Modelling and Simulation of Dynamic System
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Lecture – 37
Parameter Estimation Examples

I welcome you all in this lecture on parameter estimation examples, which is a sub model for the course modeling and simulation of dynamic systems. Last lecture, we have seen that when we derived the mathematical model, we may have some parameters in that model whose values we do not know okay.

So the aim of the parameter estimation model is to find out those values because if you want to simulate those mathematical expressions you need to know those values okay. So last lecture we have seen the ways of finding out of these parameters our estimation of parameters and we have seen the least square method. Basically these parameters are estimated with the help of the input and experimental data, which we get by giving a certain input and observing the output okay.

These input and output values are fit to our model okay, and then from that model we try to estimate the parameters. So this we have seen in the last class. Now in this class we will be taking up two examples to illustrate you how these parameters are estimated, so the first example is are the least squares estimate of two parameters.

So, suppose I am given a modern a simple model first example we are taking a very simple example say z=cx+d. Suppose this model is given where the parameter c and d will be estimated we are interested in finding out estimating the parameter of the c and d okay, and for given 3 sets of corresponding values of z and x okay. so suppose I am given these values say z=0.8, say cx1+d.

So, this is my one expression, which is given to me the other one is say 3.0=cx2+d and another expression is say 4.0=c in known to me and these expressions I do not know as I said the values of c's and d's. So these values of C and B are to be estimated. So here what we do is first right these 3 equation in the standard form which we have defined in the previous lecture okay.

So, here first I write the y vector as a 0.8, 3.0, 4.0, so this is my y vector and then we had a phi of matrix and that Phi matrix and then we had the last or column vector as the unknown vector okay. So in this case our c and d are unknown, so I am writing this c and d as a unknown vector here and the faster expression these elements of the matrix I can find out by looking at these expressions.

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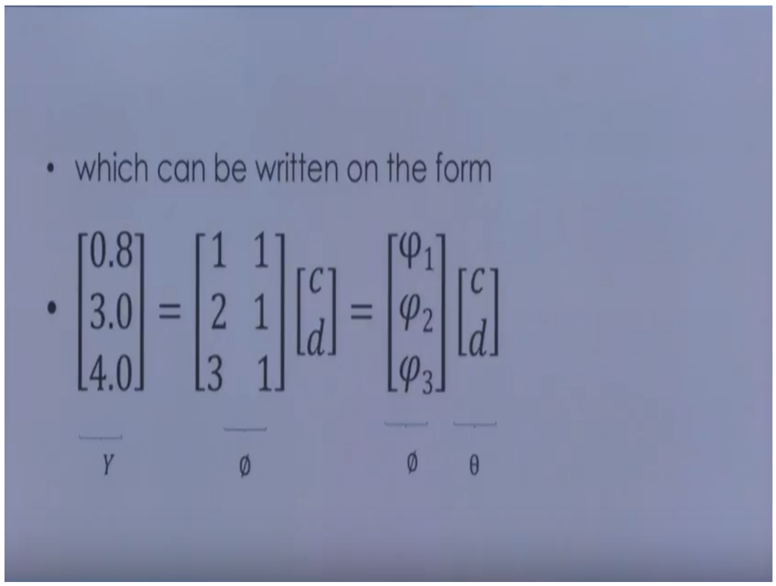

$$\begin{bmatrix}0.8\\3.0\\4.0\end{bmatrix} = \begin{bmatrix}1 & 1\\2 & 1\\3 & 1\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix} = \begin{bmatrix}\varphi_1\\\varphi_2\\\varphi_3\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}$$

So, the first one will be 1,1 okay and the second one will be 2,1 and the third one will be 3,1. So this way we can find out so this value is actually my Y value and this value is actually my Phi value okay and this value is basically my theta value as I am talking to you with respect to the least square method which we have seen in our previous lecture okay.

Now as you remember our last lecture we represented these rows in a short form and, so I am just writing them as say phi 1, phi 2, phi 3. So these are basically the three rows and then we had the unknown vector column vector here that is the cd. So, this is basically my theta and this is my the same phi okay.

So, this way I have represented it. Now the least square value the relation for the least square value which are we have seen so the theta least square this value was our phi transpose phi inverse okay x phi transpose and y okay. So this is what we had, so I can write this is our Phi value.

So, I can write the phi transpose here again so what will be Phi transpose for this it will be 1 1 or it will be 2 1 and then this will be 3 1. So this is my phi transpose which are written here okay. So this is my phi transpose then we have the Phi. So Phi I can reproduce this as it is, so this is my 1 1, 2 1 and 3 1 and we need to take, we need to multiply this and then we need to take inverse of it okay.

So, this is what are we have to do and then again we have the Phi transpose so it will be basically the same thing or which will be coming here okay so i can just write it as 1, 2, 3 and 1, 1, 1, so this is my this parameter here and then Y Y, we have this vector I have already written. So this vector I can write as 0.8, 3.0, and 4.0 okay, so this way I complete this one alright.

Now what we can do is that we can multiply these 2 matrices okay, and whatever the result comes I take the inverse of it okay and then I multiply these two matrices again and whatever is there that is multiplied with this column vector okay so, this are when we do or that so this is what we get okay.

