Modelling and Simulation of Dynamic System
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Lecture – 34
Simulation of Wheeled Mobile Robots

I welcome you all in this lecture on simulation of wheeled mobile robots, so here essentially we will be looking at a brief about on the wheeled mobile robots and then we will be mostly interested in knowing the say kinematics sand dynamics of single wheel, 2 wheel and omni directional wheeled mobile robot.

So these wheeled mobile robots are part of what we call it as mobile robots and under mobile robots basically, there are 2 these types, these types are so we have the mobile robots and this mobile robots could be wheeled and legged, so the beautiful thing about wheeled mobile robots are that one can achieve a greater speed okay.

As compared to the legged robot, who are usually slow, but the drawback for the wheeled mobile robots are, they are not suitable for uneven terrain, whereas legged robot can be used in uneven terrain, so this is say these are the feature of wheeled and the legged mobile robot not used for uneven terrain, they are useful fine.

So with this small background, let us see the wheeled mobile robots, so several robots are now design to have mobility and of course they can move on surface, water, as underwater robots or in air okay, the one which you are seeing these days that is quadcopters okay.

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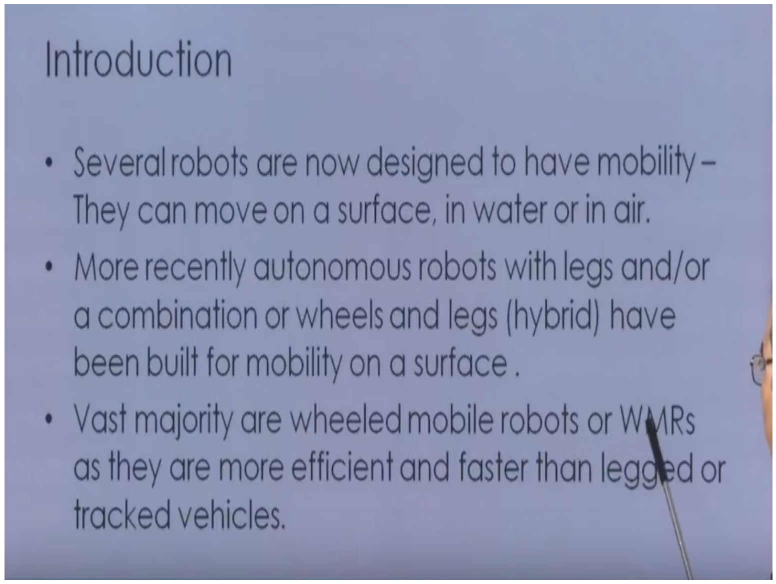

So these are there now more recently autonomous robots with legs and/or a combination of wheel and legs that we call it as the hybrid one have been built for mobility on a surface. The vast majority or wheeled mobile robots are in short they are written as WMR as they are more efficient and faster than legged or tracked vehicle as I explained you here.

These autonomous vehicles are used in industrial environment to move material from one place to another and they are also used in hazardous environment such as inside the nuclear reactor or of course in the deep sea bed, also used for planetary exploration. Now analysis of these wheeled mobile robots involves kinematics, dynamics okay.

These robots moving on a flat surface or in a plane of course there are many more thing for example, we can study the control aspect also, but in this course I will be limiting my discussions to kinematics and dynamics.

So wheeled mobile robots are model that is we can look for the modeling of wheels and these wheels are modeled as thin disc, ruling or sliding on a flat surface and of course there has been many authors who have worked a lot in this area and few of them are Muir and Newman then Alexander and Maddocks and from India professor Ghosal has done a lot of work in this area.

