Modelling and Simulation of Dynamic System
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Lecture – 39
Introduction to Optimization

I welcome you all in this lecture on introduction to optimization and this sub-module is part of the modelling and simulation of dynamic system course, which you are going through. In the last site, where we have last lecture, which was on the system identification, we have seen that while identifying the system dynamics of the system many times, we required in identifying various parameters okay and one of the way of finding out those parameters could be the optimization.

So our next 2 lectures, we will be looking at the optimization method in general and some specific optimization techniques, which could be used in the whole exercise of modelling and simulation of the dynamic systems, so optimization if we talk about a basic definition of it, then it is the mathematical process of obtaining the best result under given circumstances okay, while mathematical process could be anything, but as far as engineers are concerned, we use mathematical tools okay.

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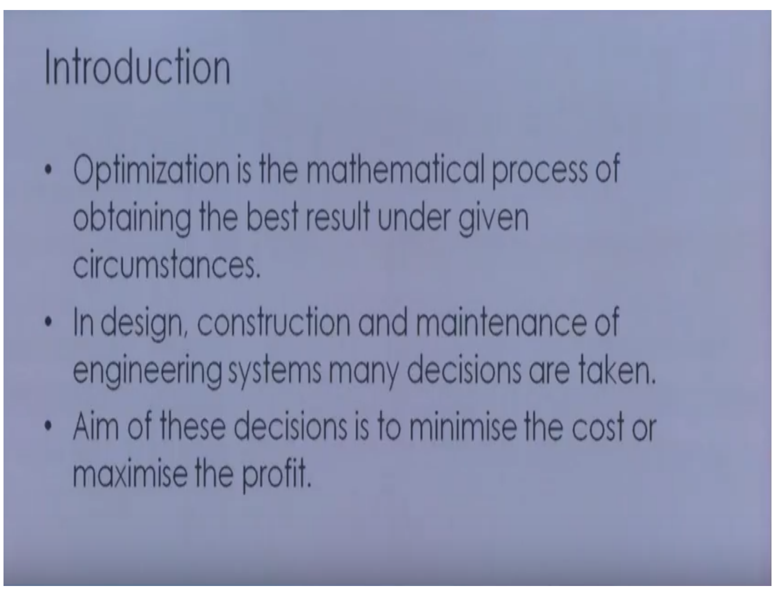

In order to get the best results under given circumstances. Now in many activities, engineering activities such as design, construction, maintenance of engineering system, we take so many decisions okay and aim of these decisions is to say either, we want to minimize the cost in order to make things economical or we want to maximize the profit okay, so that company financial health is good and it is able to run okay, so the aim here is either minimization of the cost or the maximization of the profit.

So what I want to say is that the aim is either minimization of something or the maximization of something. So, I was telling you if you want to ask the definition for optimization, then I can give that optimization is defined as the mathematical process of obtaining a set of conditions that gives the maximum or minimum value of a function okay, so here the things, which you need to note is obtaining the set of conditions.

So the optimization process is basically getting these set of conditions and these set of conditions result in the maximum or minimum value of a function okay and as you know that maximization can always be converted into minimization okay, for example if I have to maximize some function dx then I can always write as minus of minimization of –dx okay.

So this way we can treat any problem as the minimization problem because if some maximization problem is there that problem can be retained as a minimization problem, so the tools, optimization tools are usually the bob for the minimization problem okay, so the optimization problem can be taken to be a minimization problem always.

Now, the optimization has got number of applications in engineering design okay, say if you talk about designing of aircraft and aerospace structure, we know that aircraft or aerospace structure has one of the principal constraint as the minimum weight okay, so if the weight is minimum, they can take the maximum amount of pay load okay.

