Modelling and Simulation of Dynamic System
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Lecture – 40
Optimization with Modeling of Engineering Problems

I welcome you all in this lecture on optimization with modelling of engineering problem and this is a sub-module for the course modelling and simulation of dynamic systems through, which you are going through. This is the last lecture among the 40 lectures, which we have covered and in this lecture, my aim is to take up some examples optimization on optimization some engineering examples of optimization, the concept of which we have seen in the previous lecture okay.

So, this is what I intend to do here, so we begin with by taking a simple problem and we will see that from a given problem statement, how can we formulate an optimization problem okay and after that, we will be seeing the 2 ways that is the graphical method and one of the search method to see how can we solve an optimization problem.

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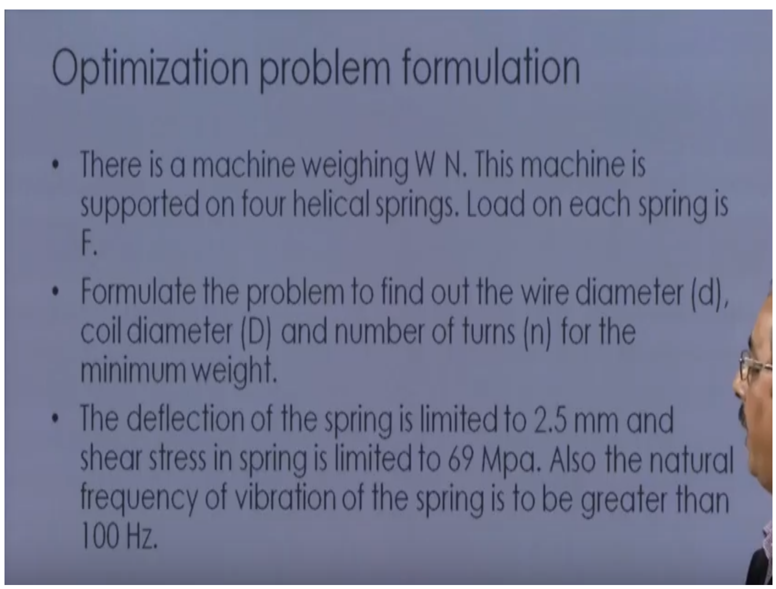

So, coming back to the optimization problem formulation, we can take a simple example say that there is a machine weighing say WN and this machine is supported on 4 helical spring and load on each string is F, which will be nothing but say W/4, now the question is to formulate the problem, to find out say the wire diameter, the coil diameter and number of turns for the minimum weight.

So, it is like this basically say we have a machine and this machine is supported on say 4 such strings, so I am just showing that 2 here alright and one of the spring I can show in detail say something like this okay, so here and certain load F is say applied from here, now in this one if we look at say the area of cross section of the wire is this one, so this has got a diameter d, the diameter of the wire and the coil diameter.

We can take say coil diameter is given by this one and so this is D and n is the number of turns. I have exaggerated a little for you to understand it. We have given the deflection of the spring is limited to 2.5 mm and say the shear stress in a spring is limited to 69 mega pascal and the natural frequency of vibration of the spring is to be greater than say 100 hertz.

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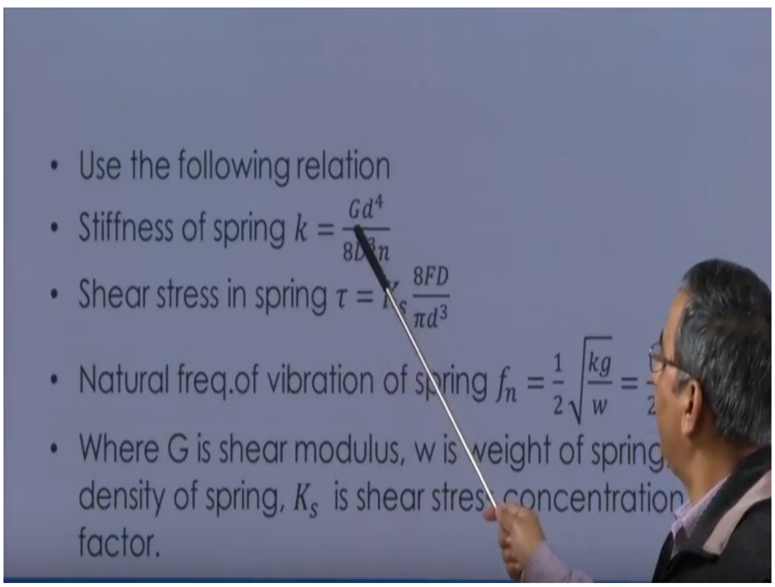

