

**Modelling and Simulation of Dynamic System**  
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**Lecture – 38**  
**System Identifications**

I welcome you all are in this lecture on System identification which is a sub model for the course on modeling and simulation of dynamic systems. So doing all these modeling activities or we may have instances where we need to identify the system that is we have sufficient ample amount of input and output data and the task made may be to identify the system okay.

That is we do not have the model of the system and we are interested in deriving the model of the system based on the input and output data and this is what we will be seen in the system identification lecture so we saw this figure at the beginning of this course and from this figure I would like to explain you what do you mean by the system identification. So here suppose we have a system okay.

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Introduction

Input Variables  $U$

Dynamic System,  $S$   
State Variables,  $X$

Output Variables  $Y$

- Identification.
- Given time histories of  $U$  and  $Y$  through experimentation otherwise find a model  $S$  and state variable  $X$  consistent with  $U$  and  $Y$ .
- A good model is consistent with a range of values of  $U$  and  $Y$ .

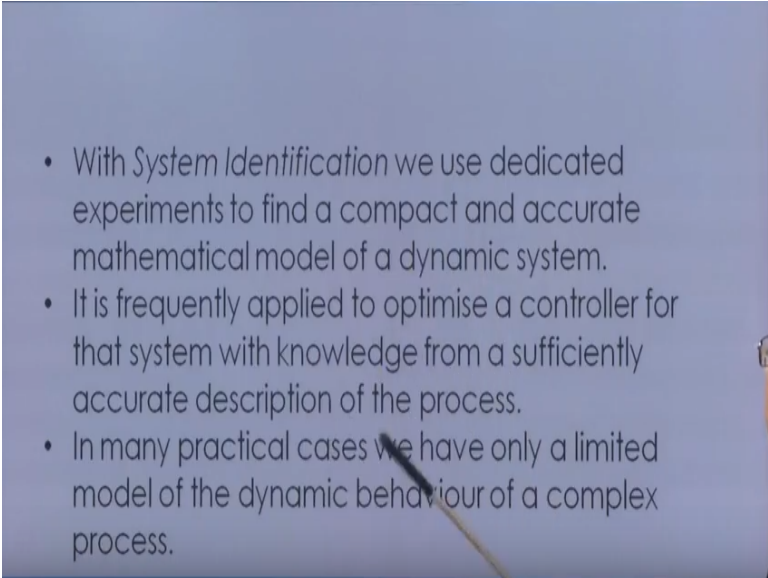
The dynamic system are represented by the expression  $S$  and these expressions are in terms of state variable  $X$  okay and this dynamic system is given certain inputs are there are certain input variables, which are varied and we corresponding we get the output and that output is represented by the output variable say  $Y$  okay. Now what do you mean by identification system identification okay.

So the system identification actually means that if we are given that time histories of  $U$  and  $Y$  through experimentation principally it is through experimentation okay. That is from experimentation we know that time history of  $U$  and the corresponding time history of  $Y$  that is we know the inputs given inputs and correspondingly what output we are getting from system that is also known.

So through experimentation or otherwise and as I said it is principally the experimentation okay. So the identification means that find a model that is find the model of the dynamic system as and the state variable  $X$  consistent with  $U$  and  $Y$ . So the identification means that we are interested in finding out this dynamic system as  $S$  I am the state variable  $X$  for the given input and the output variable or for the given input time.

Histories and the corresponding output time histories and the model which we want to identify sorry the model which we want to identify that is considered good or consists that is considered good if it is consistent with a range of values of  $U$  and  $Y$ . That is if your model is acceptable for a range of values of you and why then we say that this model is good.

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- With *System Identification* we use dedicated experiments to find a compact and accurate mathematical model of a dynamic system.
  - It is frequently applied to optimise a controller for that system with knowledge from a sufficiently accurate description of the process.
  - In many practical cases we have only a limited model of the dynamic behaviour of a complex process.