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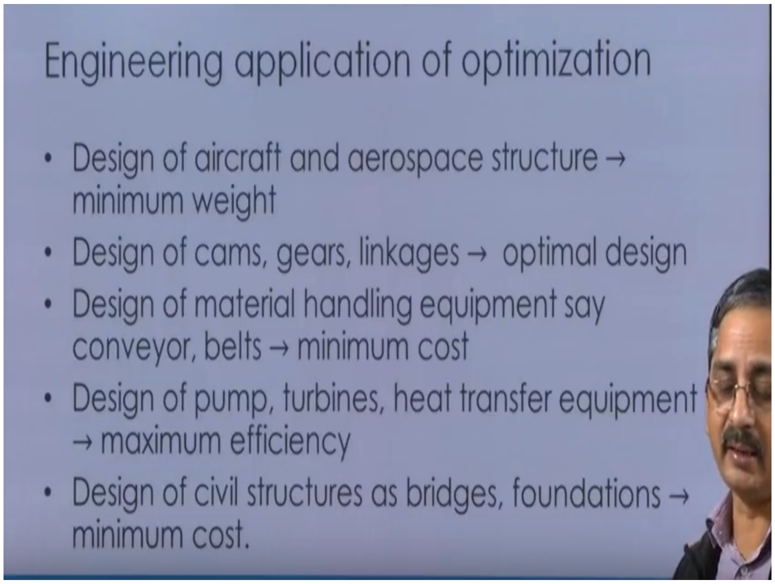

So the minimization of the weight could be one of the optimization criteria for the aircraft and the aerospace structure. Similarly, if I talk about say other mechanical components say if it is a cam, or gear or linkages, the optimization problem could be the design, optimal design okay, so optimal design for these mechanical components okay, now if say I am talking about design of material handling equipment in an industry okay, say conveyors.

If I am talking about or belt I am talking about then the M could be the minimization of the cost fine and if I am talking about say components such as pump okay or turbine or if I am interested in heat transfer equipment such as heat exchanger, then here the M could be the maximum efficiency okay.

If I talk about some structure in the civil engineering domain say if I am talking about bridges, foundation then of course, the M of the optimal design could be minimization of the cost okay, so these are few examples, which I have listed where engineering application of optimization can be there, of course there are many more, which I have not listed here. Now how does the optimization problem looks like alright.

So if we talk about optimization problem, so as I said the optimization problem is to find out a certain set of variables, which either maximize or minimize an objective okay, so mathematically if I want to write it, then the optimization problem could be stated as finding out a vector x okay and this component could be x1, x2 till xn.

so optimization problem is finding out of these values of x, which minimizes effects, so as I said maximization problem can also be converted into minimization problem, so we will be talking about only minimization problem okay, so it is the finding out of these n number of variables okay x1 to xn, which minimizes fx okay.

So this is the optimization problem. Of course, this could be subjected to the that is these design variables could be subjected to certain constraints, say gjx should be less than or equal to 0 for say j=1, 2, 3, 4 till m okay, and so this is one set of constraint, we can have another set of constraints, I will come back to these on when I talk about the type of constraints ljx could be equal to 0, where J is again 1, 2, say up to p, okay.

So these 2 are the constraints amongst these variables okay. Here x is n cross 1, which we call it as the design vector because we are interested in finding out these vectors okay and fx, this fx we call it as the objective function okay and gjx here as it involves inequality sign here these are called inequality constraint and this ljx=0 as here it involves an equal sign, these type of constraints are called equality constraints okay.

And n here these n are basically the number of variables okay and as you can see from here, m and p are basically the number of constraints okay and these type of problems where we have that is the design variable is subjected to certain constraints, this type of problem is called the constraint optimization problem okay, so the constraint optimization problem have the constraints in them okay.

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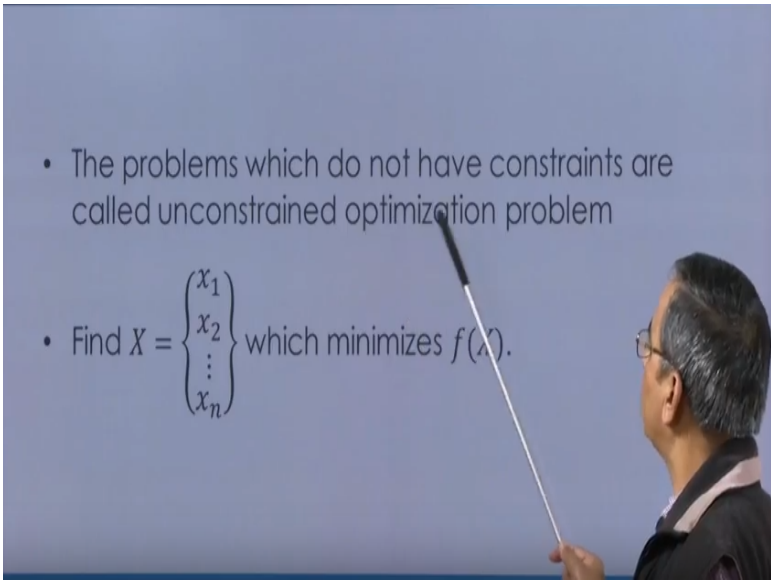

Now there could be the problem, which do not have the constants okay and these are called unconstrained optimization problem okay and it could look like this that is find x say x1 to xn, these are our design variables which minimizes the objective function fx okay and here there are no constraints, so we call it as the unconstrained optimization problem.

