

Modelling and Simulation of Dynamic Systems
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Lecture - 6
Bond Graphs modelling

In this lecture on bond graph modeling which is sub module for modeling and simulation, for dynamic system course which you are going through the bond graph modeling concept arise because of the need of two things that is the need to model systems which are in the different energy domains as well as because of the way to find out how we can generated the system equations for the system in an algorithmic manner.

So this was the principle driving forces behind the invention of the bond graphs modeling. So, as we are dealing with the diverse systems or the diverse field, the system any modeling system should be able to take care of that diverse field.

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Introduction

- Today we must develop capabilities to deal with variety.
- Only way to deal with variety and diversity is to achieve conceptual unification.
- The requirement of a unified approach to modeling, simulation and synthesis of physical systems residing in multi-energy domain may be stated as:

Now, one way to deal with diverse field or diversity is to achieve conceptual unification so if, there is some conceptual unification we can deal with the diversity now the requirement for a unified approach for modeling simulation as well as synthesis of physical systems which resides in multi energy domain should be summarized into these.

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The requirement

- The language for a modeling should have concise lexicon valid over a large variety of energy domain.
- It should allow the modeler to portray the exchange with in and across the domain.
- The portraits so created should algorithmically lead to mathematical or logical models.
- These models may then be subjected to predictive or deductive processes.

So these are the requirement of the tool that is the language for modeling should have concise lexicon, valid over a large variety of energy domain so, whatever language we are going to use for modeling it should be valid over large energy domains and it should allow the modeler to portray the exchange within as well as and across the domain.

So, this is another requirement and I told, u portraits whatever as been created this way should algorithmically lead to mathematical or logical models and of course these models can then be subjected to predictive or deductive processes. These are the above three are the basic requirement for the model modeling method.

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- In physical systems it is energy which plays the role of common currency of exchange between various domains and sustains the business of dynamics.
- The idea of bond graph was proposed by H. M. Paynter (at MIT).
- He used a graphical language for representation of physical system in multi-energy domain through creating portraits of exchange of power.

Now, in the physical system in which we are more interested in because we are talking about, discussing about of course modeling on simulation on domain system so the physical system in which we are interested in is the energy; which plays the role of common currency of exchange between various domains and that sustains the business of the dynamics the idea of born graph was the proposed by professor painter at and he used the graphical language for representation of physical system in multi energy domain through creating the exchange of power portray.

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- These portrays were further augmented by imposition of reference direction of power flow and causal relations or information exchange.
- In India Prof Amalendu Mukherjee of IIT Kharagpur worked extensively in bond graph modelling.
- The significant value of these portrays is that one may arrive at mathematical or logical models in algorithmic manner.

So, he good creative portray of exchange of power through systems which are lying in a multi energy domain now these portrays were further augmented by imposition of reference direction of power flow and causal relations or what will call information as the exchange they has been in a lot of work in borngraph modeling, as a tool and across Europe or USA or Asia, there has been many resources.

Who have been working in this one in India Professor Amalendu Mukherjee at IIT., Kharagpur, he worked extensively in born graph modeling and in fact, is a group develop a modeling software which is very popular with the name as symbols the significant values of the portrays that may arise as the portrays is that here one may arise mathematical or logical models in algorithmic manner.

So, as I have been talking to you in our introductory classes that the ultimate aim of any modeling exercise is to get the system equations and once we have the system equations.

We can use the simulation tools to simulate those equations. So, the born graph helps in developing the system equations in an algorithmic manner.

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- This brought both algorithmisation and unification.
- For engineering analysis and synthesis, computers could be deployed to be our deductive partners and also could be entrusted to perform simulation.

