**MModelling and Simulation of Dynamic System**
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**Lecture – 36**
**Parameter Estimation Methods**

I welcome you all in this lecture on parameter estimation methods using simulations or in simulation. When we are going to simulate a model there may be instances when we do not have the values of the parameters okay. So role of or the parameter estimation is paramount in carrying out the simulation studies, so in this lecture we will be looking at how we can estimate the parameters okay fine.

So mathematical models are the basis of simulators and theoretical analysis of dynamic system or say as a control system when these models are used and these models as all of you know could be in the form of say differential equations developed from the physical principle okay or from transfer function models which can be regarded as a black box model which Express the input output property of the system.

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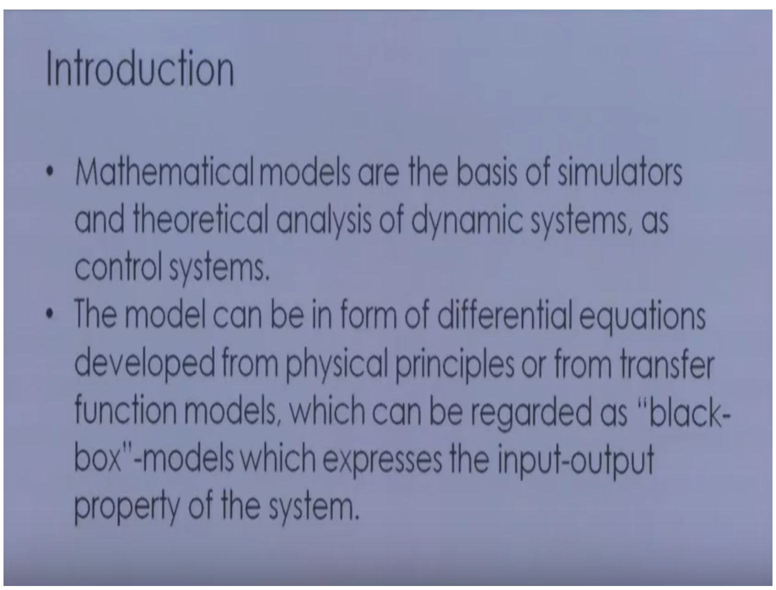


Why black box model because we know only input output property of the system. Now some of the parameters of the model can have unknown on earth uncertain values okay. Now for example a heat transfer coefficient in a thermal process or in that time or the time constant in a transfer function model could be unknown to us that is we are not aware of the values of those parameters.

So, we can try to estimate such parameters from measurements taking during experiments on the system because estimation of these parameters are important until unless you have those parameters available to you, you cannot go for the simulation. So, this estimation of such parameters is possible to measurements taken during the experiments on the system okay.

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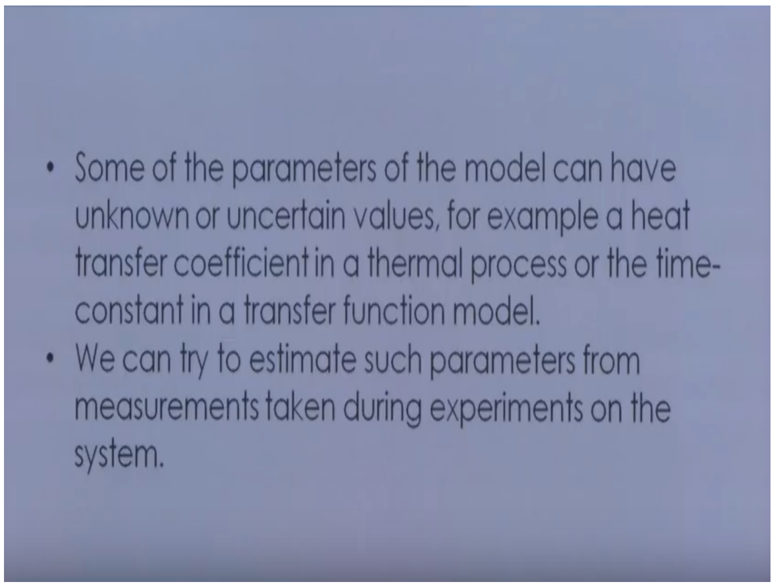


So perceive the thing is like this we have a physical system and on this physical system certain input signal is given and we get a certain output signal. Now with the help of these input signal and output signal we can have the parameter estimator for the mathematical model okay and from here we can find out the model parameters okay. That is the parameter vector can be identified.