Now let us define and explore whatever variables or whatever parameters has been associated with the optimization problem, so first and the principal one is the design vector okay, as the m of the optimization problem is to find out the value of the optimal value of the design vector, which minimizes your objective function.

So an engineering optimization model consists of parameters whose numerical values are to be determined in order to achieve the optimum design okay and these parameters are what we call as the design variables okay, so these parameters could be say size, weight of the part, number of teeth in a gear, number of coils in the spring, and so on and when I am writing this design vector collectively together in a vector form, this is what is called as the design vector.

When I am writing the design variable in the collective form, it is called the design vector. For example, if say I talk about say the 2 gear teeth okay, 2 gear teeth in mesh that is say we have a situation like this, that is we have a gear teeth and we have say another gear teeth okay and when they are in mesh okay, so here suppose these are the number of teeth t1 and t2 and say b is the width of the teeth okay.

So if b is the width of the teeth, then this is what we call it as the design variables, so bt1t2 these all the design variables of the gears, now design constraints okay, so these are the mathematical functions, which define the interaction between the design variables okay, so these design constraints are written in terms of design variables okay as I defined it, they basically represents the interaction between the different design variables.

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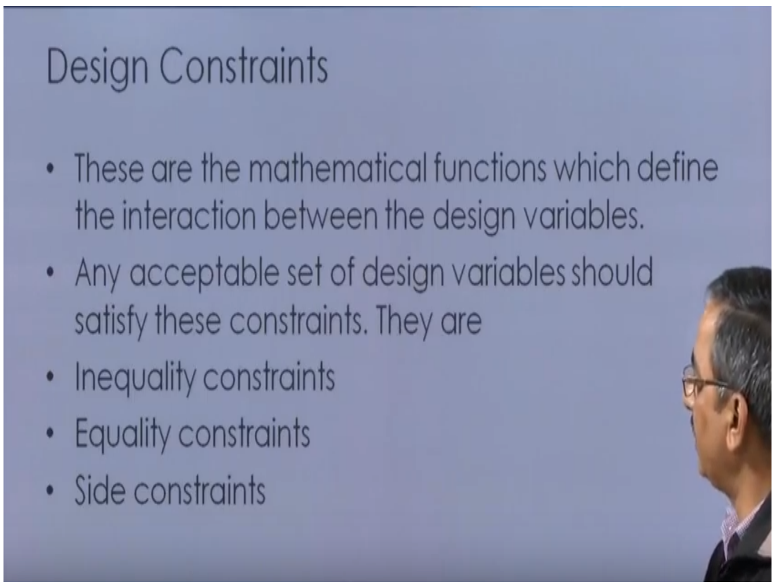

Any acceptable set of design variables should satisfy these constraints okay and these constraints could be say inequality constraints or the side constraints okay, so we will have a look at what type of constraints these are, so the inequality constraints actually they ensure safety against a failure mode okay or satisfactory behavior under the given loading condition okay, so these type of constraints in engineering design are called inequality constraints.

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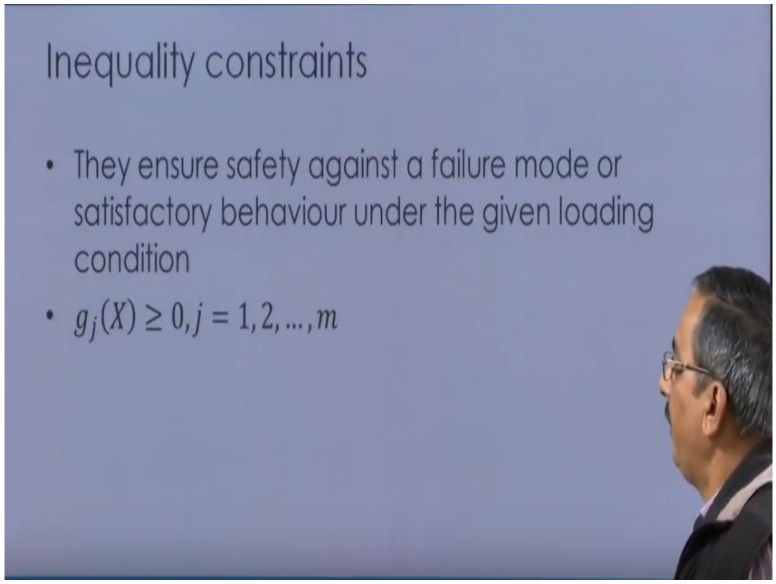

They are written usually say gjX > or =0 for j=1 to m, okay so in general, they are written like this or they could also be written as less than or equal to okay, so this type of representation can be given for the inequality constraints, then we have the equality constraints and these type of constraints must be satisfied for design to be acceptable okay.