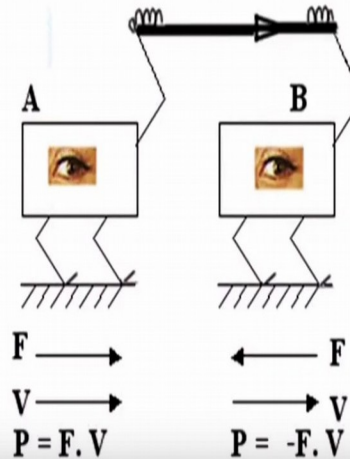
So, here in this modeling method brought both algorithmation and unification identification because, we can model systems which are there in the multi energy domain so, it unifies the systems in the different domains, and algorithmisation means that the system equations regenerated using certain logic or certain algorithm and thus, we have both algorithmisation and the unification and this fulfills the very important need of any modeling exercise to be carried out now for engineering analysis and synthesis.

Computers could be deployed as our deductive partners and also could be entrusted to do the simulation and in fact, the software which I am talking about and I will be talking to you in the coming lecture in fact, this software can do both things they can model a physical system and of course the whatever System equation or generator at the end of the modeling process those system equitation can be taken as further numerical simulation.

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An invariant nature of power exchange

- The direction of power flow at any moment is a system invariant.
- The force (effort) and the velocity (flow) are its factors.

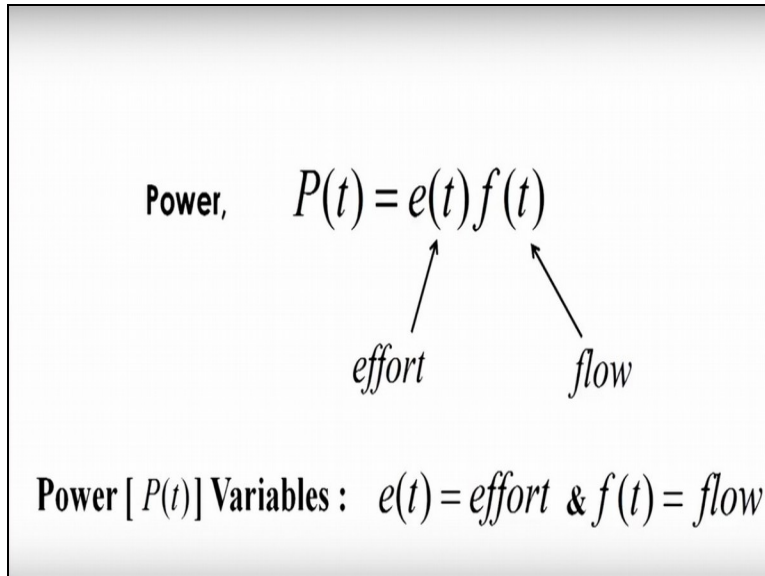


Now, before we proceed ahead let us see, nature of the power, the exchange which is the very principle what I can call it as the back bone of the bond graph modeling. So, here invariant nature of power exchange the direction of power flow at any moment is a system invariant. So for system, given a system to a willing with the direction of power flow is system invariant.

What this basically means is, let us take a simple example here there is and observe word a and another observe word is b and both are holding here and a is string to apply the a flows in this directions aha because of that it is moving this direction similarly bees stream apply the flows this direction but, is moving in topside direction here now the pulsation of a and b will be different the pulsation will be a that a is pushing r2 they are words be similarly the b pulsation will be pushing.

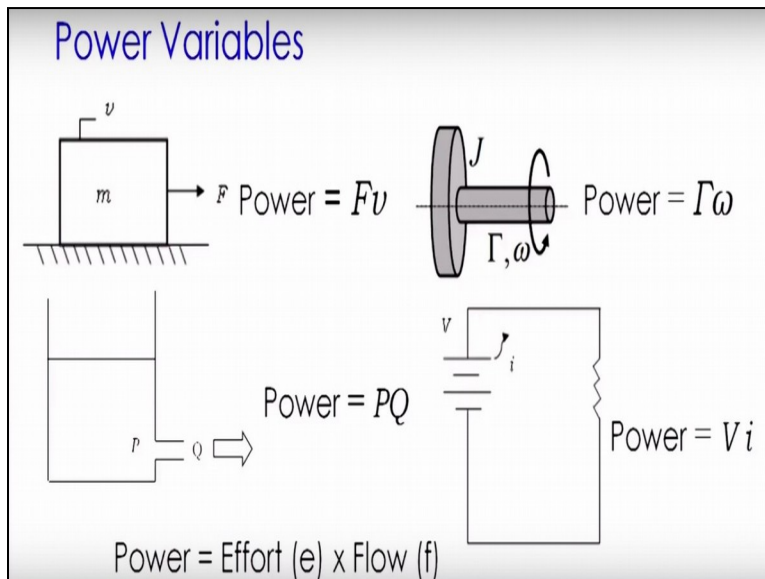
Where are to a bunt, the fact is that there is moving from direction a to direction b. So, whatever to be pulsation of a and b the power flow from system a to system b; I mean by that, the power flow at any moment it is system it where in the flows a what we call in bond graph came at the genesis effort and velocity which is team the bond graph as the flow in facts so I power basically.

