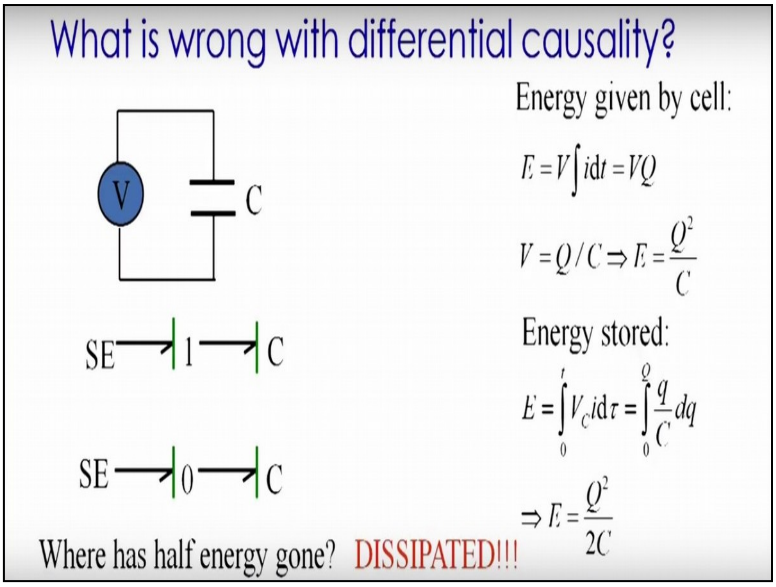
(Refer Slide Time: 29:01)



Now through castling here we can analyze our model as we know that the casual analysis shows the dependents and we can derive the state equations directly from the bond graph model that in fact, we will be looking at one full half an hour lecture on the derivation of the systematic.

We can derive the signal flow diagrams and transfer functions using the masons rule and the bond graph, and we can do the control ability, as well as observe ability analysis from here. Now, what is wrong with differential causality has been talking to you about integral and differential causality. So let us see what is wrong in during in the modeling process if, you get some differential causality.

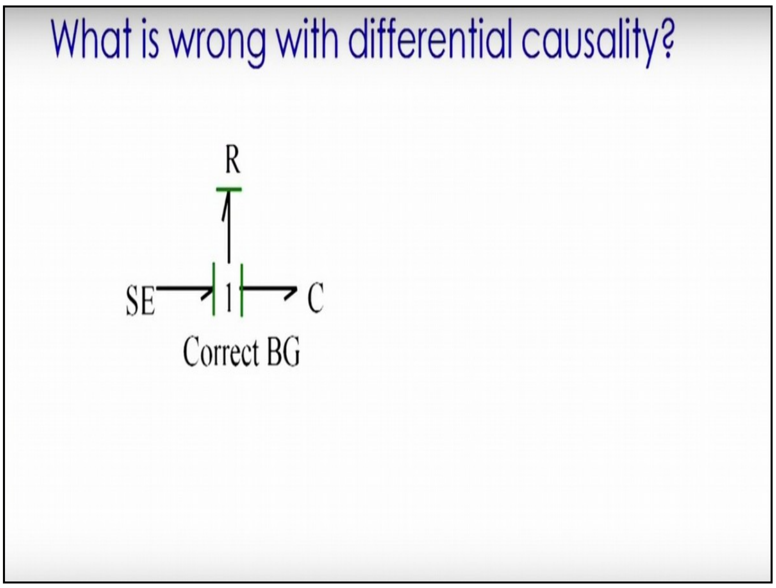
(Refer Slide Time: 29:43)



Let us take an example of charging of a cell, by so charging of a cell by it volt is source. here, we have the source. Volt a source and here capacitor, so I can model it this way or that way sorry, this is charging so a capacitor find. So I can model it either this way or that way. Now, let us see what is the spinning to the causality? So, the causalities here is stroke and in this case c element.