So, these are basically, these all statements from here to here are the constraint okay, and it is given that use the following relation say stiffness of the spring given by Gd to the power 4/d cube n, where G is the shear modulus, d is the diameter of the wire, D is the diameter of the coil and as I said n is the number of turns in the spring and the shear stress in the spring tow is given by Ks 8FD/pie d cube, where F is the load, which is coming on the individual f spring.

And of course, other terms have already defined and ks is the shear stress concentration factor. The natural frequency of vibration of spring fn is given by 1/2 under root kg/w and here if you substitute for k from here, this is the equation, which we get here also we are substituting for w as well.

So suppose we are given these things, so now the question is that how we are going to formulate the optimization problem. Now, the Optimization problem essentially consist of 3 things, the first thing is that design vector okay and then we have the objective function and we have the constraints, so these are the 3 things, which we have to write from the problem statement.

So in this case, as I said our interest is to find out d that is the wire diameter, D that is the coil diameter and N the number of turn, so d, D and N, these are the design vector okay and the objective function F X is as I said we want to minimize the weight, so objective function is given by this equation, where pie D square/4 is the area of cross section for wire and of course this length goes up to pie D.

So far one turn this is going to be the total volume and N is the number of turns, so we will have the total volume of the spring okay, so this is what is to be the objective function. Now coming to the constraints as given in the problem, first constraint is the deflection of the spring, which is given by F/K, so this del = F/K that we can find out from that equation and this has to be less than equal to 2.5.

So from here I can write it in another form that is G 1 X that is I want to write this less than equal to, less than in the form greater than okay, so I reverse it okay, so this is what I get, so this is one of the constraint based on the deflection of spring. The other constraint is the shear stress constraint, so I write expression for shear stress that is Ks 8FD/ pie D cube and this should be less than or equal to 69 mega pascal.

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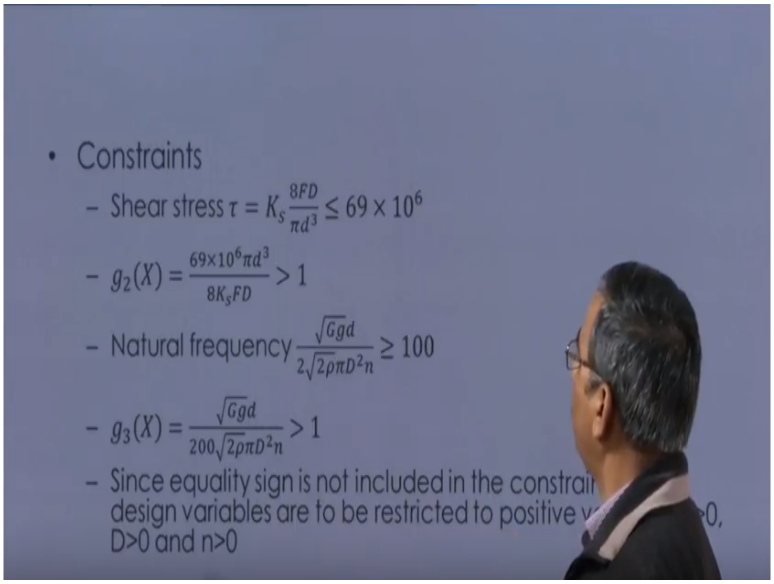

So I am writing 69x10 to the power 6 okay and again if I want to write it in the greater than form, then this is how it is to be written, so I take denominator in the numerator and numerator and denominator by reversing it, so this is what I get, so g2X, this factor has to be > 1, then coming to the natural frequency, it is given that this natural frequency has to be > or = 100 hertz. So this is the expression, which is given to us and I make it > or = 100 okay, so the greater than criteria is g3X, this root Ggd/200 and root 2 rho pie D square n okay.

