

**Modelling and Simulation of Dynamic System**  
**Prof. Dr. Pushparaj Mani Pathak**  
**Indian Institute of Technology- Roorkee**

**Lecture – 35**  
**Validation and Verification of Simulation Models**

I welcome you all in this lecture on validation and verification of simulation models, which is a sub model for the course on modeling and simulation of dynamic systems okay. See one question is always asked from the simulation experts that whatever simulation results you are getting whether it is correct or not or how do you ensure that the simulation results are correct okay.

So in order to do that we need to just have a little background of what these validation and verification of simulation models we mean by, see the best thing would always be that whatever results we are getting through simulation. If we have some experimental data or some other source or say if you have the analytical results okay.

And if you can compare that is the best way of verifying your model all right but let us see how we can do that, so as I told you one of the most difficult problem facing the simulation in a list is that determining whether the simulation model is accurate representation of the actual system being studied and as I said this is the question which is often asked to the modeling expert okay.

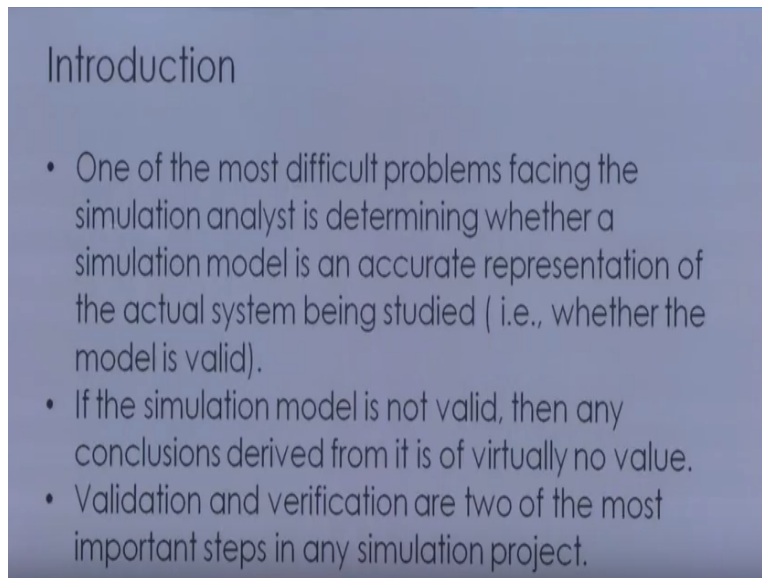
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### Introduction

- One of the most difficult problems facing the simulation analyst is determining whether a simulation model is an accurate representation of the actual system being studied ( i.e., whether the model is valid).
- If the simulation model is not valid, then any conclusions derived from it is of virtually no value.
- Validation and verification are two of the most important steps in any simulation project.

Whether your model is a valid model or not and if of course if your simulation model is not valid then naturally whatever results you are interpreting okay those results are not going to be correct, so your interpretation will be wrong and so validation and verification are the 2 most important steps in any type of simulation activity which one carries out.

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Introduction

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- Validation and verification are two of the most important steps in any simulation project.

Now if we look at the definition of validation and verification see validation is the process of determining whether the conceptual model is accurate representation of the actual system being analyzed and here that is whatever system you are analyzing okay or say whatever your physical system is there and from that physical system you have, you have replaced that physical system with an engineering system.

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- **Validation** is the process of determining whether the conceptual model is an accurate representation of the actual system being analyzed.
- **Verification** is the process of determining whether a simulation computer program works as intended (i.e., debugging the computer program).

So whether your replacement is proper or not okay or whether the engineering system which you have made okay or the model which you have prepared. Whether it is correctly is presenting the physical system or not that is what is the validation is the process of determining whether the conceptual model is an accurate representation of the actual system being analyzed okay.