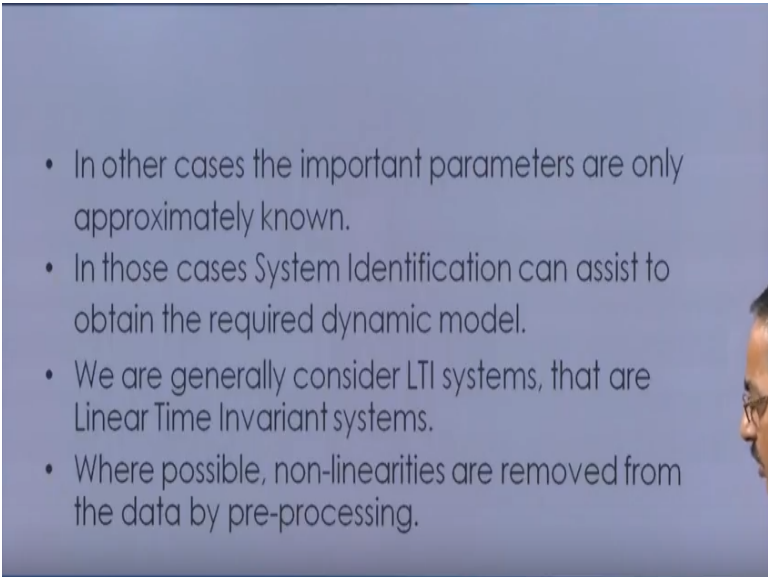
So with system identification we use dedicated experiments as I said that it is the principally it is the experiments okay. We serve us in identifying the system. So we use that dedicated

experiments to find a compact and accurate mathematical model of the dynamic system, it is frequently applied to optimize a controller that system will with knowledge from sufficiently accurate description of the process okay.

So it is frequently optimized applied to optimize a controller in our coming lecture. We will be looking at how this optimization could be done and what are the various methods of optimization okay. In many practical cases we have only a limited model of the dynamic behavior of a complex process.

Because of this limited model available or limited knowledge about the model available system identification becomes one of the important task so in other cases the important parameters are only approximately known so this may also be a situation or a need for the system identification okay.

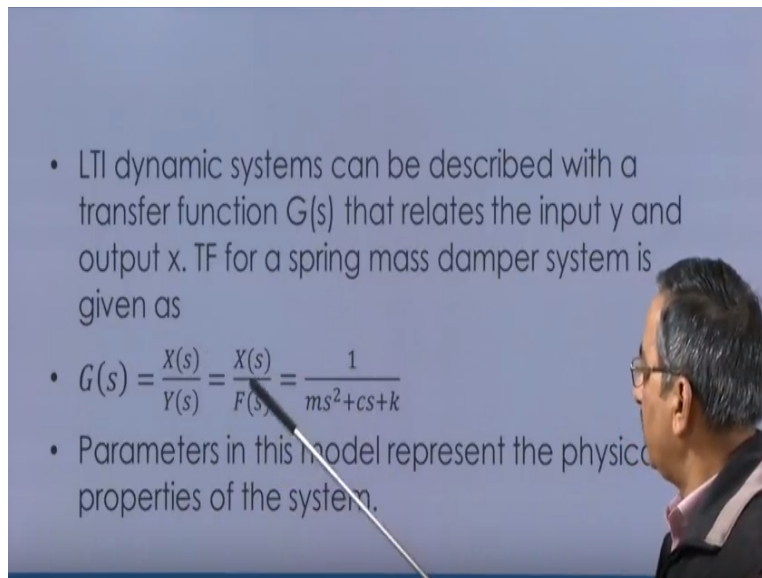
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- In other cases the important parameters are only approximately known.
  - In those cases System Identification can assist to obtain the required dynamic model.
  - We are generally consider LTI systems, that are Linear Time Invariant systems.
  - Where possible, non-linearities are removed from the data by pre-processing.

So in these cases the System identification can assist to obtain the required dynamic model. So we are generally concerned about linear time-invariant systems okay, that is LTI systems okay and wherever path nonlinearities are removed from the data by pre-processing. So the LTI dynamic system can be described as we have seen a lot with the help of a transfer function okay.

GSA that relates the input Y and the output X we have seen the discipline mass damper system okay. Where the output has been the displacement of the spring and the input has been the Y S okay. So if you remember we have seen this system the spring mass damper system okay, so this is K and this is B this is M and here is the output say X and say the input is F okay.