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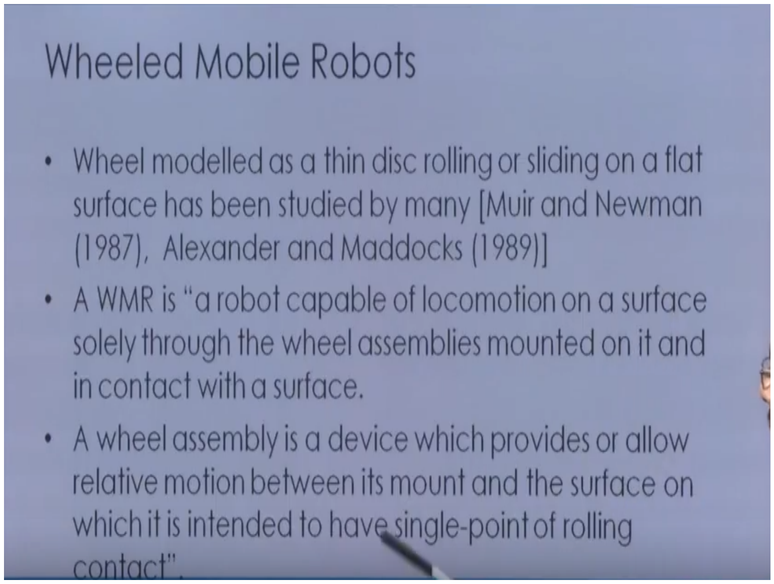

In fact most part of my lecture here covers from the textbook on Robotics written by him. A wheeled mobile robot is a robot capable of locomotion on a surface slowly through the wheel assemblies mountain on it and in contact with the surface okay, and of course wheel assembly is a device, which provides or allow relative motion between the mount and the surface on which it is intended to have a single point of rolling contact.

In practical, wheels due to deformation, there are area contact and we really find the point contact between the wheel and surface. There are different types of wheels, which are used in wheeled mobile robots. These are conventional wheels, omni directional wheels and the ball wheels.

Let us see one by one, this is an example of the conventional way, where you can see that here is the axis about with this wheel rotate and we can put actuators here and that actuator can be mounted on this frame and the shaft can be engaged to the wheel here okay, and of course from here, there can be connection to the robot body.

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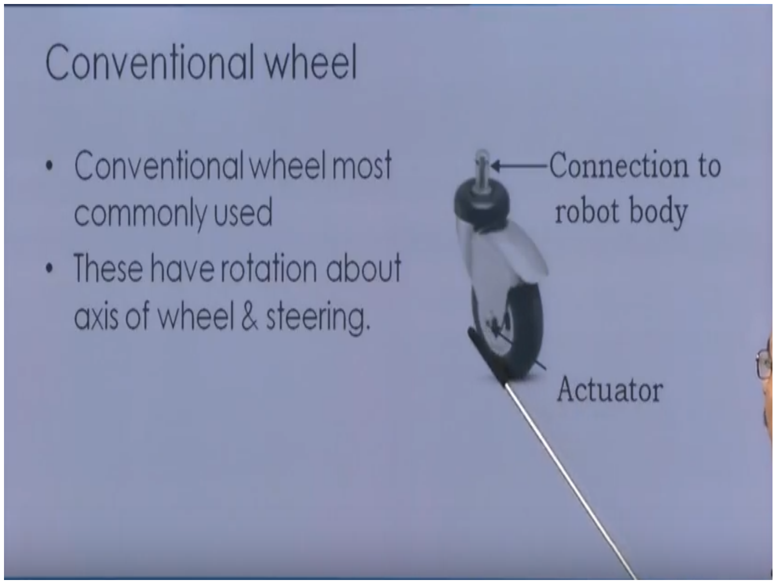

So these conventional wheels are most commonly used wheels and these have rotation about axis of the wheel, so they rotate about the axis, which is passing from here, as well as the steering is also possible that is you can turn about this axis also. Next is the omni directional wheels as you can see in this figure, these wheels are also called Swedish wheels.

In this, wheels you can see that there are barrels in the periphery okay, and because of these barrels what happens, these wheels are able to slide in the lateral direction also okay, and here is the axis along this one, here axis of the shaft which passes through this hole and the wheel turns about that axis.

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So here barrel can rotate at an angle to the wheel rotation of axis and here you can see that these barrels are perpendicular barrel axis or perpendicular to the wheel rotation axis okay, and these barrel rotations are usually not actuated and barrel rotation lead to the sliding of the wheel, so here we have the2 degree of freedom in which wheel and of course the steering.