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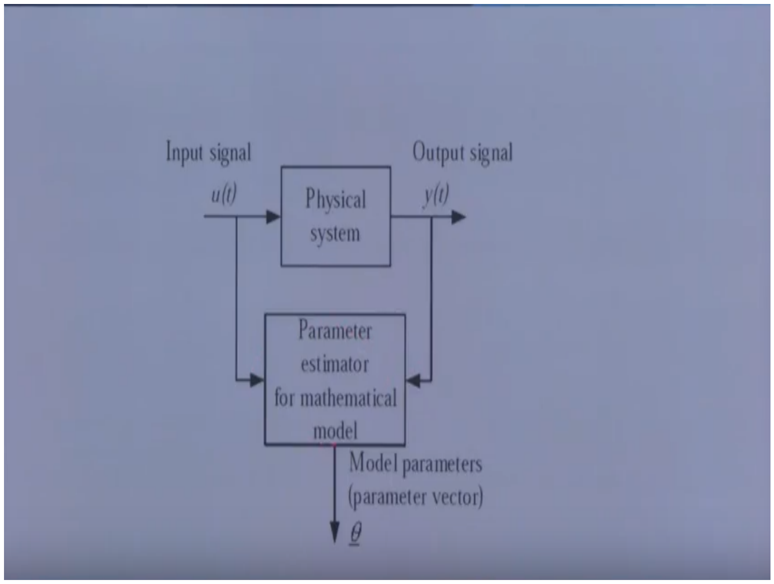


So this is what we are going to see in this lecture that how can we identify through this estimator for mathematical model by giving the input and output signal. How can we modify measure the model parameters. So here in this lecture we will discuss the popular least square method or in short or what is called as LS method for the parameter estimation okay.

The least square method can be used for parameters estimation of both static model as well as the dynamic models of a given structure. So what does the how does the least square method looks like so let us see that how we use the how we use the least square method for the estimation of the parameter okay.

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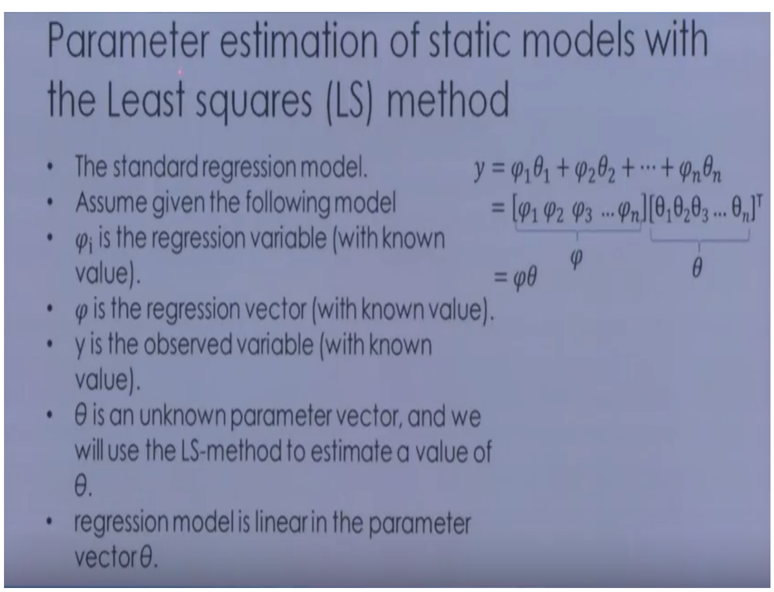

So for this actually are the standard regression model use and we can assume a model as it is indicated here or we can assume a model say Y = Phi 1 theta 1 okay + Phi 2 theta 2 and so on + Phi N theta N theta and all right. Now here what we do we segregate the Phi's and we segregate the thetas.