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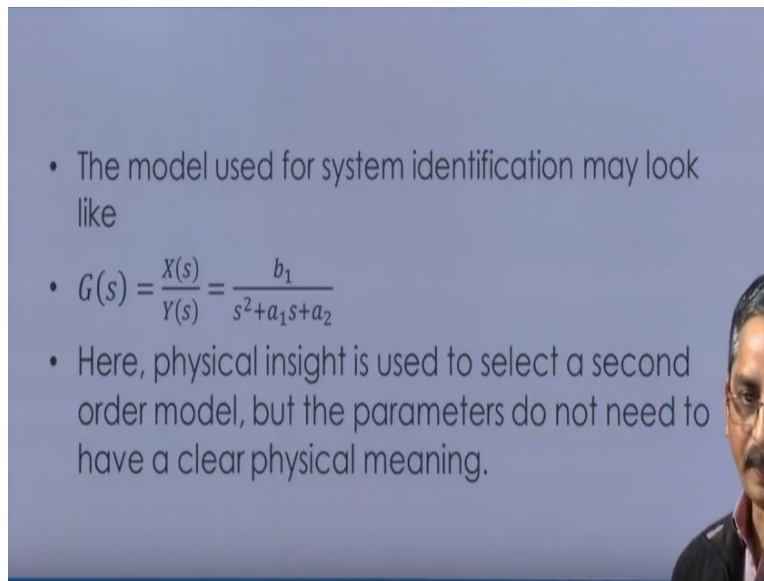
- LTI dynamic systems can be described with a transfer function  $G(s)$  that relates the input  $y$  and output  $x$ . TF for a spring mass damper system is given as
- $$G(s) = \frac{X(s)}{Y(s)} = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$
- Parameters in this model represent the physical properties of the system.

So for this type of system or the transfer function  $G_s$  is given by  $X_s$  upon  $Y_s$  or this is  $X_s$  is the output and  $F_S$  is the input here. So this is  $1/ms^2 + cs + k$ , where this damping is represented by say  $c$  okay, and we know that the parameter what are the parameters in this model the parameters in this model are mass damping and stiffness and these parameters represent the physical properties of the system okay.

Now the model which we use for system identification may look like something like this one okay. That is the model  $G_s$  will be say  $X_s$  upon  $Y_s = \text{say something } b_1 \text{ upon } s^2 + a_1s + a_2$ . Now we are here  $b_1$ ,  $a_1$  and  $a_2$  are some constant. Now here for identification by having some physical insight of the system we come to know that okay.

This system is represented by a this type of transfer function okay but again the problem here is that there is no physical meaning associated with  $b_1$ ,  $a_1$  and  $a_2$  we are not sure actually what does this represent so the parameter do not need to have a clear physical meaning in this type of system okay.

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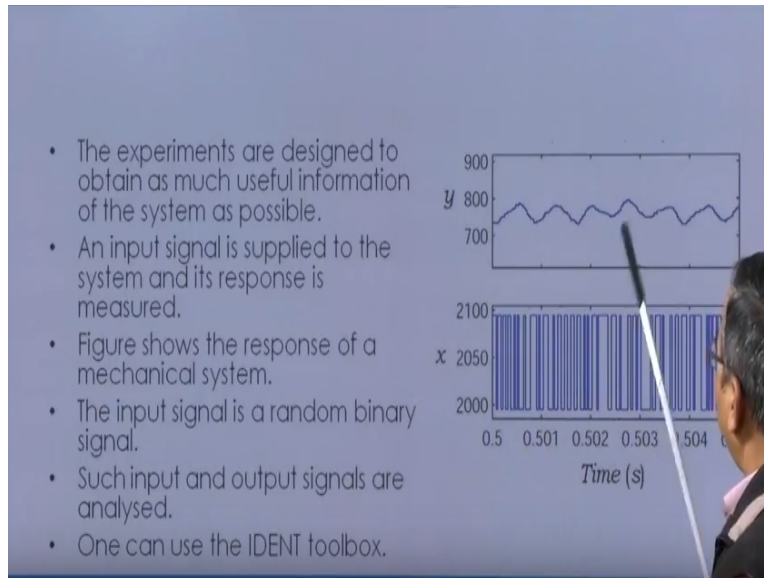


- The model used for system identification may look like
- $$G(s) = \frac{X(s)}{Y(s)} = \frac{b_1}{s^2 + a_1s + a_2}$$
- Here, physical insight is used to select a second order model, but the parameters do not need to have a clear physical meaning.

That is in the system identification so this is one of the problem in-fact with the system identification, so here we can guess about the system behavior that it is given by a second order transfer function okay. That is okay, but then we have to find out ways of finding out these parameters.