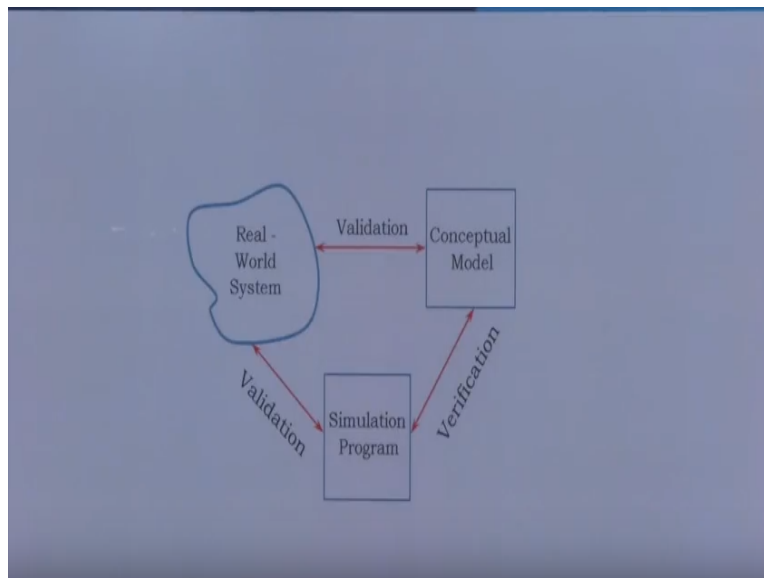
If you remember during beginning of our classes I gave you an example of a water tank being represented by okay. So we have a water tank supported on the pillars and if you could recall we represented this system as a equivalent spring mass damper system, so the valley validation.

Basically means that whether this is the true representation this is the true representation of this physical system or not and verification is the process of determining whether a simulation simulated computer programming works as intended okay. So what results you are expecting out of it okay or whether your codes are computer programs are working as intended that is what is known as the verification or what we also call it as the debugging of the computer program.

So, this is how it is so you have the real world system and this real world system we prepare a conceptual model as I am telling you here okay and this process is what we call it as validation okay and this validation could also be called that if whatever real word system you are having okay and if your simulated program simulation program is truly representing.

Whether that system or not or the behavior is truly representing the behavior as expected from the real world system or not so this is what is called as the validation and verification means that whatever conceptual model you have prepared when you are coding and of you are getting the results. So whether you are getting it correct or not, so this is what is called the verification.

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Now of techniques for verification of simulation so, let us see first the techniques to verify the simulation model the use good programming practice okay. When you are programming for something there are certain good programming practice, which one need to always follow okay. So when you are programming use as many comment statements as you feel appropriate so that the user can understand how the programming has been done okay.

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## Techniques for Verification of Simulation Models

- Use good programming practice:
  - Write and debug the computer program in subprograms.
  - In general, it is always better to start with a "moderately detailed" model, and later add extra details, if needed.
- Use "structured walk-through":
  - Get read the program by more than one person

So, write and debug computer program in 2 sub programs instead of making or writing a very huge program say of thousand lines it is always advisable to prepare the program of a small, small size okay And that is what is done by what we call it as sub programs or in some of the languages it is called function you create some small, small functions okay or sub programs.

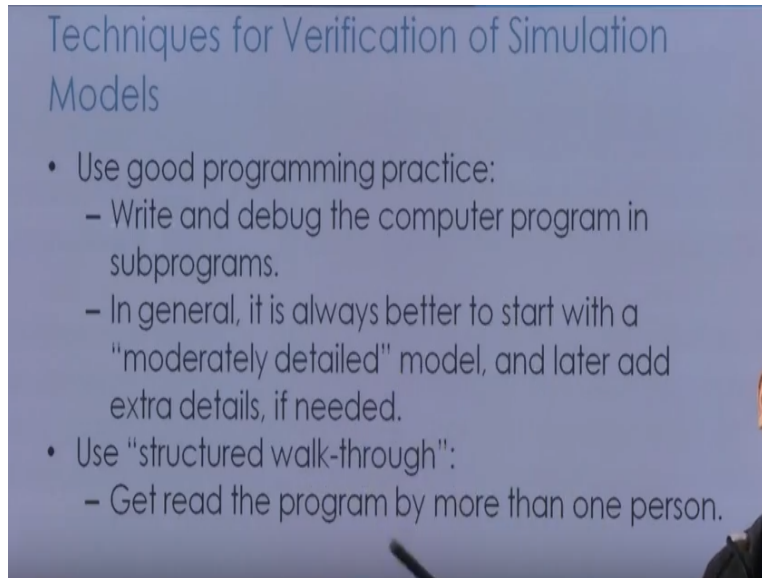
Then, you use those sub program or small functions in general it is always better to start with a moderately detailed program and letter add extra details if required okay, so instead of making your program very much detail at the beginning itself okay but it is always advisable to start with a moderately detailed program because if at the beginning itself.