So, here I can write these a Phi separately Phi 1 Phi 2 Phi N okay and again and I can write the Thetas at is theta 1 theta 2 and theta N and now here you see oh this model which we are looking at here Phi I is the regression variable which is known that is it has got the known values we know these Phi's okay.

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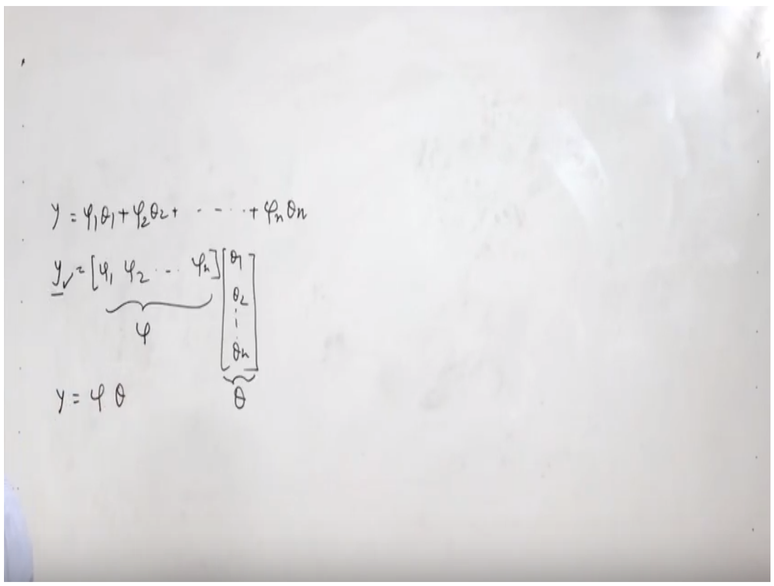


So these Phi's are known to us why is the observed variable again this Y is the observed variable and we are say no this variable it is also known and theta. Here is the unknown parameter vector and we will use the least square method to estimate these values of theta. So this can be represented in sort by psy and this in short can be represented by theta okay.

So the regression model is linear in the parameter vector theta and that is right you see that we are able to sum up do the summation. That is we can use the superposition theorem to do this summation so the regression model is linear in the parameter vector theta here okay. So in short I can write this Y, as I say Phi and theta okay.

now let us assume that we have M corresponding values of Y and Phi okay theta I am interested in finding it out okay so this theta is as I said this theta is unknown to me and I know these values of Y and Phi and this theta is not known to me okay so we assume that we have M corresponding values of Y and Phi then we can write the M expressions like this okay.

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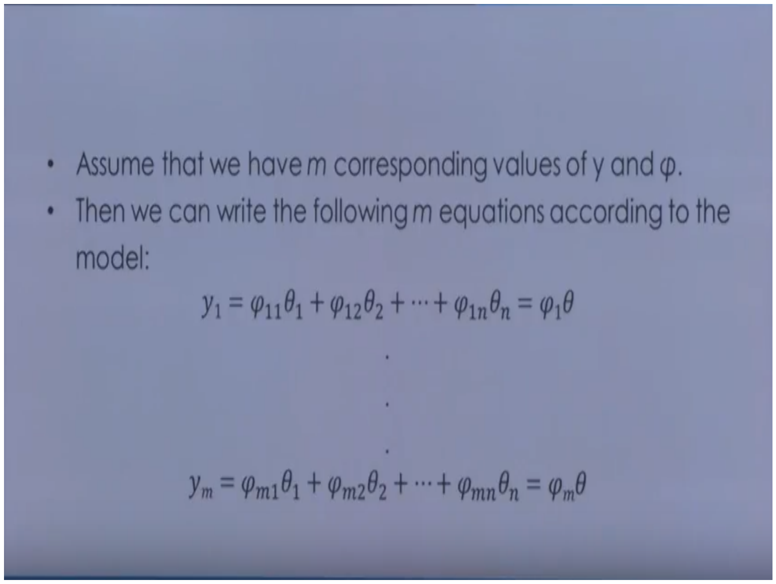