Then, we have the ball or spherical wheels, here you can see that there is a frame basically and this frame is attached to the robot body, it can be bolted from here okay, and there is a spherical ball shaped wheel which is here, okay, so these are essentially a sphere which can rotate about 2 axis and complicated drive and rotation measuring arrangement of course here, all those things have not been shown.

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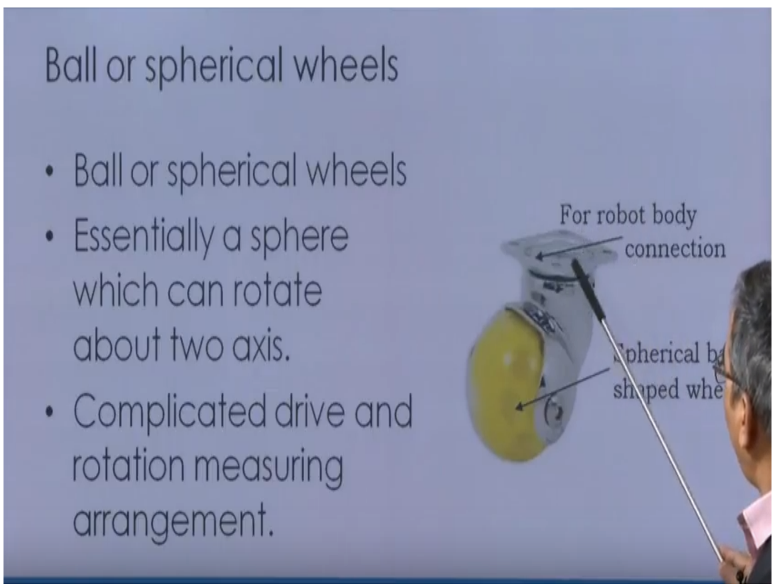

Now let us see the kinematics of a single wheel, fine so if we talk about a single wheel okay, so say this is the wheel basically and say this wheel is passing through this curved path and say this is the point of the contact and at that the point of contact where is a tangent here at x and y,I have drawn here and say this is the axis of rotation of the wheel, so this is my X direction and this is my y direction.

So, the configuration space of the wheel here consists of specifying xy and pie okay, xy are the point of contact here and pie what we call it as the steering angle okay, and say wheel are rolling without slip and that is motion is only along tangent to the path and it is not perpendicular to the path that is the motion is not there in the lateral direction, then that is what we call this also as the wheel rolling without slip, ok so if we implement without slip condition here okay.

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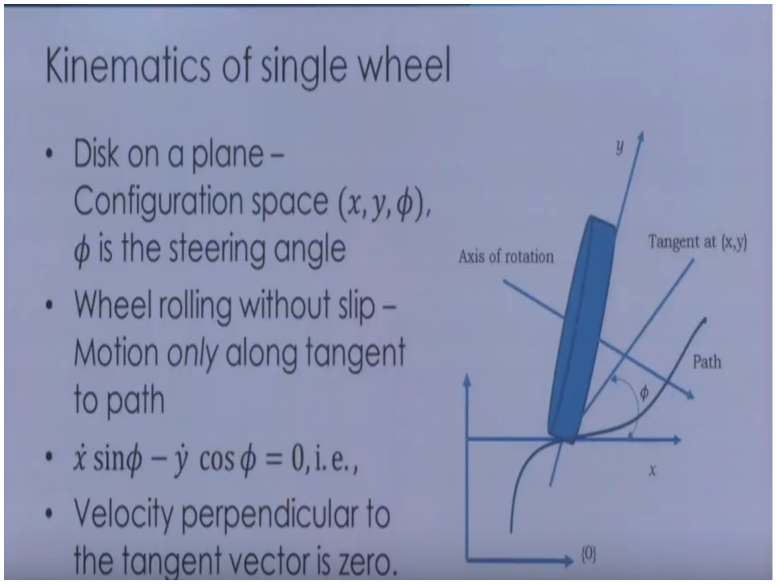

Then we can see that this is the say the direction for x dot and here is the direction for the y dot okay, then the perpendicular to the wheel here that is the perpendicular to the tangent, if we resolve these velocities then that should be equal to 0 because there is no slip ok, so this is x dot X component here.