If you will make a very huge program and during execution of that program if you come certain problem becomes or certain errors comes then the debugging will be very difficult okay and you will be spending lot of time in doing that so this is the general solution these are the good programming practices that you provide good number of comment statements right and program using small, small modules okay.

Start with a moderately detailed program and when you are confident about your program then of course you can go on adding the extra details use a structured walkthrough get ready the program by more than one get rid the program by more than one person okay. So you can ask

some other expert also to go through your program this is what do you mean by structured walkthrough okay.

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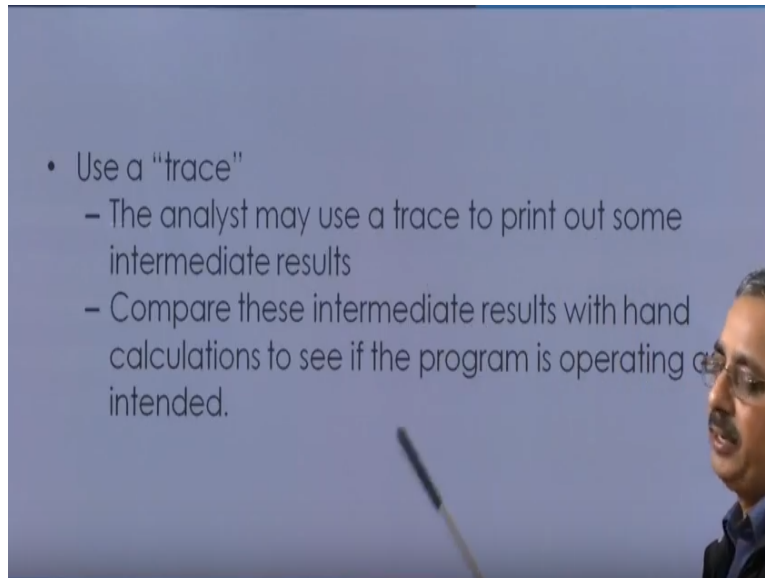


The advantage of this is that that of other expert can suggest to you the alternate means of programming okay. Our alternate interpretation of your coded program so it is always a helpful and of course his suggestions his or her suggestions are always going to be helpful for the debugging also then use a trace okay.

So actually the trace is what do you mean by trace is that we try to get some intermediate results okay. Intermediate results from the program which you have written and those intermediate results actually helps you in verifying your codes because many times if you say that you are not getting the end result correctly, but if you are getting some intermediate results correctly.

Then, you can at least identify the zone in which the programming error might be there okay. So this is the use of the trace analyst may use a trace to print a print out some intermediate results and compare these intermediate results with hand calculation or through some other means to see if the program is operating as intended check simulation output for region reasonableness.

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This is what mostly simulation people do okay. That is should have the idea one should have the intuition of what type of results one is going to get for the given conditions okay. So for example if I take the case of say simple spring mass damper system as I have shown here okay, we should have an idea that of say simple spring mass damper system the system equation.

We know  $mx \ddot{+} bx \dot{+} kx=0$  say if I am talking about the free vibration case all right. Now here we know that  $b^2-4mk$  as we have seen earlier the characteristic equation corresponding to this system will be  $ms^2+bs+k=0$ .

The values of  $s$  will be  $-B \pm \sqrt{b^2 - 4mk}$  upon  $2m$ . Now what I am telling you that we know the behavior of this equation will depend on this value  $b^2 - 4mk$ , so whether it is going to be if  $b^2 - 4mk > 0$ , then you are going to have the over damped case okay and if  $b^2 - 4mk = 0$ .

Then it is your critical damped case okay and if  $b^2 - 4mk < 0$  okay then this is going to be the under damped case so what I am telling you that we should have the intuition that okay if the parameter which I am using for  $b$ ,  $m$ , and  $k$  and this parameter values are such that they are going to be this type of condition.