So if we have that m these y's are available to us so my expressions could be as it is written there okay. So my expression could be y1 = say phi11 theta1 + phi12 theta2 and so on + phi1 and theta N okay. So this is 1 and we can have m such value so I can write ym = phi here there are since m values, so this will be m1 theta1 + phi m2 theta2 okay and so on. So, I will have phi m n and theta n and okay and again.

So I write these things in short as say a phi 1 and this is theta and this I can write as say phi m and theta okay because we have the m corresponding values of y and phi, so I can write it like this okay now what we do is that after doing this we are right we express it are in the matrix form this can be expressed in the matrix form simply as you can see here y1 or you will have y2 and then finally ym.

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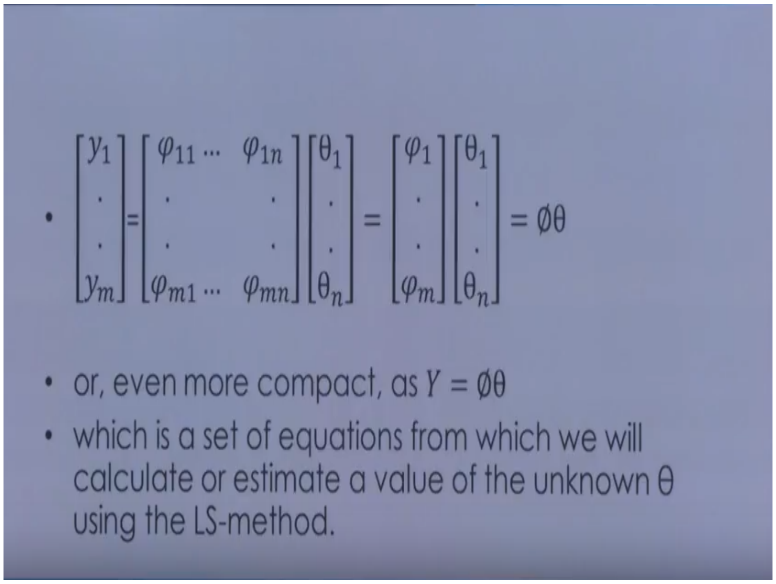


Then we will have a matrix and then we will have a vector that is a theta 1 theta 2 and theta or theta n okay and here what we will have is we will have phi11 phi12 and phi1n, okay. likewise, you will have phim1, phim2, and we will have phi mn okay. So this way we can write it or in the short or this can be written as what of phi1.

This row I am expressing by phi1, then we will have another row expressed by phi2 and then I am writing this by phi m. I am and Here I am writing this as theta1, theta2 and here theta n and okay. So this I am writing and this component, I am writing in short as phi and this as in short as theta okay. So this can be written in a very compact form as Y=phi theta.

So we write Y=phi and theta phi x theta. Now here these are set of equations from which actually as I said in the beginning itself that, here we assume that this y is known to us, phi is known to us and we are interested in finding out this value of theta okay and so from here we will calculate or estimate a value of unknown theta using the least square method.