Now here you can see that in this constraints and the previous constraint, we have not considered the equal to sign. So since it is not included in the constraint, the design variables has to be restricted to the positive values that is the diameter cannot be negative, d, D and n cannot be negative, so these values has to be taken more than 0, so this is how we can formulate the problem by writing the design vector optimization function and the constraints.

Now let us look at optimum design concept. There is a broad classification of the optimization technique okay and this classification comprises of indirect or what we call it as optimality criteria method okay and the direct, the search method. The optimality criteria are the condition, a function must satisfy at its minimum point.

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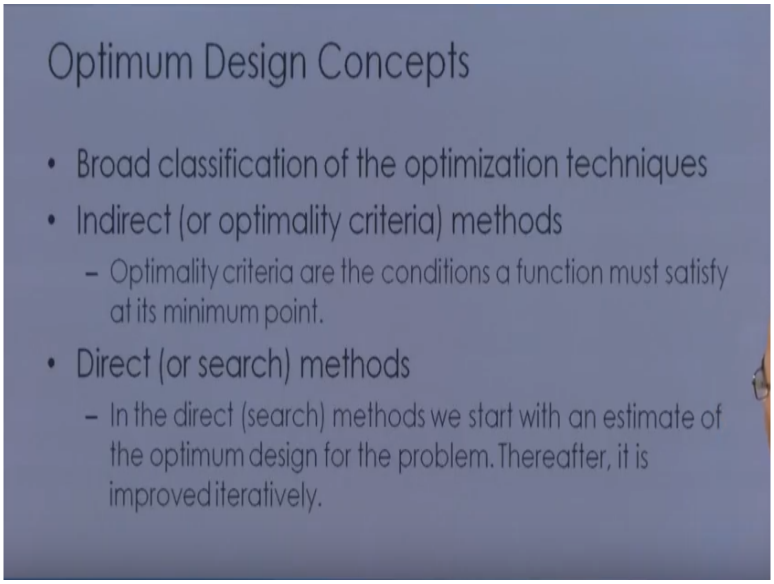

So this is based on which we decide about the optimization okay, optimum value and in case of the direct search method, what is them is that we start with an estimate of the optimum design for the problem and thereafter, we keep on narrowing down our search area and the iteratively, we try to reach to a optimum value okay.

So optimization method as I said can be classified based on optimality criteria or the search method and of course then again this can be applied for the constraint problem, as well as for the unconstraint problem. Similarly, the search method can be applied for the constraint constant problem, as well as the unconstraint problem.

Now one of the earlier ways of finding out the optimal solution has been the graphical solution, although with the invent or with the popularization of the computers, this method has become obsolete, but since it is a graphical technique it is to our understanding of how we can find out the optimal solution okay, so just for you to understand I have included this method and I will be explaining this method by taking a simple example.

So the graphical solution as I said optimization problem having 2 design variable can be solved using graphical methods okay and all constraint functions are plotted and a set of feasible design for the problem is identified using graphical methods okay and then the objective function contour are then drawn and the optimal design is determined by the visual inspection, so this is how the graphical method works okay.

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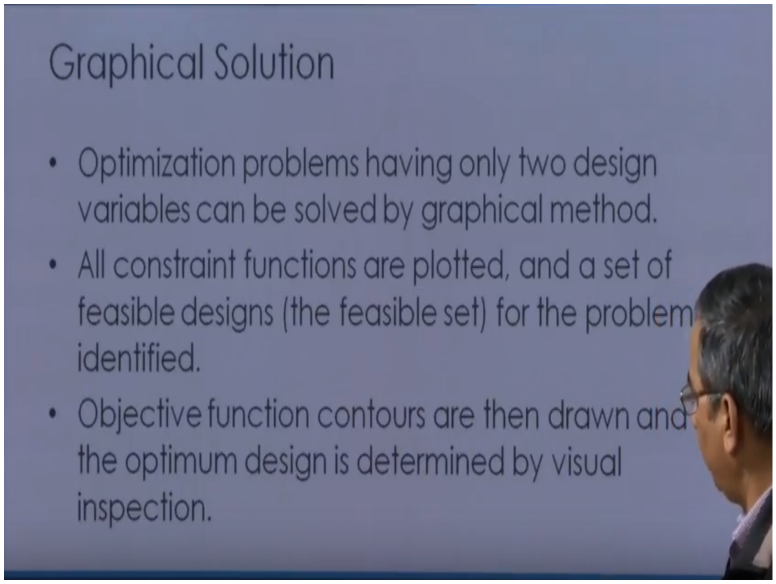

So it is applicable for the 2 design variable, first we plot all the constraints and then we plot the objective function contour and then we determine or identify the optimal design, so I am going to take a small example that is the beam design problem for optimization and as we have seen in the previous example.