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As a product of the effort and flow this I am taking general talk so the power variables could be identified as effort variable and the flow variable. Now, let us take a system across the different energy domains.

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And if you look at heresy varies models which as soon as the first one is a block which a subjected to a translation motion here in this case the power is going to be equal to the flows which is apply on the block into the velocity by which is block into moving in this case effort is force in the flow velocity.

Suppose I consider a rotary system, now in this case system suppose the rejecter, the apply and the shaft roader angular velocity ω then this case is going to be equal to tap into ω so the effort variable is stave and the flow variable is ω likewise is I take the hydraulic system here time out of liquate is fine so, the pressure here as is this charge is a then I can, I rent 5 the power I product of the pressure and this charge here basically a pressure is the effort variable and the discharge is the flow variable.

So, likewise I can take an example of a and electrical system now in this electrical system you can that there is a voltage source and of course, there is the resistor which dissipates power, now here you can see that in this case power is going to be equal to the voltage into current. So voltage is your effort variable and the current is flow variables so here I just show only flows system we can take example from system which are for many more energy domains. So, this a
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Power Variables for Different Energy Domains		
Systems	Effort (e)	Flow (f)
Mechanical	Force (F)	Velocity (v)
	Torque (τ)	Angular velocity (ω)
Electrical	Voltage (V)	Current (i)
Hydraulic	Pressure (P)	Volume flow rate (dQ/dt)
Thermal	Temperature (T)	Entropy change rate (ds/dt)
	Pressure (P)	Volume change rate (dV/dt)
Magnetic	Magneto-motive force (e_m)	Magnetic flux rate (d ϕ /dt)

This table basically summarizes the power variable for different energy domains, Now, here you can see that taken example from the various system mechanical, electrical, hydraulic, thermal and magnetic, and here the effort variables would be elated and this column the flow variables as been elapsed.

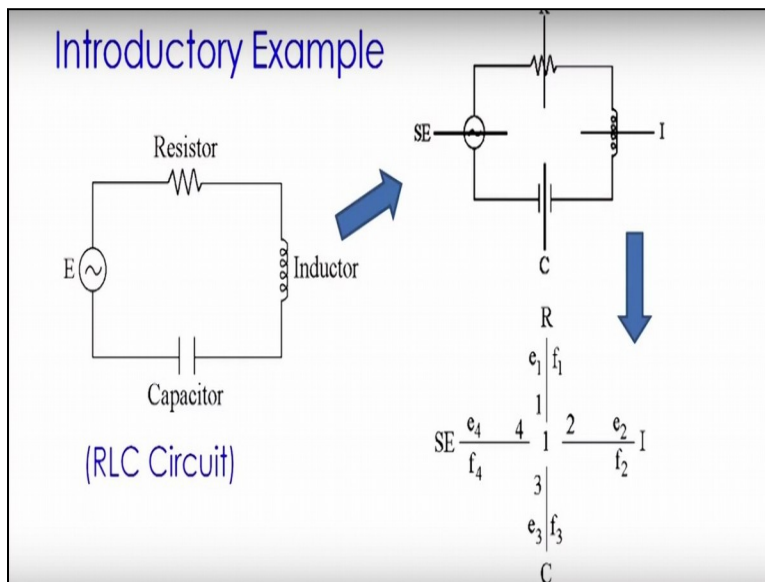
So, for mechanical system mechanical translator system of force is the effort variable and velocity is the flow variable like for mechanical rotary system torque is the effort variable, and

angular velocity is the flow variable. Now, if I talk about electrical system in this case voltage is the effort variable and current is the flow variable for hydraulic system pressure is effort variable and volume flow rate is the flow variable.