So the experiments are designed to obtain as much useful information of the system as possible okay. So let us take a certain system say but this is the input to the system and this is the output response for a general system. Which I am talking about with respect to time, so the input signal is supplied to system and its response is measured.

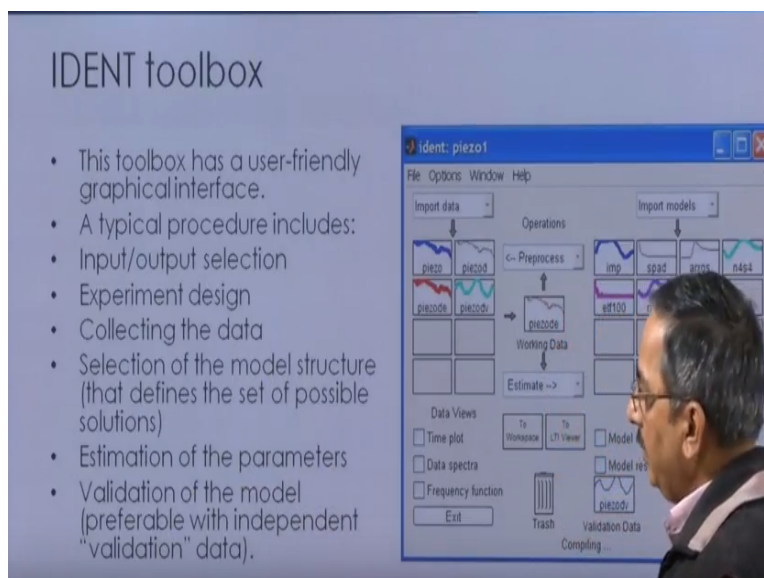
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So this is the response corresponding to this input system and suppose this is the response for a mechanical system okay, as I telling you that our role here in System identification is to identify the system transfer function or the system with the help of these 2 type of signals, that is you know the input you know the output and your role is to identify the system okay.

So the here as I said the figure shows the response of a mechanical system the input signal is random binary signal here so and we have certain output signal okay, and one can use the IDENT toolbox from the Matlab to identify the system. This is how the IDENT toolbox looks like this toolbox has got a user-friendly graphical interface.

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The typical procedure which includes is that of the input output selection first you have to do that then experiment design has to be done then our collection of data has to be done, and after this we have to do the selection of the model structure okay and that in fact defines the set of possible solution.

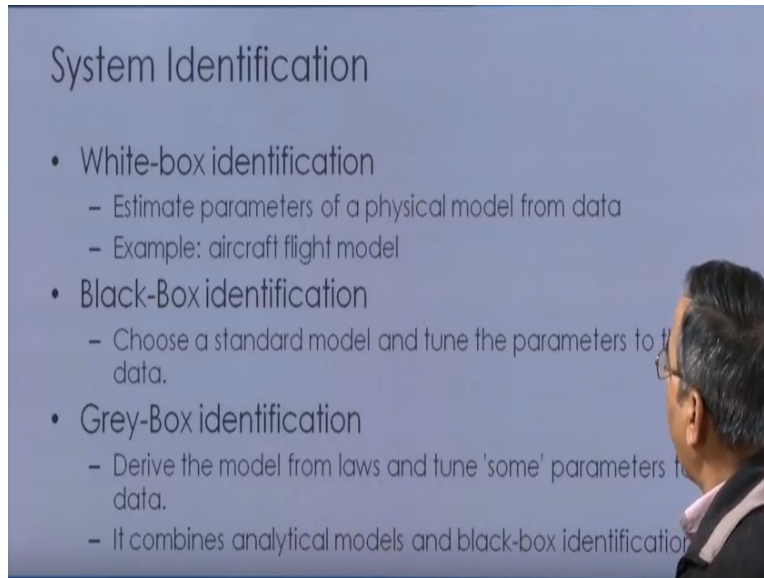
Then we have to go for estimation of the parameters and then finally the validation of the model okay and this is preferable with independent validation data this can be done okay. So this is how this user interface GUI looks like where we have the various types of models which can be imported the operations can be performed.

The data import data and can be here import models can be here okay and then we have the working data can be are here from this working data can be there and the data can we can view the time plot data spectra and frequency function also can be viewed and likewise here we can model VS.