First I give a problem statement and then the data and information collection, we will be identifying what are the design variable, objective function and what are the constraints and finally will be going for the graphical solution, so let us see a problem statement. The problem is that a beam of rectangular cross section is subjected to say bending moment and a maximum shear force.

Now the bending stress in the beam is calculated by say 6m/bd square pascal okay and the average shear stress is calculated as tow is 3V/2bd Pascal, where b is the width and d is the depth of the beam, now the allowable stresses in bending and shear are given say 10 Newton per millimeter square which is same as that of mega pascal and the 2 Newton per millimeter square respectively.

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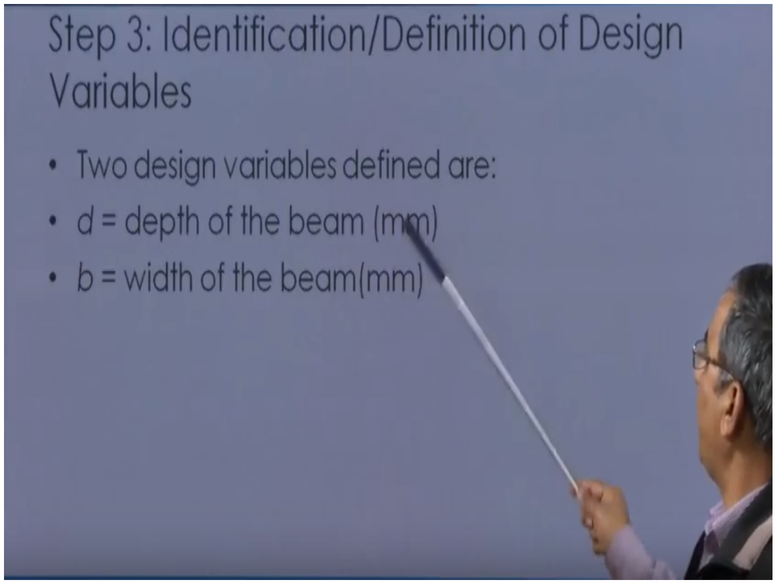

Now, it is also desirable that the depth of the beam should not exceed twice its width and that the cross sectional area of the beam is minimized, so this is our problem statement, so from this problem statement, you can see that we are talking about the true design variable and these design variables are the b and d that is width of the beam and the depth of the beam and the objective function is the cross sectional area of the beam is to be minimized.

So this is the objective function and other things are the constraint, for example the bending and shear stress values are given, so this is constraint, similarly the relationship between depth and the width of the beam is given, so that is also a constraint, so the data and information which are given to us, say the bending moment apply is 40 Newton meter and the shear force say 150 kilo Newton okay.

So, although I am not going to solve this problem completely, but I will be telling you the entire process. Then is the identification definition of the design variable, so that the 2 design variables are defined as I said and these design variables are the depth of the beam and the width of the beam okay, both are given in a consistent unit.

So millimeter here, then as I said the objective function for the problem is the cross-sectional area, which is expressed as bxd, bd, then identification of the constant, so as I said there are 3 constraints and these are the bending stress, the shear stress and depth to width ratio, which is given, so we can find out these values, so bending stress 6m/bd square.

So M is given to us, so 40 so that is converted into a consistent unit, so this is the bending stress, then we have the shear stress 3v/2bd, so I substitute the value of v here, so I get an expression for the shear stress and the allowable bending stress value is known to me that is 10 mega pascal and allowable shear stress value is also given as 2 omega pascal okay.