These set of constraints are of this form say ljX=0, where j varies from 1 to, or j can have values 1 to 2, say till p. Now, example for this equality constraints could be given from the structure mechanics problem okay, where basically the constraints can be represented like this, here we know that the K is the stiffness matrix, v is the displacement vector, and p is the root.

So, these type of constraints, which involve the equal sign are called equality constraint, then we have the third type of constraint that is called the side constraint okay, here upper and lower bounds on the design variables, these are referred as side constraints okay. These constraints are also known as the geometry constraint and example could be the upper and lower bound on the number of teeth in a gear mesh okay.

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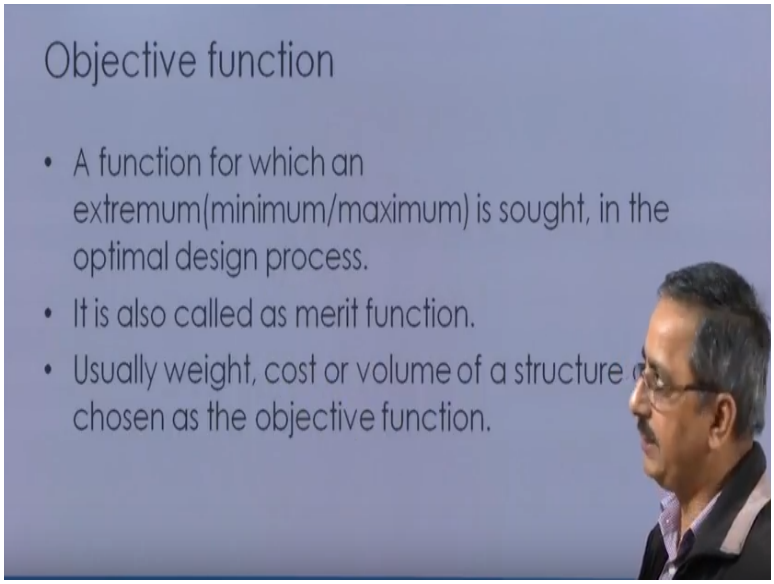

So these are what we call as the side constraints, then we have after seeing the design variable and the constraints, next is the objective function okay. Now, a function for which and extremum and extremum as I said it could be minimum or maximum, in fact we can convert the maximum into a minimum problem as well is sought okay, so a function for which an extremum is sought in the optimal design process that function is called the objective function okay.

There is another great name which is used in literature and that is called the merit function, then usually say weight or cost or volume of structure are chosen as the objective function. We could have other objective function as well then let us have some idea about design space. This design space is basically defined as the total reason or the domain defined all design variable in the objective function okay.

so the total reason or domain defined by all the design variables, which are there in the objective function is called the design space okay and this design space is limited to constraints okay, otherwise we may have unbounded design space. If there are no constraints, we will have the unbounded design space and for which, of course no feasible solution may exist okay.

As you may be knowing as the constraints increases, our life becomes easier, easier in the sense that the number of solutions keep on reducing okay, so that way it is good for the decision process, so if there is no constraint, we have unbounded design space, then there will be no feasible solution may exist. Constraints are useful as I said in restricting the reason in which the search for minimum or maximum design variable to be done and that make way, it helps us in finding out the optimal solution, then the local and global maximum.

To explain you this concept, the local and global maxima actually suppose we have a certain function, fx and say if you plot X here, then suppose the value of fx is something like this, say this is point a, b, c, d okay, so we can see that this a, fx value at a is highest near to the proximity of this one, near the proximity of a next value is highest and so this we call it as the local maxima okay.

Similarly here the v value is highest, so this is again called the local maxima and c value is highest nearby, so this is called local maxima and this is called the local maxima, but among these local maxima, d has the highest value, so we will call this d as the global maxima okay, so the local and global maxima.