So the value which we are expected to get that is 1.6 and -0.6, I am not doing those calculation for you please just it is simple multiplication and inverse. So that you can work out and what I was telling you that this theta ls is actually our unknown values okay. In which we are interested in alright. So we are trying to estimate these theta values okay and what is this theta this theta is basically c and d values.

So whatever we have got here this is nothing but our c least square and this is d least square okay. so this is what are we get and so the c least square value will be 1.6 and d least square value will be minus 0.6 okay. So this is how we can estimate these values of c and d ok so our general expression could be our after estimation of these parameters could be $z = 1.6x+-0.6$ or we can say that $1.6x-0.6$.

So this is what is going to be our expression and using these three results and the least square method of estimation of the parameters we could identify these unknown parameters c and d okay. So this is one example and another example which I want to take is or this one where we would like to estimate the kv okay, so that we can do.

So let us try to understand the problem and then we will try to estimate the parameter okay. So you can look at this figure it is a basically valve okay. So in this example we want to estimate the wall parameter we do not know the valve parameter and we are interested in finding out the value of the valve parameter okay.

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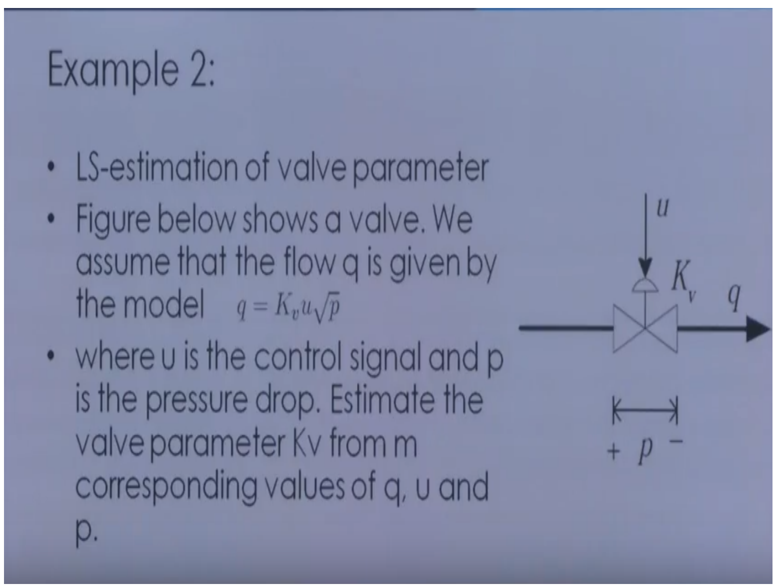

Now in this valve you can see that this is the direction of flow say a pipe okay through which a flow q and here is your valve all right. now in this valve there is a control signal u coming here okay and because of this control signals there is say when the say water passes through this valve okay. There is a drop in pressure p okay.

So we assume that the flow q is given by this mathematical model okay. so the mathematical model is q=kv okay and u underscore p. Now you see that here this kv is unknown to us we are not aware about the value of this kv okay. So here as I was telling you u is our control signal and p is the pressure drop our aim is to estimate the value parameters kv from m corresponding values of qu and p okay.

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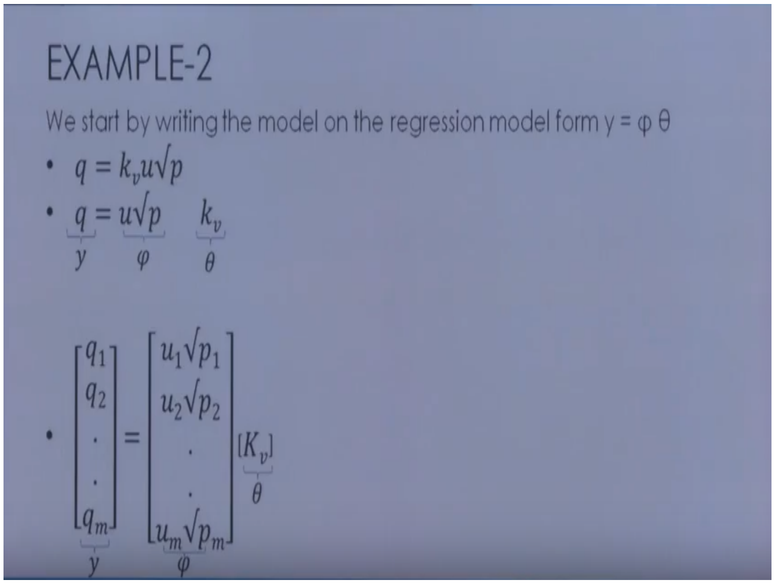

So I know I have M responding values of this q and corresponding values of say this u and also m corresponding values. So these are known to me and I am interested in finding out that what is value of this kv okay so this is my problem okay problem statement for this example that I am interested in finding out the kv.

It is a very real practical problem to find these valve parameters which are usually unknown and they are determined by this way only the way, which I am just going to explain to you okay. Fine so we start by writing the model on the regression model of the form y=phi theta. So you remember we had the model y=phi theta and remember in this y=phi theta.