They are going to create this type of condition. Then it is going to be an over damped case okay. So from these values we should have the intuition or intuition or we should know that what type of result we are going to get okay and that helps a simulation expert a lot so this is what I mean by that check simulation output for reason or reasonableness.

whether the result simulation result of which you are getting it is reasonable or not run the simulation model for a variety of input scenarios you can run it for different input conditions and check to see if the output is reasonable or not okay and in some instances certain measure of performance can be computed exactly and used for the comparison and these what could be this measure of performances.

Those we have already seen in our lectures earlier lectures that is the performance measure for the first order system and performance measures for the second order system. So these performance measures can be computed you are exactly and they can be used for the comparison another a very beautiful thing, which one can do to verify the result is the animation these animations are usually carried out using your simulated data.

So whatever simulation data you are seeing okay. Those simulation data are those data you can take export and you can use it in the animation software okay, and you can see the actual behavior visually by creating the videos are by seeing the different frames animation frames you can judge how your system is behaving okay and let me tell you that this animations helps us many times.

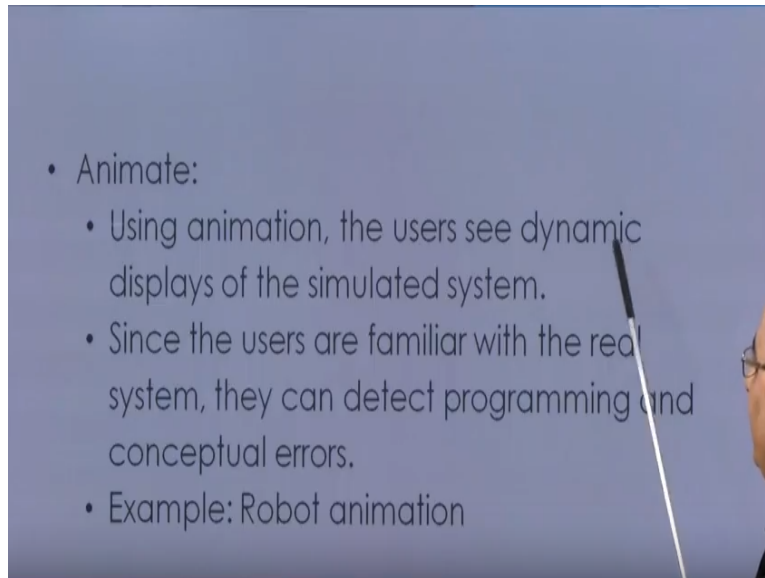
That is whatever inside is not given by the simulation result because many times it is difficult to interpret your simulation result. I can give you an example of say animation for the robot motion okay. Then how links are moving how joints are moving okay, where your end effector is going alright all those things if you want to simulate then simulation may not be giving the very nice representation of what actually is happening.

Rather, if you do the animation of it then you will be observing that what actually is happening with your system, so using animation the users see the dynamic display of the simulated system



and since the users are familiar with the real system they can detect the programming and the conception errors.

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What programming errors are there what conceptual errors are there so all those programming and conceptual errors can be checked and verified and as example I gave you the animation for the robots okay. So how the different links are moving where the end effector is going okay, all those things one can see an image and through animation.

It is a very good visual representation and it, it gives some sort of scenario or environment as if your real system is working. So it really helps many times in checking your model then compare final simulation output with analytical results of course, if those results are available they verify the simulation response by running a simplified version of simulation program with a known analytical results.

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- Compare final simulation output with analytical results:
  - May verify the simulation response by running a simplified version of the simulation program with a known analytical result.
  - If the results of the simulation do not deviate significantly from the known mean response, then the true distributions can then be used.

Of course, if those analytical results are available to you it helps many times in verifying our simulation results and if the results of simulation do not deviate significantly from the known mean responses then the true distribution can be used. Now after seeing the techniques for verification, now let us look at the techniques for validation of the simulation model.

Now, if you want to validate the simulation model one can adopt a 3 step approach for developing a valid and creditable model these approaches are that is the first is develop a model with high face validity okay. The objective of this step is to develop a model that seems reasonable to the people who are familiar with the system under the study okay.

