Modelling and Simulation of Dynamic System
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Lecture – 30
Bode Plot

I welcome you all in this lecture on Bode plot. Bode plot is a method of studying the frequency response of a system and that is basically the response of dynamic system when it is subjected to a sinusoidal input okay, so the frequency response as we can talk is that if we plot a set of values of the magnitude say Gj Omega and phase angle Phi that occurs when a sinusoidal input signal is varied over a range of frequencies.

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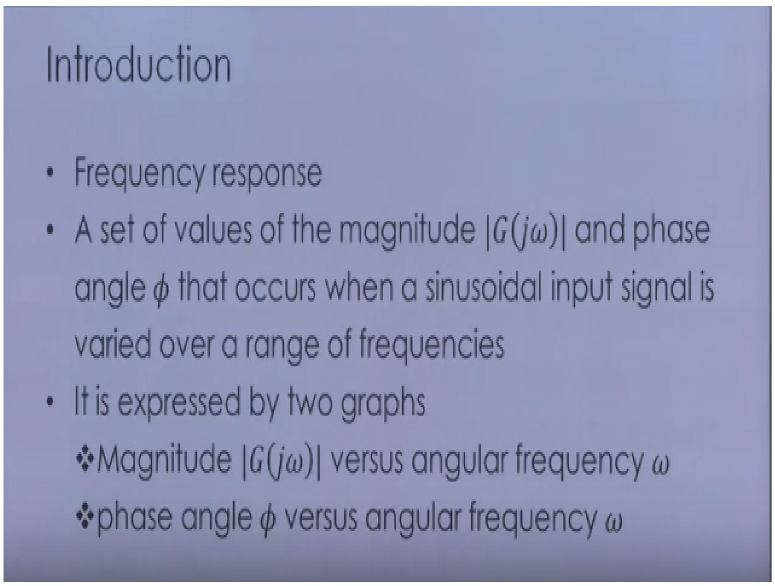

So, when sinusoidal input signal is varied over a range of frequencies then if we plot the magnitude and the phase angle, so this is what is termed as the frequency response. Now this frequency response can be represented with the help of 2 graphs and these graphs are the first is the magnitude versus angular frequency because here we are interested in varying the frequency and seeing the variation of the magnitude and the second one is the phase angle versus angular frequency.

So by varying the phase angular frequency, we are interested in seeing the variation of the phase angle, so these magnitude and angular frequency are plotted using logarithmic scale okay and such a plot is called the Bode plot okay and in this Bode plot, magnitude is expressed in decibel units and this is how the expression for the decibel unit is that is Gj Omega magnitude in decibel=20 log on the base 10 Gj Omega magnitude okay, so this is how it is expressed.

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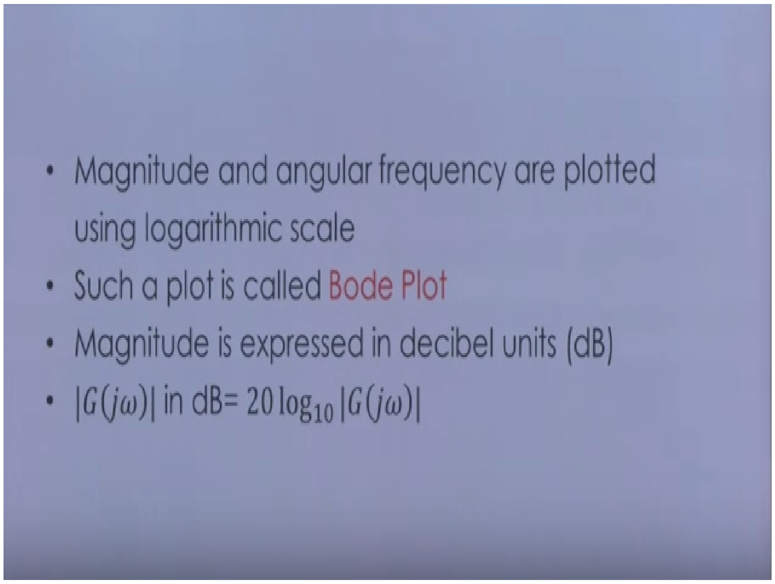

Now, let us take example of Bode plot, so first we will be taking an example for a system having a transfer function say Gs= K, where K is a constant okay, so the transfer function Gs=K is here and here, the K is constant. Now, I can get the frequency response function from this transfer function by replacing s by j Omega okay, so I get Gj Omega=K because I get the same thing because here K is a constant okay, so what we mean the magnitude of Gj Omega.