We assume that we know the value of y we know the value of Phi and we wanted to find out this value of theta okay. So in this expression the expression which we have we identify what is our y what is our phi and what is our theta okay so first we need to find out those identify those parameters okay.

So here basically I can write this q as u_under root pxkv okay. Now you see this kv we are interested in finding out this kv value. So I write this as theta and u underscore p and these both are known to me and they are there on the right hand side so i just write them as fine and this qi am writing it as y okay.

So this is how I am doing it so we have we have here why we have here phi and we have here theta okay. So as I said we have we can assume that I know m such values okay. I know m such values so I can write these expressions for each value and up to m such values okay. So what will be my expression my expression will be basically here q1, q2, qm, we know are these m values okay.

And here are these will be basically say u1 root p1 okay. Likewise I can write u2 root p2 and um root pm okay. So this is my Phi and then of course I have got the kv okay. So this is my as I was telling you this is my Y and this is my Phi and this is my theta okay and our interest here is to find out this value of kv.

So here what we have done is that first we are writing this mathematical expression which has give been given to us in the regression a model form okay. Then the least square estimate of kv that is kv least square can be calculated using whatever of the standard value, which we have evaluated the unknown value least square as we have evaluated that was phi transpose phi whole inverse okay x it was phi transpose y okay.

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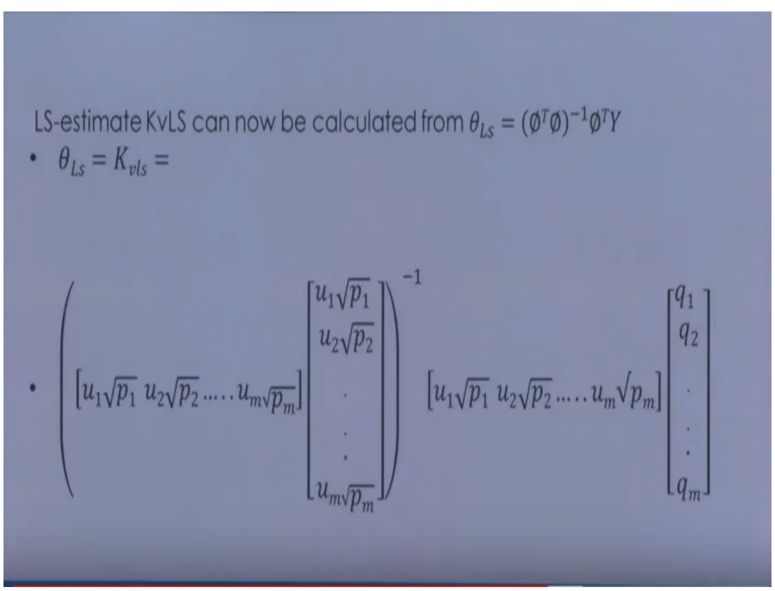

So we can substitute these expressions here and then we can get the theta least square okay. so what our values are here basically, so this if I take the phi transpose here then this will be this will be basically like this that is under root u1 under root p1 okay and u2 under root p2.and so on um under root pm this is my first, first term here okay. So this is my first term then what I have is phi, so phi will be as it is so that is u1 root p1 u2 root p2 and then we will have you m root pm.

So this is there and then whole to the power –1. Here I inverse into what we have is again this transpose. So I can again write that u1 root p1 u2 root p2 and um root pm. So it is this value and then the y value okay so and a y value are as you can see here it is base q, so I know that okay. So these things are known to be so q1, q2 and qm okay.

So this is what this is basically theta ls theta least square and what is that theta least square that is basically the kv least square, so this is nothing, but this is kv least square okay. So what we can do is now we can simplify this expression in order to estimate the value of or this kv ls okay. So this kv ls that is kv least square can be written as now you see when you are going to multiply this with this one okay.

What you will be having actually is u1 root p1 square plus okay we will be having u2 root p2 square um root pm square. So this is what we are going to get okay and to the power – 1here since it will be - 1 and then of course we will be having these components that is u1 q1 root p1+u2 q2 root p2+um qm root pm. okay. So basically this is what we are going to get so this is actually what we have this is actually there are a one row and m column okay.

Here and here we have M rows and one column. So when we do the multiplication here i will be getting one row one column only that is why i am getting it here like this okay so this one i can write kv ls as u1 q1 root p1+u2 q2 root p2+ um qm root pm, of course I can just take this to the denominator okay. So this is what I will have u1 root p1 square+u2 root p2 square okay+um root pm square okay.

So this is what we are going to get and this way I can estimate this value of kv ls, which is unknown for us then the in this case alright and once we have this value of kv ls okay. I can write the expression for my q = kv lsu under root p. So this way I am able to estimate this value of k, v, left, s and this value is now known to me which was unknown earlier okay.

Now but here you see that for this estimation I need all these values okay, that is I need the values are say all the inputs that is u1, u2 are um. I need a the pressure drop for all the m cases p1, p2, pm and not only this I need these m set of these q values also so q1 q2 and q1 so these things I need to know then I can estimate this value of the unknown parameter k, v, l, s okay. So, this with this I will conclude this lecture. Thank you for your attention.