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## Techniques for Validation of Simulation Models

- A three-step approach for developing a valid and credible model:
  - 1 Develop a model with high face validity:
    - The objective of this step is to develop a model that, seems reasonable to people who are familiar with the system under study.
    - This step can be achieved through discussions with system experts, observing the system, or the use of intuition.
    - It is important for the modeler to perform a structured walk-through of the conceptual model before key people to ensure the correctness of model's assumptions.

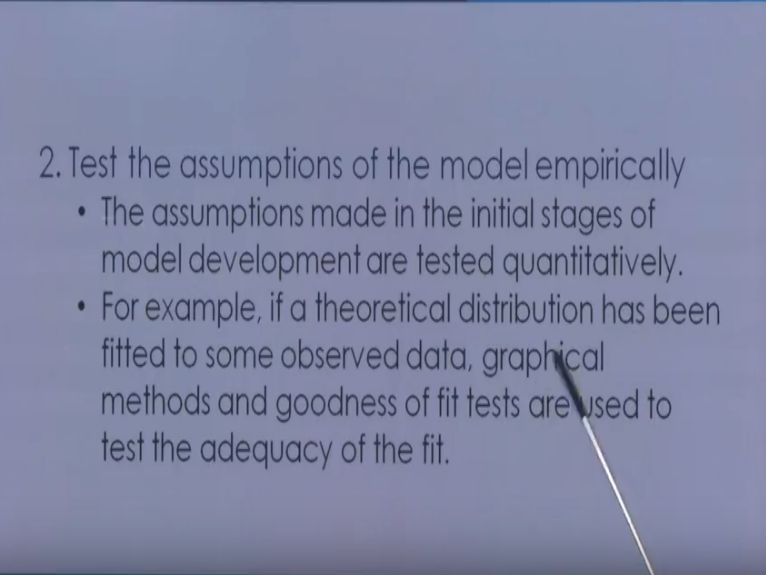
So, what does this mean is that whatever model you have developed that model should be reasonable convincing okay. And this step can be achieved through discussion with system experts observing the system or the use of intuition as I was telling you intuition plays a very big role in your modeling and simulation activity okay.

So, this can be discussed with the system experts and observing the system are the use of intuition it is important for the modular to perform a structured walkthrough of the conceptual model before people to ensure the correctness of the model assumptions as I said the, the model models are always replicate replicating the real world but real world has got so many features that you cannot replicate everything in your model okay.

So, the feature which is going to play a significant role in the analysis which we are doing okay we consider only those features in our model okay and we can carry out a structured walkthrough. Basically, that is to present the model in front of other experts to see the important assumptions are know about the correctness of the model the second step is the test the assumption of the model empirically okay.

The assumption made in the initial stages of the model development are tested quantitatively and for example if some theoretical distribution has been fitted to some observed data graphical method and goodness of the fit tests are used to check for the adequacy of the feed of course this is what is done in the data analysis.

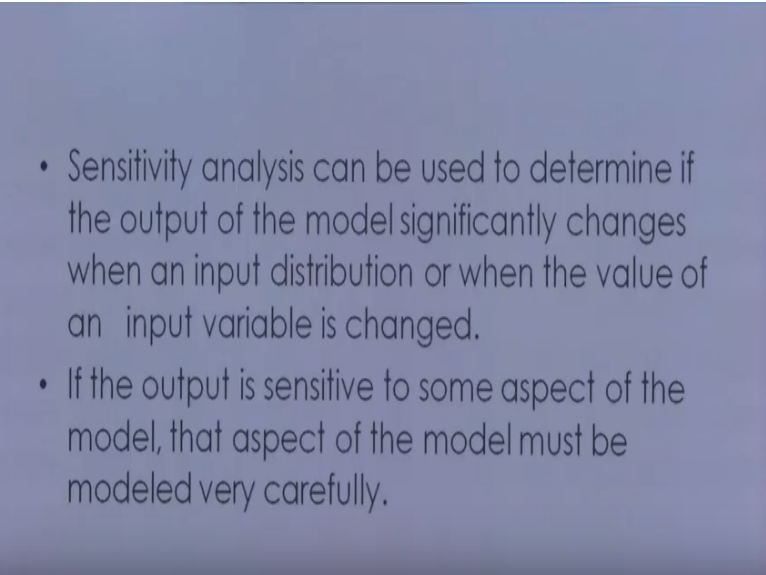
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2. Test the assumptions of the model empirically
- The assumptions made in the initial stages of model development are tested quantitatively.
  - For example, if a theoretical distribution has been fitted to some observed data, graphical methods and goodness of fit tests are used to test the adequacy of the fit.