The point in design space that is higher than all other points within its immediate vicinity and this is what is called the local maxima okay and the highest of all local maxima is called the global maxima. Now let us try to see how can we classify the optimization problem, so till now we have seen how to define an optimization problem and what are the factors associated with the optimization problem such as objective function, design variables and constraints okay.

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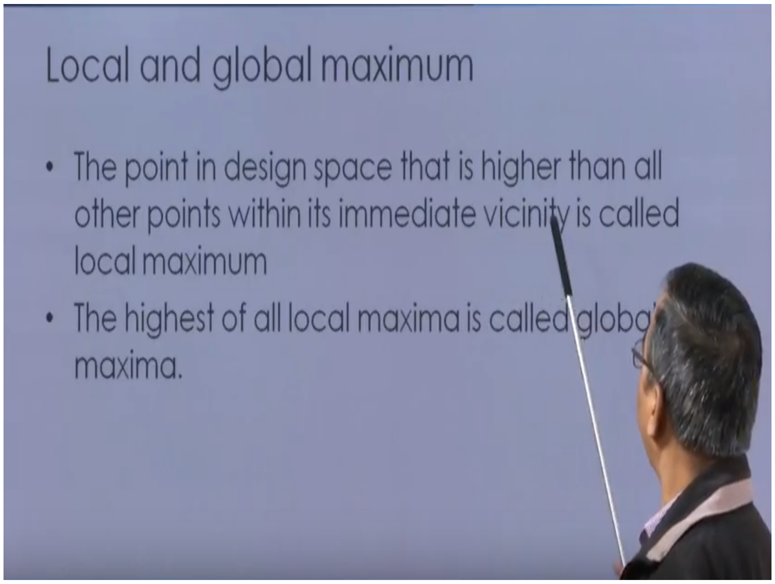

Again constraints are 3 types, that is the side constraints, inequality constraints, and equality constraints okay, so after finding the optimization problem, now let us see the classification of the optimization problem. There are various ways of classifying the optimization problem okay and these classification or not mean to be mutually exclusive, but to describe the mathematical characteristic possessed by the problems.

First classification is based on the type of constraints okay, so the optimization problem is said to be unconstrained problem, optimization problem if there are no constraint or it could be a constraint optimization problem if one or more conditions are present or one or more constraints are present, then we call it as the constraint optimization problems and if there are no constraint, then we call it as the unconstrained optimization problem.

Then number of independent design variables okay, based on this, we can classify an optimization problem, so we can have a single variable optimization problem and the multivariable optimization problem. In case of single variable optimization, the objective function is function of one variable and of course constraint okay.

And in case of multivariable optimization problem, objective function is going to be function of 2 or more number of variables then other classification could be time dependent or not okay, so if in case of static optimization problem, dependence on time is not there okay, and in case of dynamic optimization problem, dependent on time exist.

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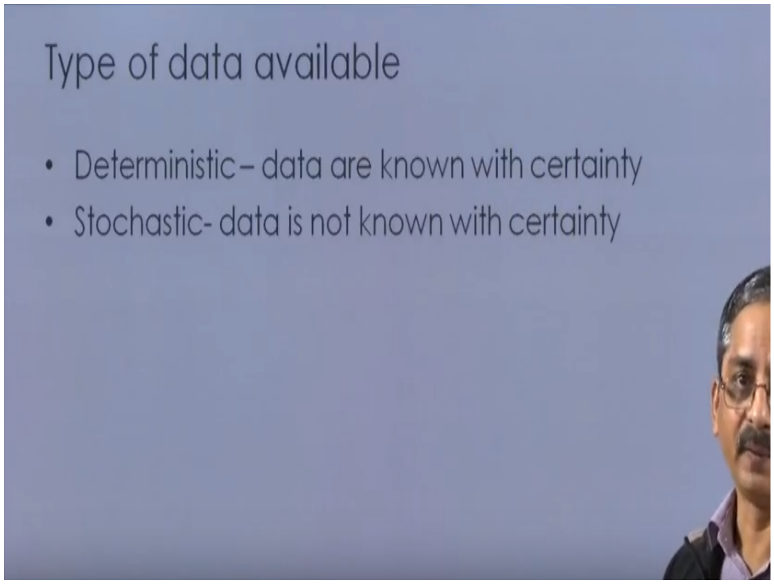

Then what type of data are available okay, so it could be a deterministic optimization problem, where the data are known with certainity okay and it could be stochastic optimization problem, where data is not known with certainity, then we could have classification as number of objective functions okay.