Likewise, the thermal system I can define the temperature is the effort variable and entropy change rate as the flow variable like pressure is the Effort variable and volume change rate the flow variable. I can also define the effort and flow variable, for the magnetic system so the magneto motive force is the effort variable and magnetic flux rate is the flow variable. So, this way we can identify the effort and flow variable for any system, which a line in different energy domain and once, we can identify the effort and flow variables then our bond graph modeling becomes easy.

Let us take very interactive example what causally bond graph is and how the bond graph look like all of you must be our simple rlc circuit which that in shape first year of 2 year of year under graduates classes.

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So here, this figure shows the simple RLC circuit, this consists of voltage sources, resistor, inductor, and capacitor. Now it is up to you to draw the bond graph model for this circuit. How to proceed, how to draw the bond graph model for this system. Now, let's assume that, as I am telling you is system in Variant here in this are in bond graph.

So, here there is visual power pipe line throw which power comes to this circuit, so the visual power pipes line on here alright I am writing here se because volt is sources of effort is which just in the privies lint in electrical systems now, then we have a resistor a and all of a know that resistor respect power.

So, again assume that the visual power pipe line throw which displaced by the resistor then we have inductor so for inductor here again inductor is stroke power so letter zoom there is visual power pipe line throw which inductor and here there is the capacitor again is toque for let zoom that there visual power in pipes line through, which power goes to the capacitor .so here basically at first stage what we had done.

We have represent how the power forces a cross all the forces elements in this physical model now if I remove the physical circuit from the back ground of the finger then, this is what we get a source of a effort we have a resistor we have an inductor and we have a capacitor find so here what is done basically, we have the physically circuit by it components and true be this bonds we have just indicators that, how are that is the flow of power to this elements alright now after this is basically what constitute it we have so many elements in the system and we have resented both elements.

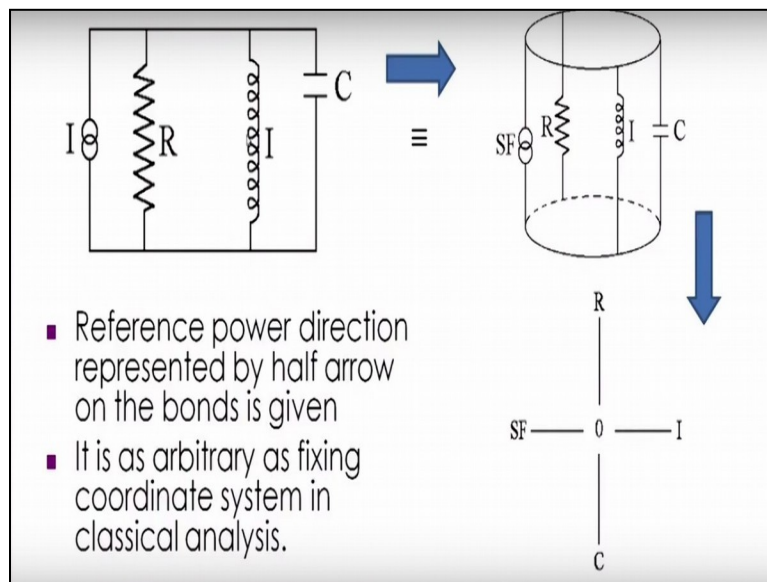
Certain components now the next time are how these elements are constant here. So, if a look at the constant this is a simple rlc circuit and this rlc circuit ah it is series rlc circuit we can see here the current is going to be same as current a case of electrical system it a flow variable so the flow is going to same across all this four elements so that think we ah put here are diet identification that flow is same across all the elements is done by putting here the number one. So, this way it completes the bond graph.