Then, we can carry out the sensitivity analysis to see that what are the inputs which are going to affect your output significantly and then we should take care of those input so sensitivity analysis can be used to determine if the output of the model significantly changes when an input distribution or when the value of the input variable is changed okay

If the output is sensitive to some aspect of the model that aspect should be taken care more carefully and the third is the step is that determined. How representative the simulation output data are okay. The most definitely test of the model valid validity is determining. How closely the simulation output resembles the output from the real system again.

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- Sensitivity analysis can be used to determine if the output of the model significantly changes when an input distribution or when the value of an input variable is changed.
  - If the output is sensitive to some aspect of the model, that aspect of the model must be modeled very carefully.

So, this is the most representative of the simulation output data okay. One can go through the Turing test can be used to compare the simulation output with the output from the real system. Then output data from the simulation can be presented to experts who knows a about the system in the same exact form as of the actual system data and of course the experts can differentiate between the simulation.

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3 Determine how representative the simulation output data are:

- The most definitive test of a model's validity is determining how closely the simulation output resembles the output from the real system.
- The *Turing test* can be used to compare the simulation output with the output from the real system.

The system output their explanation of how they did should improve the model in your further iterations okay there are statistical method available for comparing the output from the simulation model as well as those from the real world systems. So statistical method for comparing real world observation with the simulation data these are the 2 basic approaches which are followed one is the inspection approach.

Another one is the confidence interval approach okay. So in inspection approach basically we run the simulation model with the historical system input data instead of sampling from the input probability distribution and we compare the system and the model output data so both the output data are compared by comparing with the or by giving the historical system input data okay.

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## Inspection Approach

- Run the simulation model with historical system input data instead of sampling from the input probability distributions, and compare the system and model output data.
- The system and the model experience exactly the same observations from the input random variables.
- This approach results in model and system outputs being positively correlated.

The system and the model experience exactly the same observation from the input random variables and this approach results in model and system output being positively correlated. So what is basically done here is that you have the historical system input data for the both the cases you feed this historical input data to the actual system as well as to the simulation model and the system output data can be observed.

Similarly, you can have the model output data and then one can compare the system output data and the model output data then there is another approach of what is called as the confidence interval approach this is a more reliable approach for comparing the simulation model with the real world system and of course this requires a large amount of data okay.

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## Confidence-Interval Approach

- A more reliable approach for comparing a simulation model with the real-world system.
- Requires a large amount of data.
- Suppose we collect  $m$  independent sets of data from the system and  $n$  independent sets of data from the model ( $m$  and  $n$  can be equal).
- Let  $X_j$  be the average of observations in the  $j$ th set of system data with mean  $\mu_x = E(X_j)$  and let  $Y_j$  be the output from the  $j$ th replication of the simulation model with  $\mu_y = E(Y_j)$
- The objective is to build a confidence-interval for  $\zeta = \mu_x - \mu_y$

Suppose we collect  $m$  independent set of data from the system and  $n$  independent set of data from the model okay  $m$  and  $n$  can be equal. now let  $x$  be the average of observation of in the  $j$ th set of the subsystem data with certain mean same  $\mu_x$  and let  $y$  be the output from the  $j$ th replication of the simulation model say with mean  $\mu_y$  okay. So the object is to build a confidence interval that is the difference between the  $\mu_x$  and  $\mu_y$ .

So, this is what is done in the confidence interval approach okay, so basically we try to find out the difference between this means of the system data and that of the output data and this is what is called as the confidence interval approach in fact this may be only feasible approach because of the severe limitation of the amount of data available on the operation of some real word system.

And here, one must take care in interpreting the result of this approach okay. So this is a very short presentation on how we are going to or what care we must take in validating and verifying our simulation data. Thank you.