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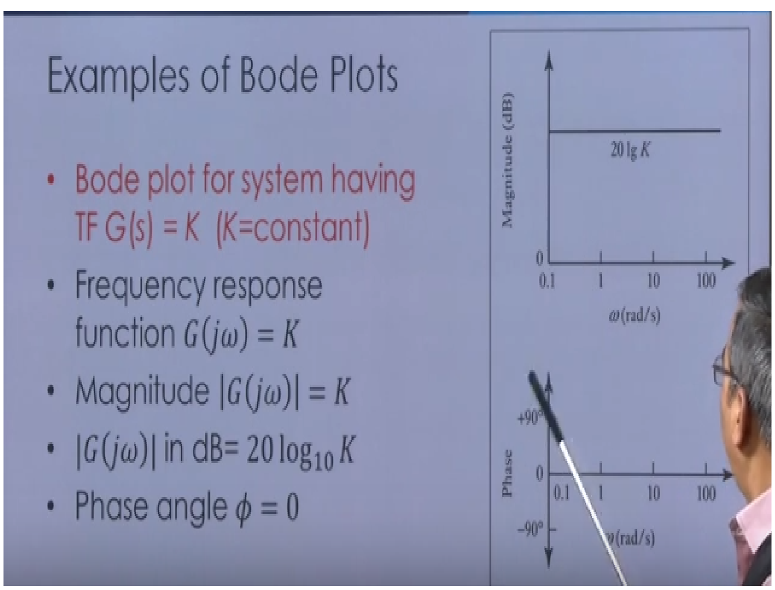

This magnitude will be actually under root x square+y square okay, where x and y are the real and imaginary component of Gj Omega okay, so here the real component is K and imaginary component is 0, so here this will be K only okay, so this magnitude is K okay, and if I write it in terms of decibel, this is 20 log 10 K fine and the phase angle here is going to be 0 because tan Phi is actually y/x and the y component here is 0.

So your phase angle is going to be 0 okay, so if you see if we see the variation of the magnitude and phase angle with angular frequency then here you can see that the magnitude variation with Omega okay and this is what we are plotting that is 20 log K, so 20 log this is on base 10, so 20 log K and you can see that this is a constant okay, whereas the phase angle is 0 okay and so, here we are plotting for Omega=say 0.1, 1, 10 and 100 that is these are numbers or multiple of 10, okay.

So, this is what how the Bode plot looks like for a transfer function Gs=K now let us look at Bode plot for a system having transfer function GS=say 1 by S okay, so again we can get the frequency response function for GS=1/S by replacing S/j omega, so Gj Omega is 1/j Omega okay, I am replacing this S with j Omega, so this Gj Omega I can write as –j/Omega, this is how we get because our G jOmega is 1/jOmega.

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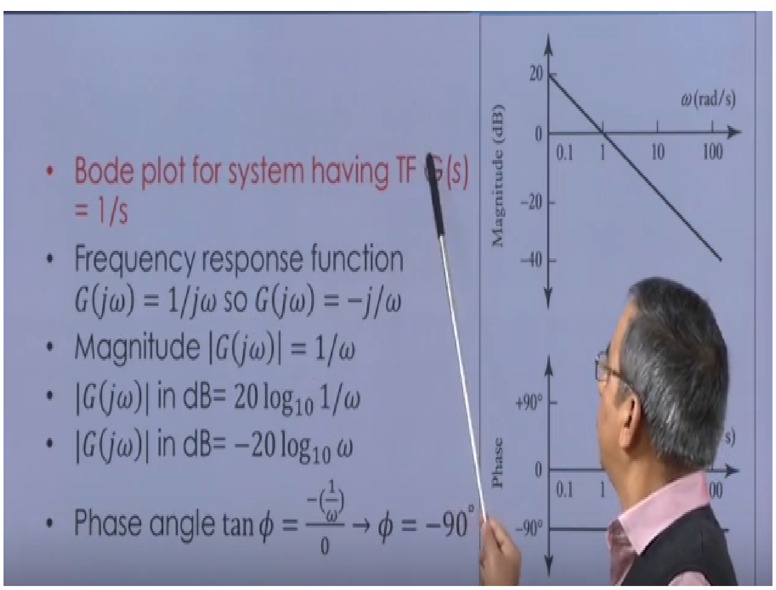

So to express it in terms of the coefficient of j what is done is that, we multiply the numerator and denominator by J, so here this is j/j square Omega and this j square is -1, so this is –j/Omega, so this is what we get –j/Omega, now the magnitude of 8 is equal to what you see Gj Omega is expressed as x+jy, so here the magnitude of j Omega is given as under root x square+y square.

So in this case this will be under root 1/Omega square or this is 1/Omega okay, so this is what is the magnitude is 1/Omega and in decibel this magnitude can be written as 20 log 10, 1/Omega and this is basically 20 log 1–20 log Omega and log 1 is 0, so what we have finally is -20 log Omega okay.