We can see that transient response frequency response poles and zeros Liza spectrum and so on. So using this identical box we can carry out the identification process. So here we deal here in dynamic system so the time dependency of the signal plays a very important role and it also means that the model describing the system will include the differential equation in this the time derivatives or the discrete time equivalent differences are going to be present.

Now the system identification can be classified at 3 types one is the white box identification, then we have the black box identification, and the gray box identification. The white box identification actually here are we estimate parameters of the physical model from the given data there are some example say aircraft flight model.

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We can do that, then we have the black box identification where what we do is that we choose a standard model and then we tune the parameters to the data okay. So that the input and output response can be matched and then we have the grey box identification here what we do is here we derive the model from the laws and tune some parameter to the data okay.

So here basically, since of the derivation from the model is concerned this method, basically combines the analytical model as black as well as the black box identification. So this is what we call it as the gray box identification method. Now the system identification procedure actually it consists of as you said first of all we have to collect the data okay.

So that is choose the input signal such that the data has the maximum information, then we have to choose the model structure use application of knowledge and the engineering inclusion to find out the structure of the model and then we choose the identification approach how would a good model look like and choose best model in model structure that is for that we need to do the optimization or estimation and then finally we validate the model okay.

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## The System Identification Procedure

- Collect Data. If possible choose the input signal such that the data has maximally informative
- Choose Model Structure. Use application knowledge and engineering intuition.
- Choose Identification Approach. How would a good model look like?
- Choose best model in model structure (Optimization or estimation)
- Model Validation. Is the model good enough for our purpose?

So, good enough for the purpose as well as I said that we need to have certain data for the validation purpose so through this process, we can identify a system now there are certain questions associated when we are doing the system identification by these questions could be how to design the experiment okay. How many data samples to collect this is always question arise in the minds of the experimentalist how many data sample to collect okay.

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## Some questions

- How to design the experiment. How many data samples to collect?
- How to choose the model structure?
- How to deal with noise?
- How to measure the quality of a model?
- What is the purpose of the model?
- How do we handle nonlinear and time-varying effects?

How to choose the model structure what type of structure, whether it is a first order systems, again the order system. What type of model the structure it is going to have then how to deal with noise these signals output signals usually contain noise okay. So the question is how to deal with this type of noises okay.

How to measure the quality of a model whatever model you have got how to measure its quality and what is the purpose of the model? How do we handle nonlinear and time in variant effect? okay. In doing the model identification, so these are some of the questions which always come to the mind of the modular now coming to the system identification methods truly speaking.

There are 2 types of methods one is the nonparametric method and the other one is the parametric method. So system can be identified using any of these two types of method. The nonparametric method okay, here in the nonparametric method the results are in the form of graphs tables etc.

These methods are simple to apply they give basic information about system such as you can get the time delay. You can get the time constant of the system okay, and this way you can identify your system then we have the parametric methods are here the results are values of the parameters in the model okay.

These may provide better accuracy or you can say that the more information but are often computationally more intensive. So I am going to discuss a little detail about the nonparametric method. The nonparametric method can be explained by taking a simple example, so I let there be a system with transfer function  $G(s)$  with known input  $u$  okay.

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## Some questions

- How to design the experiment. How many data samples to collect?
- How to choose the model structure?
- How to deal with noise?
- How to measure the quality of a model?
- What is the purpose of the model?
- How do we handle nonlinear and time-varying effects?

And suppose, an unknown disturbance  $v$  is added to the output of  $g_0$  to obtain a miserable output signal  $y$  okay. The description of the system  $g_0$  somehow has to account for the way you are input you affects the miserable output  $y$  and this is what we always intend to do that, so the remaining part of  $y$  which equals to the disturbance has no relation to this in with this input  $u$ .

So this is what I was telling to you that say we have the system with say transfer function  $g_0z$  and here I am going to supply a certain  $u$  okay and suppose that is a certain disturbance as I was telling you in the output signal. We can have some noise okay. So say that disturbance or noise is represented by  $v$ .

So the output which I am getting that  $Y$  will have the contribution from  $V$  as well as contribution from  $U$  okay. So I can write this  $y_t$  as  $g_0z u + v_t$  here of course I am doing for, when are taking one assumption that my system is linear and it can be written as a discrete time function say  $G_0 Z$  alright.