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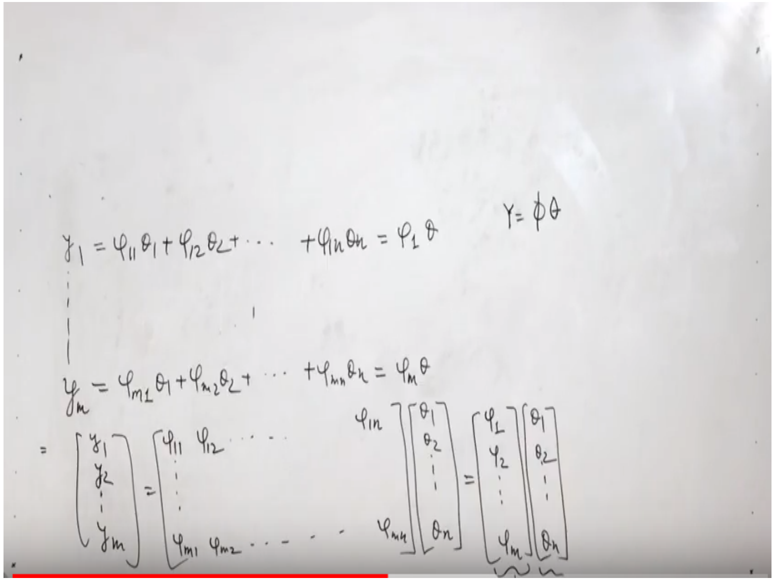

So, let us see how we do it now so in the least square problem what we do is that we define the error vector okay. So the error vector can be defined and this error vector will be basically y-phi theta. So we can arrange the error vector, so e can be put as their our error vector say e1 e2 and we will have em since there are m rows.

So we will have values up to em and this error vector can be written as say y1-say phi1 theta remember. Here this phi1 is a row vector okay, and theta is a column vector. So our multiplication here of this row, row and column vector is possible okay. So we have a row vector this I1 is a row vector and the theta is a column vector like this okay.

So we have the multiplication possible. Here all right, so it is this and then we will have here y2-phi2 theta and finally we will have ym-phi theta okay. so this is there or this e we can again put in short as say y-this is phi theta, y-phi theta, so this way we can write it.

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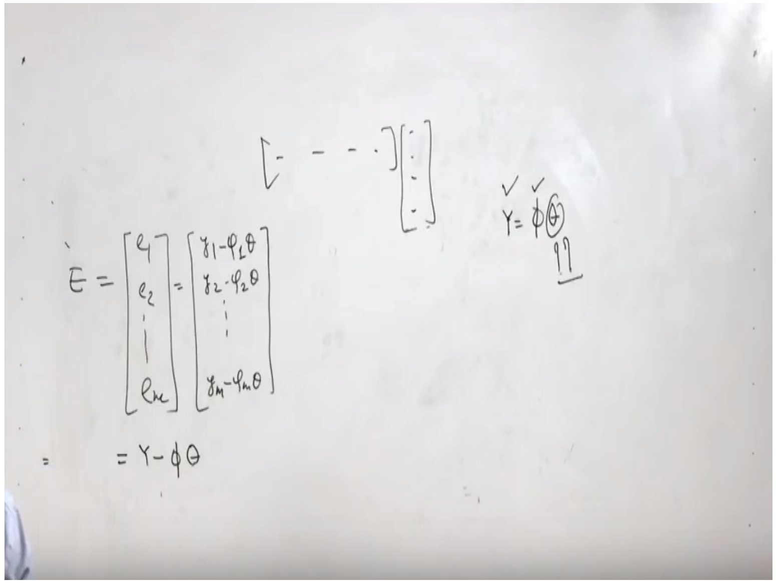

Now, the problem as I told you the problem is to calculate a value or an estimate, estimation of the unknown parameter vector theta. So that the following quadratic criteria function is minimized that is what we are basically trying to do. Here is that we want to find out the value of theta such that this of summation of the squares of the error is actually minimized okay.