What we have done, we have represented the elements oldest sources by sources of efforts resistor by r , inductor by L , capacitor by C and we have I , put the constant here that is the constant current constant by putting the number I here now for book-keeping purposes.

We can number this visual power of pipe line or what we call it in bond graph modeling, as the bonds by different numbers 1 2 3 4 and, here I just for our understanding return the different power variables in different bonds the bond it is even is the effort variable f_i is the flow variable here one basis it basically means that it is related to bond number one likewise, define the power variables for the other bonds.

So this slide must give you an idea of the bond graph modeling next letter take to another example now suppose,

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We had resistor inductor and capacitor they are in parallel, can current connected with the current sources, now here again is you go by the same concept which I discuss with you in the precise lines this circuits made of the capacitor and inductor resistor and current source. So, here what I just do it and all this elements are parallel.

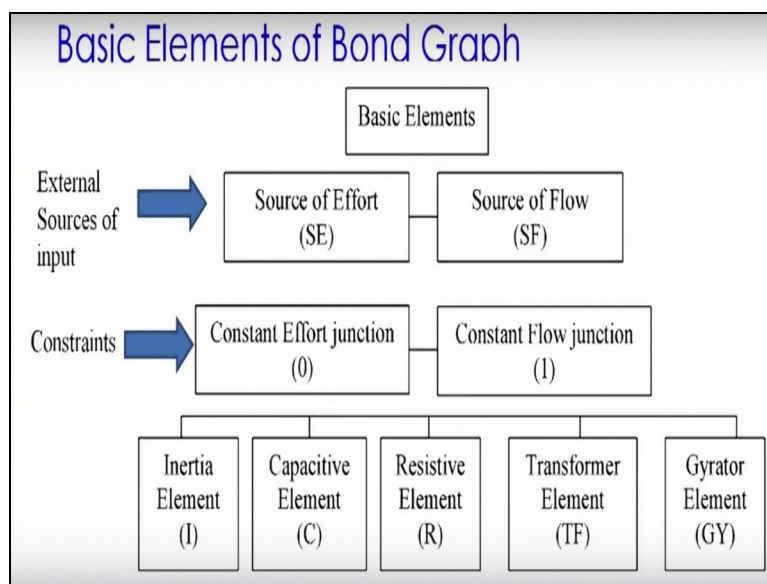
So, basically this is how I can represent here and then identify a current, is a flow variable so here what we call is that the source of current; is source of flow. I represent sf there is resistor. I represent r there is the inductor, so I represented by I and there is the capacitor so I represented by c , so again we have virtual power pipe line from the virtual power of pipe line or true bonds the power goes to all this elements so this was the dissipation of power across the elements.

Next, what we put is that how these elements are constant. So in this case you can see that voltage of same across all the elements and that represented voltage is actually or in general term we call it as an effort, so the effort cross all this four elements and that it represented by putting is o elements here this is how we draw the bond graph for this physical circuit now a in this model so whatever you see here in this model are for further augmented by carpeting the power direction.

Which is nothing that, a I we put a coordinator system in classical mechanical and also we put power direction represented the half arrow so that is how it is done why put the half arrow because full arrow is use as signal bond in the classical controller system problem. So, to the word confusion with a signal bond here we have put in the half arrow and just to indicate the are the power bonds so the basic elements of bond graph now let's come back to the theory of bondgraph modeling.

What are the basic elements of the bond graph? So, basically there are nine elements throw which we can draw the any dynamic system in fact, with the help of this nine element we can draw the bond graph model of any dynamic system, now this nine element could be class five at sources source of input.

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There are two sources the source of effort which is represented by e and the sources of flow represented by the f . So, these are the two sources, there are two constant also I, talked to you in my previous like about the two types of constant, the constant effort and constant flow.

So, this constant effort junction use be because they indicate the junction of the different bonds, so the constant effort junction is represented by 0 and constant flow junction represented by 1 and there are pipes other than five other elements parts of from this to and this out of I, we have the inertia element I , capacitive element c , these two are actually the energy storage element then we have a resistor element r which of course this dissipates power and then we have a two energy; two other elements.