So what are going to be the constraints, so the bending stress constraint is going to be that this value okay -10 should be less than or equal to 0 okay or whatever bending stresses are being developed, this value should be less than 10 okay, so this will be giving us the bending stress constraint. Similarly, I can find out the shear stress constraint okay.

So, that is that this value should be less than the given value that is the allowable shear stress, so this -2 should be less than or equal to 0. Similarly, we can have the third constraint that is the depth be no more than twice the width, depth – twice of width has to be less than or equal to 0 okay and since both design variable should be non-negative.

It means that we will have the 2 more constraints that is g4 and g5, so minus of b has to be less than equal to 0 or you can say that b has to be more than 0 because it has to be non-negative, this is another way of writing it, if you want to write all the constraint in the form of less than and equal to sign, so -b should be less than and equal to 0.

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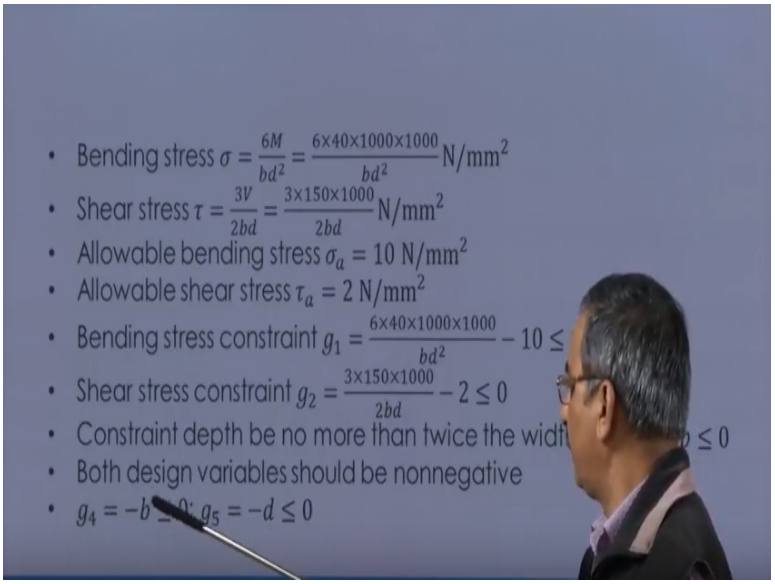

Similarly -d should be less than or equal to 0, so now we are talking about the graphical solution. After defining the design variables, objective function and the constraints, now it is to be solved okay, the optimal value is to be determined, so here what is done is that we have the 2 co-ordinate axes are drawn along say the x we are plotting the width and along Y we are plotting the depth okay so this is my objective function.

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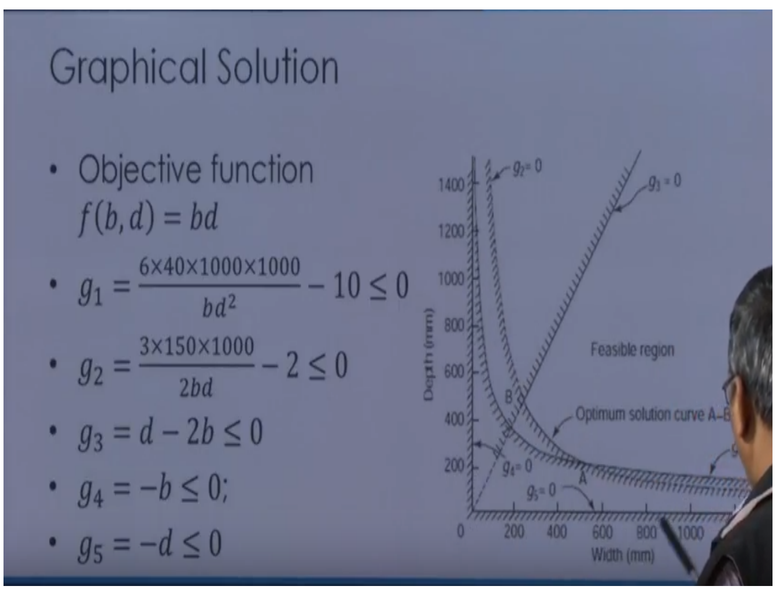

My first constraint is one, so g1=-10 should be less than or equal to 0, so for plotting purpose, we take this one =0 and by giving the different values of say B, we can find out the value of D and then we can join all those points and we can plot, so the G1 can be plotted like this, so you can see this is your plot for G1 okay.