So this is how I can write the output expression for it. Now I can do the system identification using the correlation analysis okay. So the goal of the identification as I said to determine the linear systems  $G_0z$ . So if you are able to determine the linear system  $G_0z$ , that it means that we have able to identify the system or we have been able to generate the model of the system.

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## Correlation analysis

- The goal of the identification is to determine the linear system  $G_0(z)$ .
- One way to describe the system is by using the Impulse Response  $g_0(k)$ ,  $k = 0, 1, 2, \dots$  as the output  $y$  can be written as
- $y(t) = \sum_{k=0}^{\infty} g_0(k)u(t-k) + v(t)$  ( $t = 0, 1, 2, \dots$ )
- By comparing with  $y(t) = G_0(z)u(t) + v(t)$
- $G_0(z) = \sum_{k=0}^{\infty} g_0(k)z^{-k}$

So one word to describe the system is by using the impulse response okay, so but there are ways impulse response step response I will be just discussing only the impulse response okay, so we impulse response say that is given by  $G_0k$ , where  $K$  is a  $0, 1, 2$  okay then output I can write  $y_t = G_0k$ . This is the impulse response and  $u_{t-k}$  okay, so that is represented, where  $k$  is from  $0$  to infinity + my disturbance  $v_t$  okay.

We can have  $t=0, 1, 2$  and so on so this is what I am saying that this is my impulse response okay. That is the response of the system when it is subjected to an impulse input okay. So this is my impulse response of the system. Now what I do is that I compare this impulse response with the output expression okay.

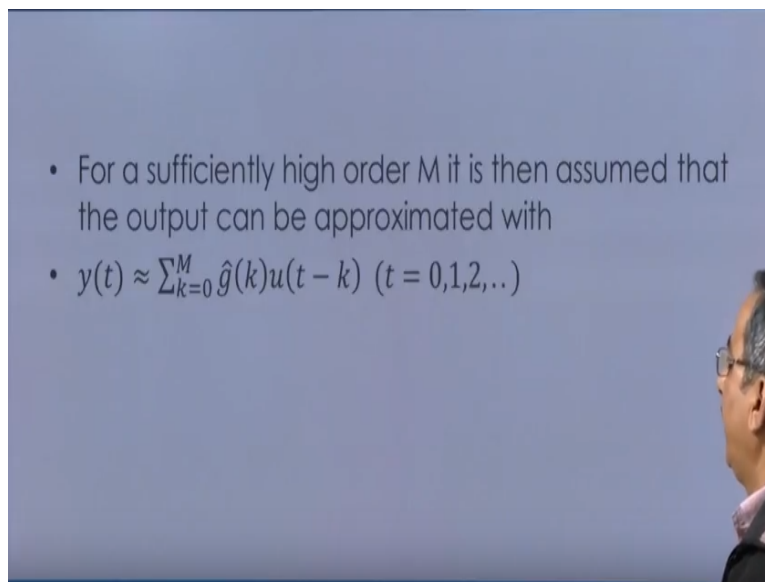
So this is my output expression that is  $y_t = G_0z u_t + v_t$ . So by if I compare these 2 terms then I can identify this  $G_0z$  as  $k_0$  to infinity  $G_0 k \times z$  to the power  $-k$ . So this way I am able to find out that transfer function  $G_0z$  which has been my objective of system identification using the correlation analysis okay.

As I said that we had the INDENT software, where of the matlab where this process can be carried out. Now the drawback of this expression is that impulse response of infinite length is used that is here I am going from  $k=0$  to infinity, so that is the drawback so for an identification

then put you and output  $y$  are recorded and these systems  $G(z)$  will be modeled with a finite impulse response okay.

So our fir that is finite impulse response  $g(k)$ ,  $k=0$  to  $m$ . We can go rather than going up to infinity okay. Here the Hat notation indicate that this response is an estimate for the system computed from the measured data so for a sufficient high order  $M$ . It is assumed that the output can be computed approximately using this expression that is  $y(t) \approx \sum_{k=0}^M g(k)u(t-k)$  okay.

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Then we can compare this expression with that of the output expression okay, and we can find out the  $g_0$  okay. So this is how we can identify the system alright. So with this, I conclude this lecture you can take several examples such as identification of PISA because if you want to identify that is determine the model for the PISA that can be carried out using the set of the data and using the Matlab toolbox in that okay. Thank you.