So we could have a single objective programming problem okay, that is we have single objective function and we could also have the multiple objective programming problem, where we can have the multiple objective functions to be there, then classification could be the nature of equations involved okay and based on this classification, it could be a linear programming problem or it could be a non-linear programming problem okay.

So the linear programming problem, there is a unique objective function and whenever a decision variable appears in either in the objective function or in one of the constant function, it must appear only as power term with exponent of one that is it should be linear and possibly multiplied by a constant. No term in the objective function or in any of the constraint can contain product of the decision variables okay.

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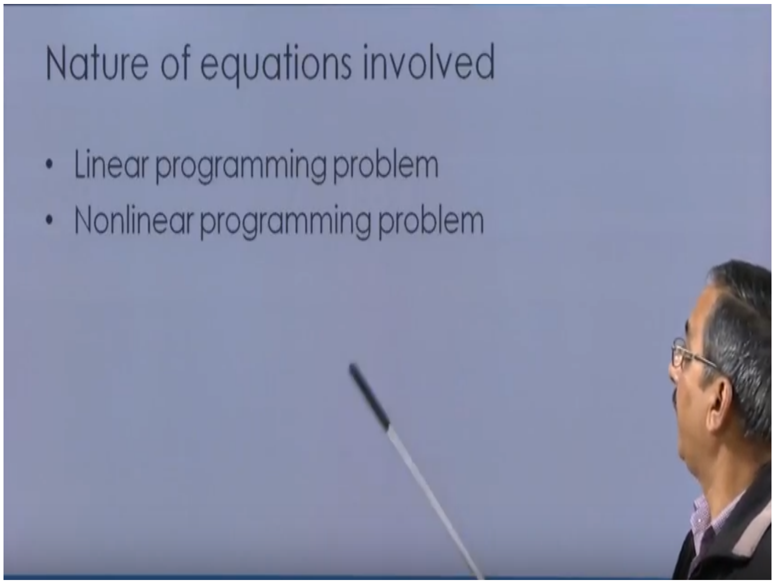

So, product should also not be there, then the coefficients of the decision variables in the objective function and each constraints and the decision variables are permitted to assume fractional, as well as integer values, so this is what is there in case of the linear programming. Then, we may also have the nonlinear programming in this case, if any function among objective function or the constraints okay, if they are non-linear then this type of problem.

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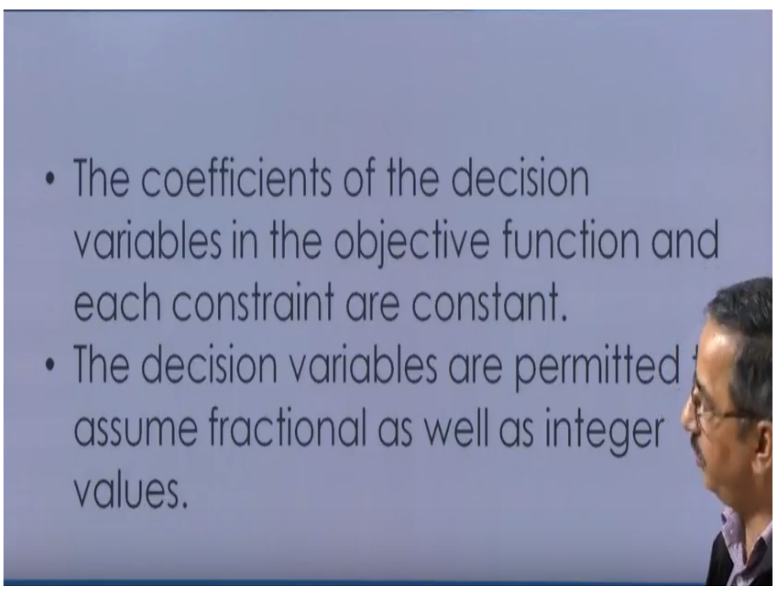

We call it as the non-linear programming problem okay and remember for every type, there are different tools available, which could be used for the optimization problem, finding out the optimal value of the decision variables and design variables, which could either minimize or maximize your objective function.

So this is all, which I wanted to tell you in this lecture. You can have further reading on a very good optimization book that is Engineering Optimization by S.S. Rao okay and next lecture we will see some examples of the optimization problem, thank you.