So here you can see that say this is my x and this is pie ok, so if this is my direction for x dot, so this is going to be x dot sine pi and likewise, we will have say this is the direction for y dot, this is y direction, so our y dot is going to be here, so here this pi so this is s90 – pi, so this is again pi, so this component is going to be y dot cos pi.

So what we have is x dot sine pie-y dot cos pi=0 ok, so x dot sine pi- y dot cos pi=0, so this is the condition for the no slip ok, so this is the conditions such that your wheel is not moving in this direction, okay wheel is not moving in that direction that is the velocity perpendicular to the tangent vector is 0 now, this type of constraint okay.

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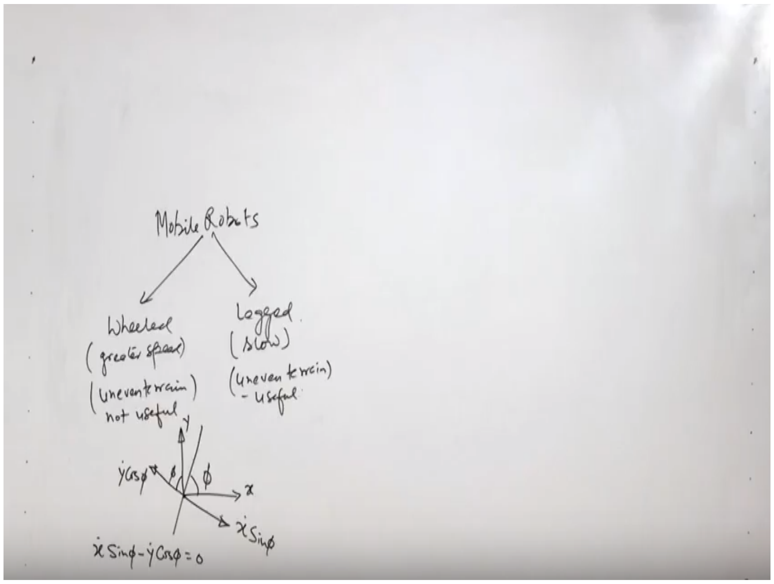

What we are seeing from here as x dot sine pi-y dot cos pi=0, this type of constraint is what is called a non-holonomic constraint. This non-holonomic because this equation cannot be integrated to obtain a function such as fxy pi=0 that is here, the constraint equation which is in terms of velocity and which cannot be integrated in order to get a constraint equation in terms of displacement that type of constraint is called the non-holonomic constraint okay.

So the constraint q dot=x dot y dot pi dot transpose and not in terms of this one, so this is the non-holonomic constraint. Now we can give the justification for the non-holonomic nature of this expression okay, so we can assume that let us see that this expression can be integrated. Now if I am able to integrate this expression and get the integrated expression in terms of displacement.

So this implies that existence of some fxy pi=0 that is the 3 unknown with one constraint equation, so here there are 3 unknowns that is xy pi, okay and fxy pi with 3 unknowns here ok, we will be having a constraint equation here, so what does this means, this means that if I know any of the 2 parameter say xy, I can find out pi, ok by choosing any of 2xy pi.

We can determine the third component. Now suppose if I take X Y = 0, we get pi value, which is a constant and of course which is determined from the constraint relation. Thus no other values of pi is possible and of course this is not true because as we have seen in this example, the disc is steerable okay.

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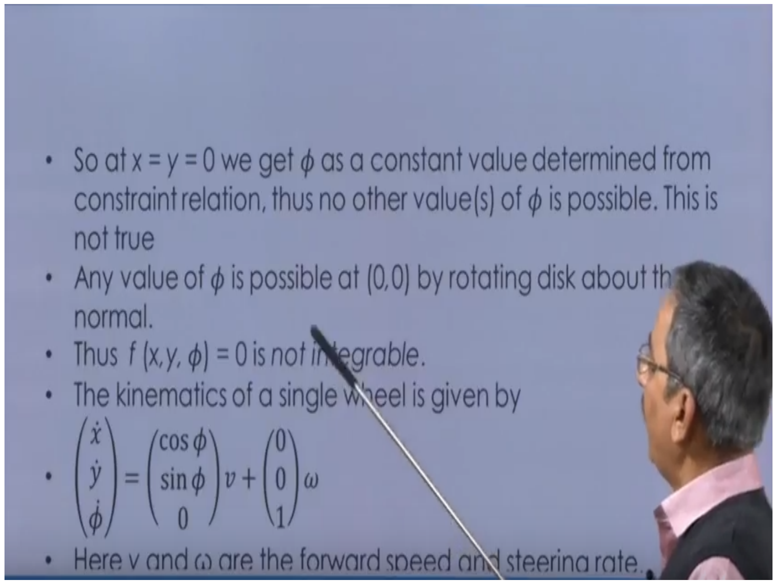