Similarly, this was our second constraint g2=this value -2 should be less than or equal to 0 okay, so again for plotting purpose, we take equal to2 given value of b and find out the value of d from this expression, we will get a point and then we plot all those points, join all those points in order to get the contour for g2.

So this is my g2 contour okay, so this is being plotted and then we have the contour for g3 that is d-2b <= 0, so here again we take equal to sign and for a given value of b, find out the value of d, so this is going to be a straight line expression passing through the origin, so this is my constraint gt=0 fine and then we have –b <= 0 and –d <= 0.

Now remember here, we are emphasizing that this value should be less than or equal to 0 okay, so we are essentially interested in the reason nearer towards the 0 side okay, towards the 0 side, so that is why you are saying these hatching allright, so this is how we do plot all the constraint here okay.

So either we do manually or we can take the help of some of the software, say Matlab and the constraints of the problem can be plotted and the feasible region is identified, now here note that the cos function is parallel to the constraint g2 because the cos function involves bd and the g2 constraint also involves bd, so the cos function is naturally going to be parallel to that of this one okay, bd = constant.

So therefore any point along the curve ab represents an optimal solution, so we are going to have infinite number of optimal design and of course this is a desirable situation, since a wide choice of optimal solution is available in order to meet the designer's need. So, from here, the optimal cross section area can be find out, this is 112.500 mm square and point b here of course, we are looking for the optimal.

As I said optimal curve, our objective function curve is going to be parallel to this g2 okay, so here the b value, we can find out, which is from here. Similarly, we can find out the another value a okay from here that is the intersection of these constraints, so these points basically represents the 2 extreme optimal solution and all other solution lies between these2 points a and b of the curve okay.

So this is how it is and this way we can find out the optimal solution, so the graphical method which you have seen this is easier, but tedious and it is limited to the problems okay and the other method that is the differential calculus method is not suitable if your derivatives are not continuous okay.

So in this scenario actually it is the search best method, which appears to be promising and direct method actually what we do is that we evaluate a function at sequence of point, say x1, x2, xn and compare values in order to reach to the optimal solution, x star, so the direct search method is done by either we go for exhaustive or the total search okay which of course is going to be less efficient.

Because we are going to do an exhaustive search or we go for some efficient search method such as golden section method or Fibonacci method in order to find out the optimal solution so we will be looking at the single variable search technique. These single variable search technique, which I just talked, they are efficient techniques okay.

They are applicable for unimodal function okay. The unimodal function are basically the functions within a certain interval if that function increases monotonically to a maximum and thereafter decreases monotonically okay that is what I want to say is that it has got only one hump or one depression within a defined interval, then we call that function as the unimodal function okay.

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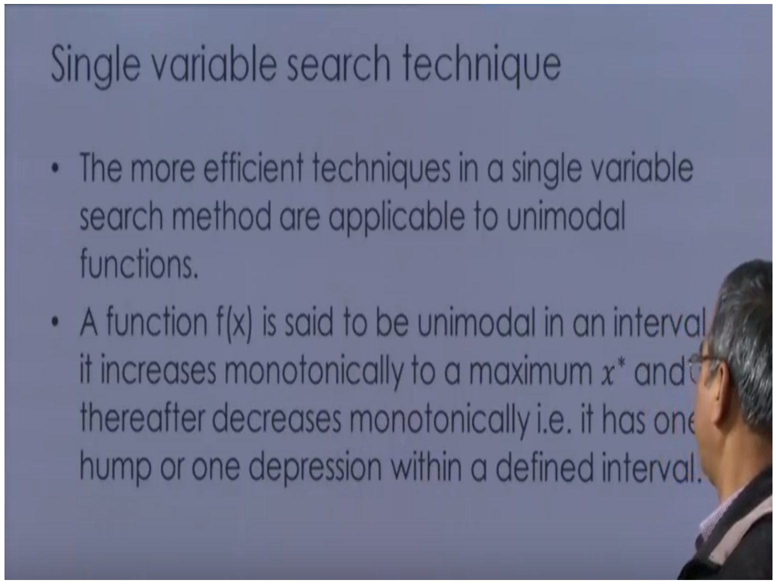

So this is what it is, now one of the very popular method the search method, which is an efficient search method okay because it does not carry out the exhaustive search is the golden section method okay, so this golden section method is based on the golden section rule and this rule actually deals with division of an interval into 2 unequal parts such that the ratio of the smaller to larger interval is equal to ratio of larger to the whole.