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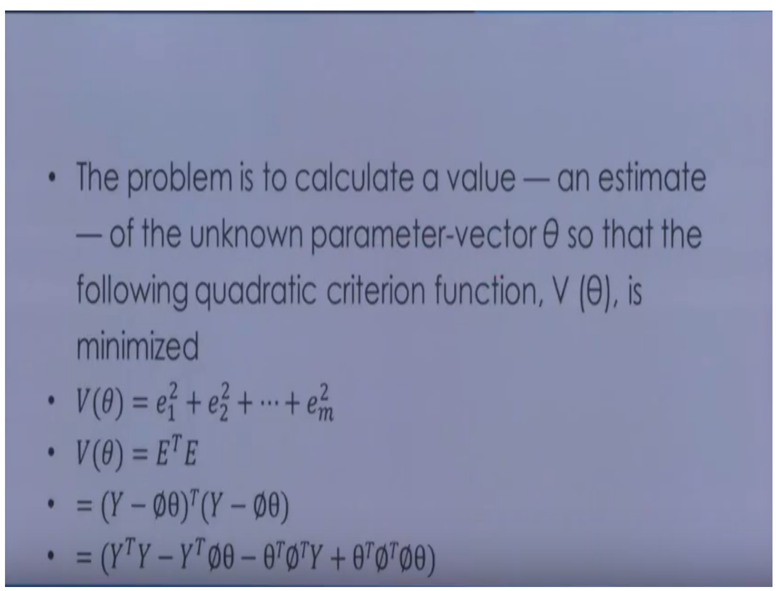


So, first we can find out the function we can find out the function which is to be minimized that is v theta okay and this is as I said we had the summation of the errors e1 square+e2 square+ we are going to have em square all right. Now in a compact form or this can be of course written as error vector transpose into error vector okay.

So, if you do this multiply these 2 then you will be getting the top of row that is summation of the is square. Now in the short form or this can be written as y-phi theta transpose xy-phi theta okay. So this is possible and now when we take the transpose for this inside what will happen is I will get y transpose and here the transpose order will change okay.

So this will be - theta transpose phi x y-phi theta so this is what we get and now we can do this our multiplication. So this will be y transpose y-y transpose phi theta - theta transpose phi y+ it will be theta transpose phi okay phi theta okay. So that is there, so this is what, here there is one small mistake.

So this is theta transpose here will also be phi transpose okay. Only order is reversed so then when we multiply this, so we have y transpose-or y transpose y-y transpose phi theta-theta transpose phi transpose y+theta transpose phi transpose phi theta. So, this is what you get, so this is the same expression which is written over here in the slide okay.

So this is what we get now what we actually want to do is that what we want to do is that this v theta which we have actually we will differentiate it with respect to theta and make that = 0 okay so if I differentiate it with respect to theta here then from here you can see there are terms theta transpose and theta.

So, if I differentiate this term then this is what I am going to get that is 2 phi transpose phi theta and these 2 term differentiation with respect to theta will lead me to -2 phi transpose y okay. So this way we can do that that is if I am trans differentiating this then this since here we are going to get the square values will be here okay.

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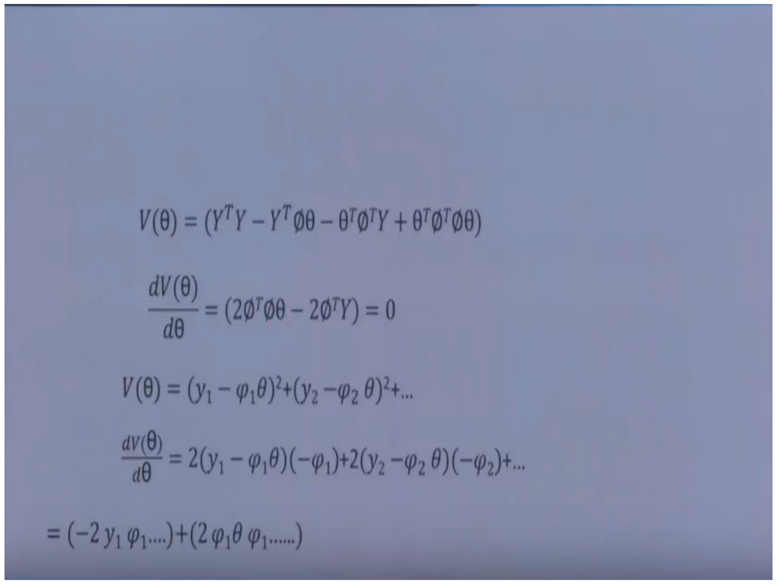