That is pie can change, it cannot remain constant, so our assumption that this expression can be integrated in order to get a function in terms of xy and pie that is totally wrong okay, so it implies that this expression is a non-holonomic expression that is by integrating this, you cannot get a displacement relationship okay, so this is not integrable.

Now the kinematics of a single wheel can be given by this expression, it is a very simple expression of course if we know that how the wheel is moving that is if you know the wheel velocity, then we can find out this kinematic equation, so suppose we have the tangential velocity of the wheel, this is v and say this is my x direction and this is my y direction okay.

This angle is pi, then we know that what is by X dot and Y dot, so X dot will be V cos pi and Y dot will be V sine pi okay, and of course the pi dot will be omega and I can write this expression in the matrix form like this X dot is V cos pi, Y dot is V sine pi and pi dot = omega okay, here V and omega are the forward speed and the steering rate okay.

So here this omega is the steering rate. Now let us see the2 wheeled robot, which is very similar to that of the bicycle model okay, so in this one say this is the2 wheeled robot and say this is the rear wheel and this one is the front wheel, now in this case there are 4 generalized coordinates, which we can observe, these coordinates are the xy point of contact of the rear wheel here okay.

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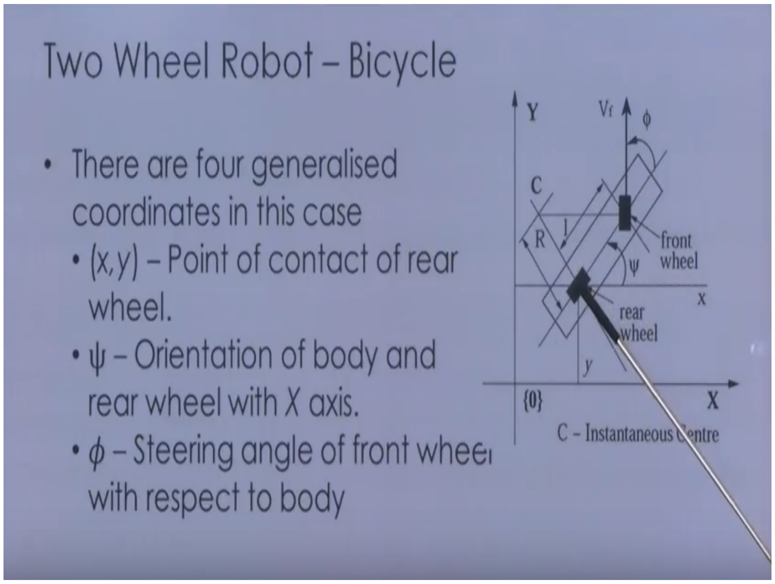

We have the sie that is orientation of body and rear wheel with x axis, this is orientation of body, as well as rear wheel with X axis and there is another generalized coordinate pie, which is the steering angle of the front wheel with respect to the body okay, so this way we can define the 4 generalized coordinates here and we have the x and y say this is 0 frame or what you can say that, this is the absolute frame okay.

So here there are 2 rolling constraints in each wheel, in this case we can work it out. The first one is this one X dot sine sie–Y dot cos sie=0, this is basically the no slip condition here, and the second one here is the no slip condition here ok, and here you can see that this is because of the translational velocity and term is because of the angular velocity that is there okay.

So these are the constraint in each wheel and the kinematic equation for this can be written like this basically here, vf is the velocity of the front wheel, so I can find out its component in the direction of the wheel axis, so this will be vf cos pie and of course this vs cos pi I can resolve in x direction, so that will be vf cos pi and cos sie, so here we have got x dot as vf cos pi cos sie.