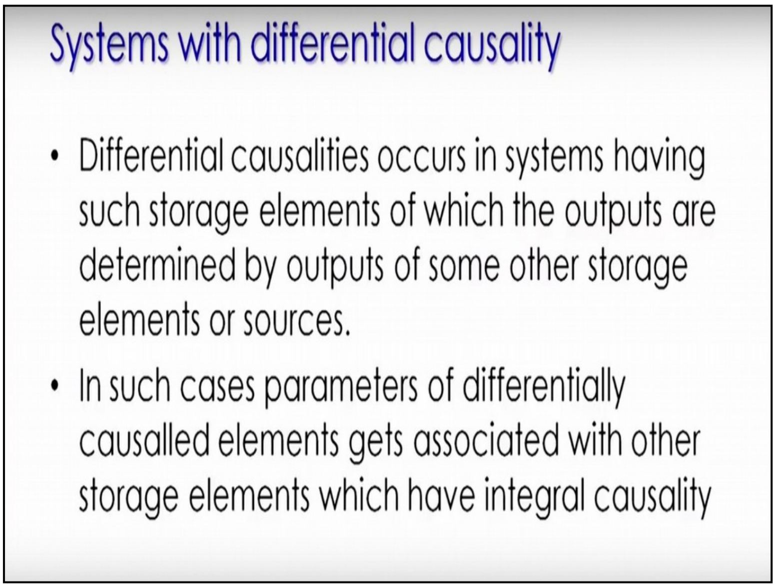
The casual is stroke, here now we see that these are differential casual. now let us look at what is the energy given by the cell; so the cell gives an in energy of q by q square by c alright, and what is the energy stroke if you look at this derivation the energy stored by the capacitor is q square by two c so energy given is q square by c and energy stored is q square by two c so where the half energy has been gone.

(Refer Slide Time: 31:12)



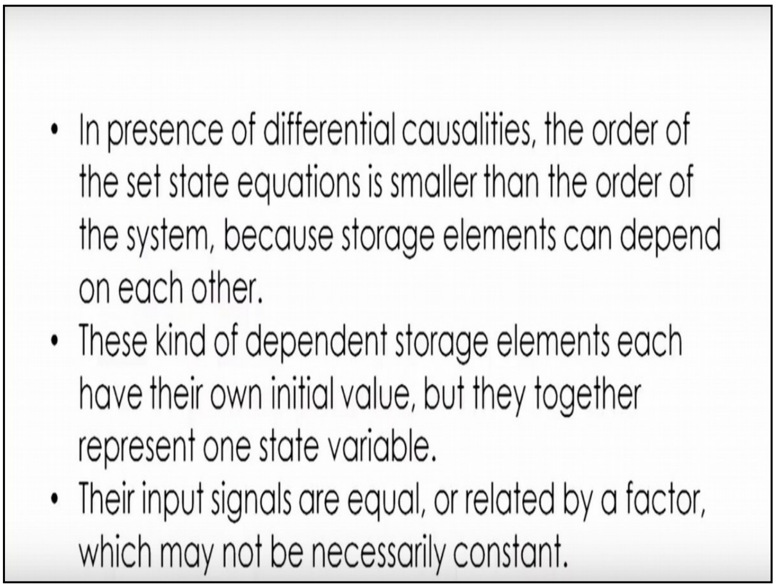
Basically, has been dissipated so, we need the model of dissipations here also then we can get rid of this differential causality so the bond graph model has to be like this so one r element being shown here to show the dissipations', so this is the correct bond graph. So neglecting this form of the model is wrong and what do you mean by system will different casualty in system modeling.

(Refer Slide Time: 31:34)



This differential causality usually occurs in system having such storage elements of which the outputs are determined by outputs of some other storage elements or sources. and in such cases the parameters of differentially casual elements get associated with other storage elements which have the integral causality.

(Refer Slide Time: 32:00)



So, in presence of differential causalities the order of the set state equations is smaller than the order of the system because, storage elements can depend on each other.

Now, these kind of dependent storage elements each have their own initial value but, they together represent one state variable and their input signals are equal, or related by a factor which may not be necessarily constant. So, this is what happened by the differential causality and these are the references. You can look at our book on the intelligent mechanical system modeling control and digenesis for the details contact. Thank you