So if you look at here say this is an interval and we divide this interval into 2 parts, say R1 and R2 then as per the golden section method, the R2/R1 will be equal to R1/R okay that is the smaller segment length divided by the bigger segment length is going to be equal to the bigger segment length divided by the whole length okay, so this is what is on which the golden section method is based.

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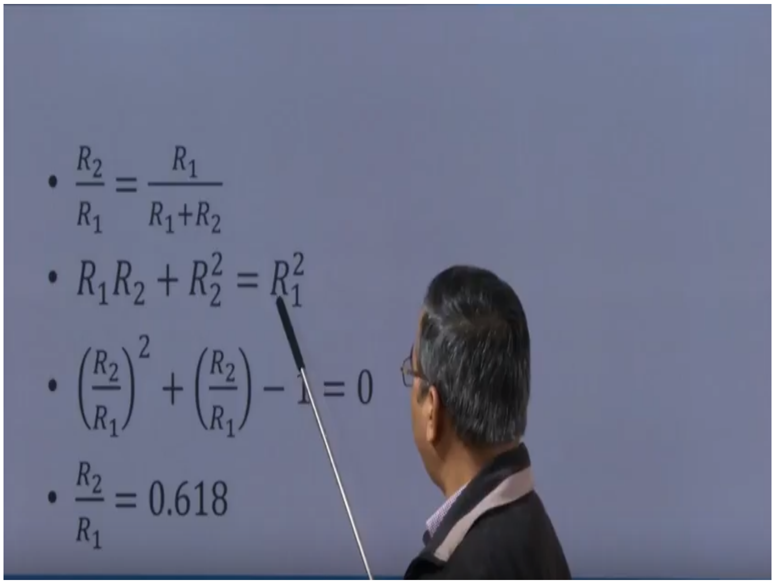

Now what we do is that in this one, we know that this R is equal to R1+R2, so I substitute this R=R1+R2 into this expression, so what I get R2/R1=R1/R1+R2, so this one is there, R2/R1=R1/R1+R2, now what I do, I do the cross multiplication, so I have R1 square here and here I have R2 R1+R2 square.

Now what I do, I divide the whole expression by R1 square, so here I will be getting 1 and then I take that to the left hand side, so I have the -1 and here I get R2/R1 whole square and here I will be getting R2/R1 square, sorry this term is R2/R1 square and here I will be having R2/R1 okay, -1=0, so this is a quadratic equation and if I solve for this quadratic equation.

I get R2/R1=0.618, so this R2/R1=0.618 is basically applied for the search, so in the golden section method, the search is applied, if the search is applied, the interval of uncertainty is reduced by a value of 0.618 times okay, this is what is meant by it, so by repeated application of the rule, the interval of uncertainty after say N evaluation is reduced to the initial value of uncertainty, I0 by one.

So In is I0 x 0.618 to the power and -1, now let us see that how this search is carried out, so suppose this curve shows the unimodal function as I said it is not only one depression, so this is the example of the unimodel function okay, now in this unimodel function, so we have the 2 boundaries, BL is the lower boundary and BU is the upper boundary okay.