Similarly the Y component I can get that is vf cos pi and is sine component, so VF cos sie cos pi and its sine component here okay, and the sie dot, I can get basically this VF will also=dot sie pi okay, so from here I can get this value of sie dot and of course pi dot will be given by omega okay, line perpendicular to the front and rear wheel velocity vector meet here at say the C.

Which is the instantaneous centre of rotation, so here we have seen the constraint equation for the wheel, as well as we have seen the kinematic equation. Likewise, similarly to the 2 wheel, we can work for the 3 wheel also, here you can see that there are 2 rear wheels okay, you have a front wheel fine and say C is the instantaneous centre of rotation and of course everything expressed with respect to a 0th frame that is X and Y frame.

Now here you can see as we have seen in the previous case, this is the front wheel is steerable here, it is steered and the speed of the rear wheels are different of course when they are making it turn okay, and we can represent the wheel speed for each wheel by R theta I dot, where of course I=1,2,3 for different wheels and R of course the radius of the Ith wheel.

Now here we can see that one rear wheel is driven and the speed of the second rear wheel must adjust so that there is a single instantaneous centre of rotation okay, and if the second rear wheel can be free that is what employs from the first statement is that the second rear wheel can be free or a differential has to be used, of course the kinematics I am not going to deal here.

It is very similar to what we have seen previously the bicycle model okay, and this C here determines relation between the R that is the distance of the robo access from the instantaneous centre, theta I that is the individual wheel rotations, radius of the wheel L that is the distance between the front and the rear wheel ok and this is the vehicle turning sie and the steering angle okay.

If there is no single C then of course as we all know from our basic kinematics understanding that we are going to have the slip. Next, we will see the omni directional wheel okay, in omni directional wheel, there are actually 6 equally spaced free barrels on the periphery as I have shown, these barrels in initial slides.

Now each barrel can rotate about its axis we have also seen and this is basically responsible for giving the lateral motion to the wheel, usually the 2 rows of barrels are used, so that one barrel is always in contact with the ground and the distance of point of contact from the vehicle centre changes.

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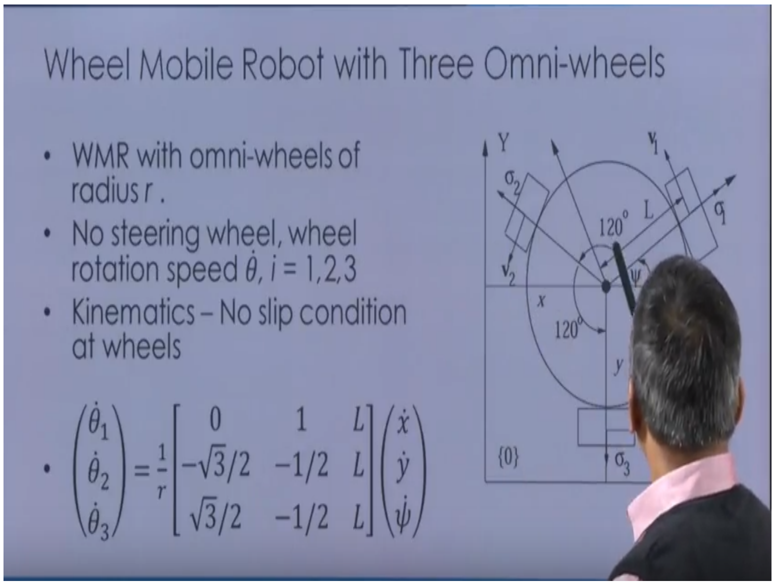

Let rotation speed is represented by theta dot and the sliding speed is represented by say sigma, so if we want to look at the kinematics of the wheel mobile robots with 3 omni wheels then here you can see that say this is our first wheel, second wheel, third wheel okay, and these wheels are placed at 120 degree each fine.

So here the tangential velocities are V1,V2,V3 and of course these are the slip velocity sigma 1, sigma 2, and sigma 3 are here okay, so no steering wheel rotation speed theta dot I, where I goes from 1to3, and we can find out the kinematics for no slip condition and we can find out this theta 1 derivative, theta 2 derivative, theta 3 derivative, okay.