So I am getting this 2 because of differentiation here and here. Since there are 2 terms and we are differentiating with respect to theta we are going to get this value. Now if you have understanding in differentiating a differentiation in this form okay, then you can take the scalar form basically that is v theta was if you remember it was the formation of the error square.

So, this is actually e1 square that is y1-phi phi1 theta square this is a second error a square term by 2-phi 2 theta square and, so on okay. Now if I differentiate this with respect to theta then naturally what you will be getting is 2y1-phi1 thetax-phi1 and 2y 2-phi2 thetax-phi2 and, so on okay. Now you see that this - and this term will be making it - here.

So, –2 y1 phi1, I will be having + this - - will +, so it will be 2phi1 theta phi1 okay. So this is basically what we are going to get so these are so this form is similar to what you have seen here okay. So here as you can see that there is a term corresponding 2phi and theta, so 2phi and theta and there is a term corresponding to 2y and phi, so 2y and phi is there. If we equate it to 0, then this is what we are going to get okay.

So, we have the phi transpose phi theta = phi transpose y remember our interest is to find out the value of theta, theta is unknown for us. So now what I do is here that I multiply both sides that is left hand side and right side of equation by a inverse of this multiplication. So here if I multiply by this, so Phi transpose Phi inverse into the same of phi transpose phi x theta.

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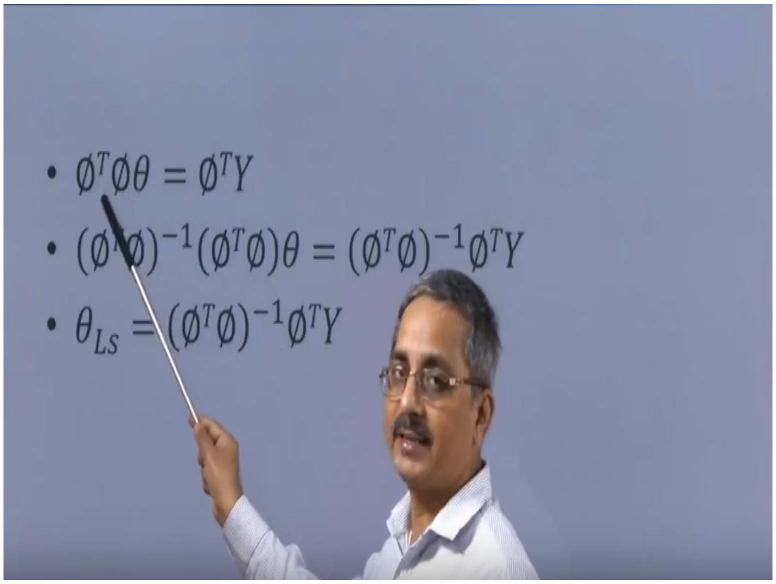


This will be equal to this I am multiplying this to the left hand as well as to the right hand side. So this I will be multiplying here to the right hand side also so this into phi transpose and y okay and now you can see that this multiplication will actually lead to you an identification sorry an identity matrix. So this multiplication can be replaced by identity matrix so what I will be getting is the theta least square okay.

So, this theta it is least square value will be phi suppose phi inverse this multiplication inverse into phi transpose y, so this is what I am going to get so this way we can find out the this theta ls value and we can identify or we can yes we can find out or we can estimate the these values of the theta okay.

So by this, I complete this lecture and in next lecture. we will take up2 examples and we will see that, how can we use this method to estimate the unknown parameter in a in an expression okay, thank you.