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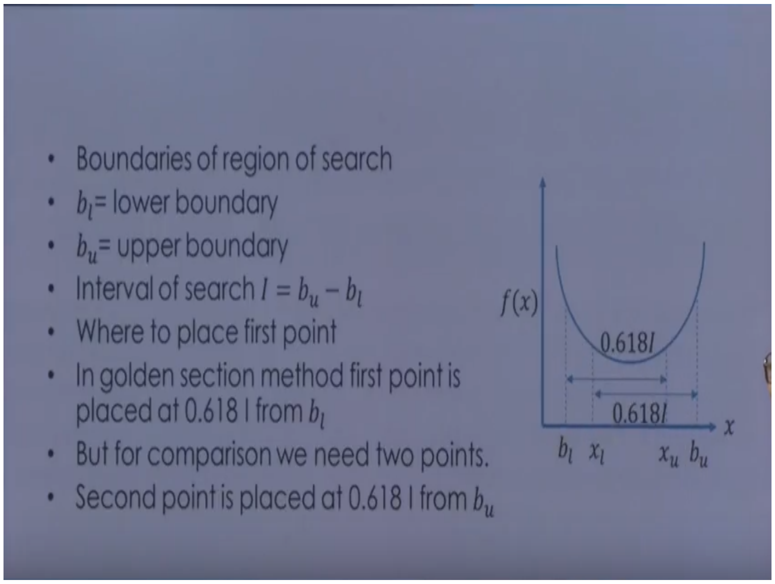

So the internal of search I which we define is going to be bu–bl that is the I will be bu–bl. Now, the question is we have to search in this region I that is bu–bl, but from where to start the search that is the question okay, so in golden section method what is done is that the first point is placed at 0.618I from bl, so from this bl, I take 0.618I and I place my first search here okay, but for comparison purpose, we need 2 points okay.

So where should we take the second point and then we should compare, so for the second point what we do is that the second point is placed again at 0.618I from bu, so from this bu, I take 0.618 I and I find out this point. So, this is basically xl and this is basically xu, so this way I find out these 2 points, I evaluate my function at these 2 points and then compare.

So my xu is 0.618I and of course this is from bl similarly my xl is this one, 0.618xI and this is from bu okay, so evaluate the function at these 2 points and then make a comparison to see if function value at xl is more than function value at xu and of course this is the condition for minimization okay, so we evaluate function value at xl and we evaluate function value at xu, now what happens if fxl is greater than fxu.

so if this value is more than this one and if you are looking for the minimization okay and we have already assumed that the function is the unimodal function, then what we do is that we remove this part of the segment for our search okay, because we have already seen that fxl is here is more than fxu, so we are not interested in finding at the left hand side area of the xl, so in this case, we confined our search to region right of xl.

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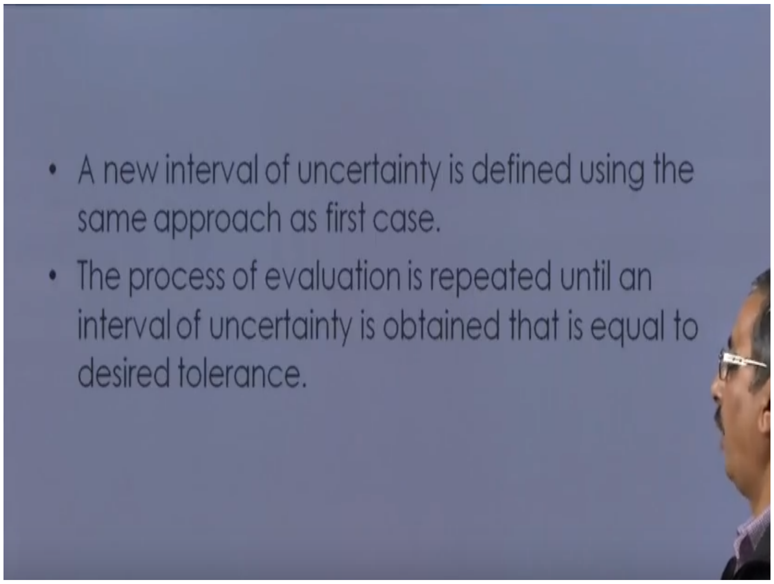

Otherwise confine our searching to the left of xu okay, so we can confine our search in this area and if this is not the case, then we can find our search in this area and then after this, we find out a new interval of uncertainty is defined using the same approach as the first case and the process of evaluation is repeated until an interval of uncertainty is obtained that is equal to the desired tolerance, which we are looking for okay.

So when there is, I will complete this lecture. You can look for these 2 books very good books by professor Arora and professor Rao on Optimization, of course these are books in itself, a full one semester course, which I have tried to cover in 2 lectures, but this will give you an idea okay, so thank you and thank you for enrolling in this course and participating in this course. Wish you all the best.