What we called it is the transformer elements and gyrator elements and this transformer and gyrator in the bond graph model calling as that two part the element because they interact with rest of the system through to bonds.

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Power and Energy variables

Power variables
 $e(t)$ and $f(t)$

Energy variables

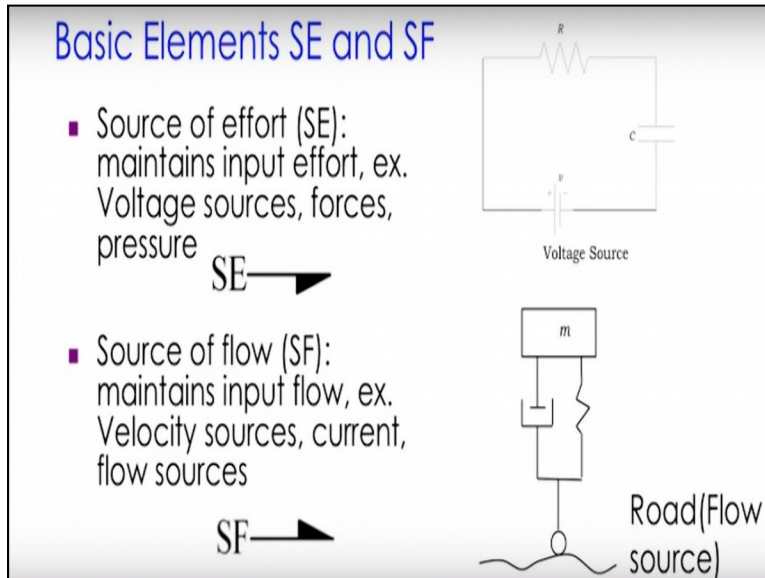
$$p(t) \equiv \int_{-\infty}^t e(\tau) d\tau = p_0 + \int_{t_0}^t e(\tau) d\tau,$$

$$q(t) \equiv \int_{-\infty}^t f(\tau) d\tau = q_0 + \int_{t_0}^t f(\tau) d\tau.$$

So the power and energy variables the power variables, as I talked to you they are effort and flow and the energy variable could be defined likewise, as the moment as this placements this moment are this define integration of effort of course, from minus infinitive to t . Likewise, to this placement to define a increment of the flow and this minus infinitive we can replay with into

about putting in initial condition as well here also. Next is the basic elements se and so so let us see a little description of these basic elements.

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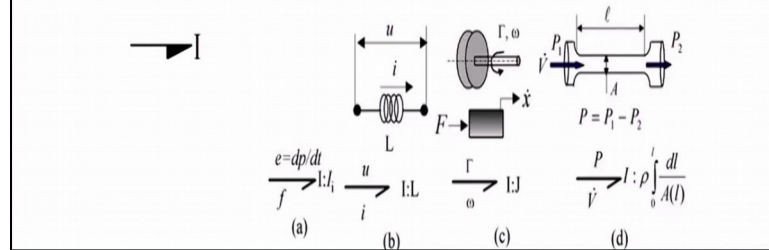


So we have I just set the two basic elements are sources what we call the source of efforts and source of flow, so the source of effort maintains input effort in the example of voltages source forces and pressure and this is how the source of effort is represented, and this example a voltage sources actually the source of effort then, we have sources of flow which maintains input flow the example could be velocity sources current and the flow sources and this is how is represented by half arrow and the source of flow is basically would be a good example can be given by the ground exapatiation by on suspension system of the car then we have energy storing element I now energy storing elements.

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Energy Storing Elements I

- Inertia/Inductance (Effort storage) $F(t) = m dv / dt$
 - Idealisation of devices like mass, inductance, inertia in mechanical, electrical, hydraulic systems respectively. $v(t) = (1/m) \int_{-\infty}^t F(t) dt$
- $$f(t) = G \int_{-\infty}^t e(t) dt$$



This elements I are used to represent the inertia inductance, and this are effort storage element why effort storage because a we know from newtons law that we ft, we write as mdv by dt and from here we as one by m integral $ft dt$ so here the genital team from Wright so I have a flow equal to same content into integration of $et dt$. So here you se that the effect is being integrated that is five called the effort storage effort storage element and the I element could be idealization of devises.