As these are certain matrix into say cartesian velocity vector okay, so this is somewhat very similar to what we have seen in robotics, industrial robots okay, where we had a matrix multiplied by joint velocities and we were getting the cartesian velocity. Here we have a matrix multiplied by the cartesian velocities in order to get the individual wheel rotations, okay.

So this can be worked out by going through the simple working out the simple kinematic equation for this one. Translational and rotational dynamics of a wheel if I try to understand then you can see that say there is a wheel here, there is a rigid ground, you have the deformable wheel, this is the forward direction say velocity is x derivative okay.

This is the rotation, a torque tow on acting here and there is a rigid ground, normal reaction is N and the traction force is FT, so the translational dynamics here consists of ft=say mass of the wheel x acceleration okay, and the rotational dynamics will be equal to that is given by tow torque and this torque will be basically what, this torque we can find out by writing torque – this traction torque.

That is FTR=J wheel x acceleration of the wheel, angular acceleration of the wheel okay. So here FT is tractive force at the wheel ground interface and tow is the torque applied as a wheel axis, so these 2 expressions gives the translational and rotational dynamics of the wheel and the last, we can similarly workout the equation of motion for the omni-directional mobile robot okay.

This equation of motion one can obtain by using Lagrange equation okay, so we can evaluate the kinetic energy, we can evaluate the potential energy, we can find out the Lagrange and then by substituting in Lagrange equation, we can get the expression. The expression looks like this one, so we have the kinetic energy of the platform is there and the kinetic energy of wheels are here, we assume that there is no potential energy as the motion is on the flat plate, okay.

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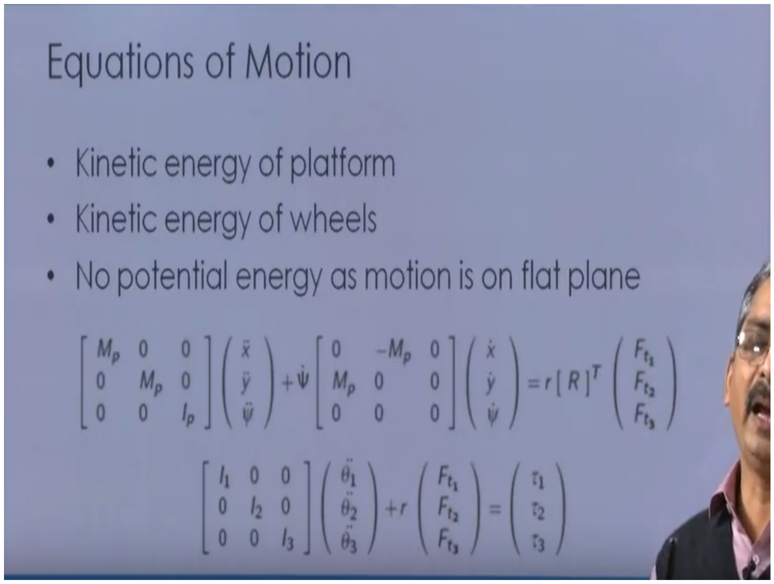

So here let us try to understand this expression, which is of this form, so here are the Mp of the mass of the platform so you have the mass of the platform into acceleration of the platform in the X direction, likewise mass of the platform and acceleration of the platform in y direction and here this is the polar moment of inertia of the platform and the angular acceleration of the platform okay.

So plus you have a matrix here and of course here we have the velocity vectors and there is also the vector sie dot and this is equal to R into this rotation matrix, which we have seen previously that transpose into the traction forces okay, these traction forces we can find out from this equation okay, and this equation is nothing but expression for the individual wheels.

So here we have the I1, I2, I3 and the wheel accelerations plus rxft1, ft2, ft3 that is these are the traction torque and these equal to the torque of the individual motors ok applied on the individual wheels okay, so now we can use these expressions and give the numerical values in order to do the simulations okay.

So I will stop here you can try simulating this expressions and for further details, please refer to Ashitava Ghosal, Robotics Fundamental Concept and Analysis, where you can find out many simulation results for the given set of parameters, Thank you.