Like mass in the factors interracial and mechanical aha electrical and hydraulic system respectively. This way this can be model and here you can see the different examples here it is inductor, here an example of root it is system here that translator system and hydraulic system so through bond graph and inductor can be model aha h inductor can be represented like this so I call in the write I we are a I is induction of the inductor similarly here for Rooty system it I write I a Equal to j we are j equal to polar moment of inertia of the rotary system like wise I can define the I element are inertia for a hydraulic system next is energy is storage element see

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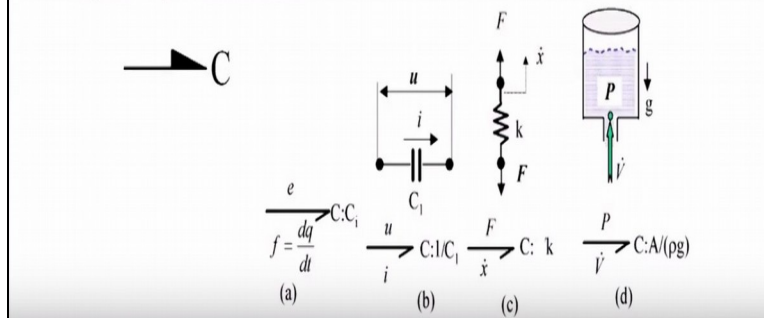
Energy Storing Elements C

- Compliance/Capacitor (Flow storage)
- Idealisation of devices as spring, capacitor, accumulators

$$Q(t) = \int_{-\infty}^t f(t) dt$$

$$e(t) = KQ(t) = K \int_{-\infty}^t f(t) dt$$

$$e(t) = G \int_{-\infty}^t f(t) dt$$



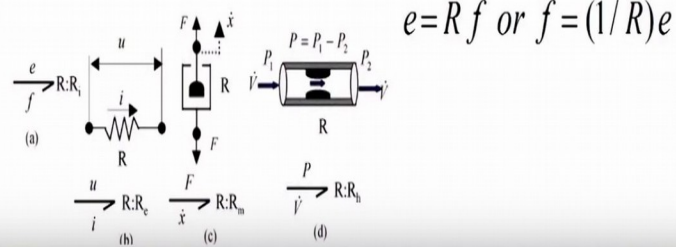
These are the compliance or capacitors, a compliance capacitor can be represented by an energy storage element. These are also called as the flow storage elements. So, these are idealizations of devices as devices such as spring capacitor accumulator and this is how because they take power. This is how they are represented on the bond graph. Why we call it flow storage? Because, you see the displays could be returned on integration of the flow that is fine we call it the flow in storage element and here you see this is fitness of the spring into displacement or k integration of the $f dt$, and this k I can general term I can represent by same constant g .

So, effort is g into integration of the flow. Alright, we can take whereas example as a set a ya I electrical system capacitor to would be model like this be in c being represented by c_1 spring can be represented like this which see being is fitness of the spring and like a hydraulic system see can be represented by ay row g similarly let's see we resistor element at this restore element are indication of devices like the tempers resistor flu caring pipe.

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A Resistive Element (R)

- Idealisation of devices like dampers, resistors, fluid carrying pipes. $\longrightarrow R$
- Dissipative element.
- Removes energy and relates effort to flow.



This is how they are represented in the bond graph, these are dissipative element and this remove energy and relates effort to flow so as we up we incuse of the dampers efforts all the floes is temping constant a into the velocity.

So here I can write e equal to re ri can write it to other way sf equal to one by r into e so I set this as example this how in general team represented to with effort and flow being return as ef the to power variable and here are semicolon of r one so for electrical system this r 1 is r resistor here for a mechanical system this is the damping constant here for hydraulic system here this is return as the hydraulic resistor ri like wise we can define the other two elements. So I stop here and will be continues in the next lecture. Thank you.