So, we can see this is the magnitude in decibel and now if I plot it magnitude in decibel, then you can see that when I put the Omega value as 1, this is what I am going to get that is 0, so the magnitude is 0 corresponding to Omega=1, similarly if I put Omega=0.1, this magnitude from here is going to 20 and similarly I can plot this for the other value of magnitudes of Omega.

Now coming back to the phase angle Phi, we know tan Phi is y/x from here and Phi will be y/x and so, y here is -1/Omega, y component from here and x component is 0, so from here we get Phi is equal to -90 degree okay, so I can plot the phase here that is variation for it with Omega as constant and that equal to -90 degree okay. Now next let us see the Bode plot for the first-order system okay.

So, let us assume the system have got a transfer function Gs=1/tau s+1, where tau is the time constant okay. We get the frequency response function from this transfer function by replacing s/j Omega, so Gj Omega=1/j Omega tau +1, now I can represent it in the conventional complex number representation of x+jy, I multiply the numerator and denominator by 1-j Omega tau okay.
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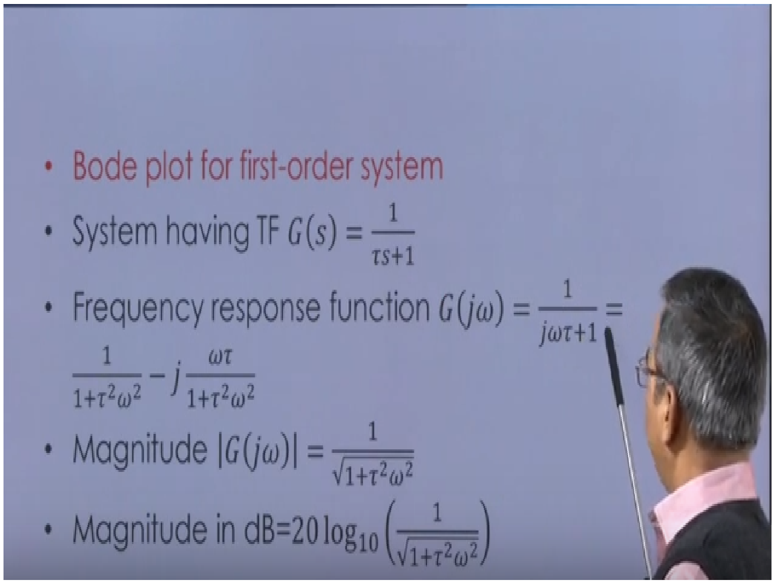

So this is what I get okay, so 1-j Omega tau in the numerator and the denominator, I will be getting 1+Omega square tau square, so 1+Omega square tau square in the denominator, so this is what I am going to get okay, so we have this portion is actually the x component and this portion is the y component, so I can find out the magnitude here=under root x square+y square and if I do that this is what I get 1 by under root 1+tau square Omega square.

Now, I can write this magnitude in decibel as 20 log 10 1/under root 1+tau square Omega square, so this magnitude is magnitude in decibel is this 20 log 10 1/under root 1+tau square Omega square okay, now we can see that when Omega tau, this is very less than very much less than 1 or Omega is very much less than 1/tau, then what happens I can neglect this value because it is very much less than as compared to 1, so this whole value becomes 1 okay.

So what we get magnitude in dB as 0, so Omega tau or tau Omega very much less than 1/tau, this is what value of magnitude we get that value is=0 okay, now there can be another case when Omega tau is very much >1 or say Omega is very much >1/tau, then in that case this value will be basically I can neglect this one in that case, so what I will have is 1/root of tau square Omega square or 1/ it will be tau Omega.

So I have got 1/tau Omega or which I can write as -20 log 10 tau Omega okay, so this is what I can write, so if I plot this okay, then this is what I am going to get, so this is the straight line approximation for plot of this portion and this is a the plot for this portion okay and you can see that the intersection of these 2 straight line what we call as the break point or the corner frequency.

So this is what it is called. Now, next let us look at the frequency response function as I was telling you we had this value of the frequency response function and this value is my x value and this value is my y value okay, so the phase of the first order system from here I can write the phase tan Phi=y/x, so this is my y component with a minus sign and this is my x component, so what I get is minus Omega tau okay.

Now, you can see that at low frequency say Omega less than 0.1/tau, this value is going to be very near to 0, so tan Phi=0, so we have the Phi=0 okay, so if I plot this phase with Omega say Omega less than 0.1/tau up to here this value is almost 0 okay. Similarly at higher frequencies that is say Omega > 10/tau okay or Omega tau >10, this Phi value will be equal to -90 degree okay.

So here you can see that Omega value > 10/tau, this value is going to be equal to -90 degree and between these 2 frequencies, phase can be approximated as a straight line, so this way we can draw the Bode plot for the first-order system. Now, let us see Bode plot for a second-order system okay, so if we have a second order system say with the transfer function Gs=Omega n square/s square+2 Zeta Omega ns+Omega n square.

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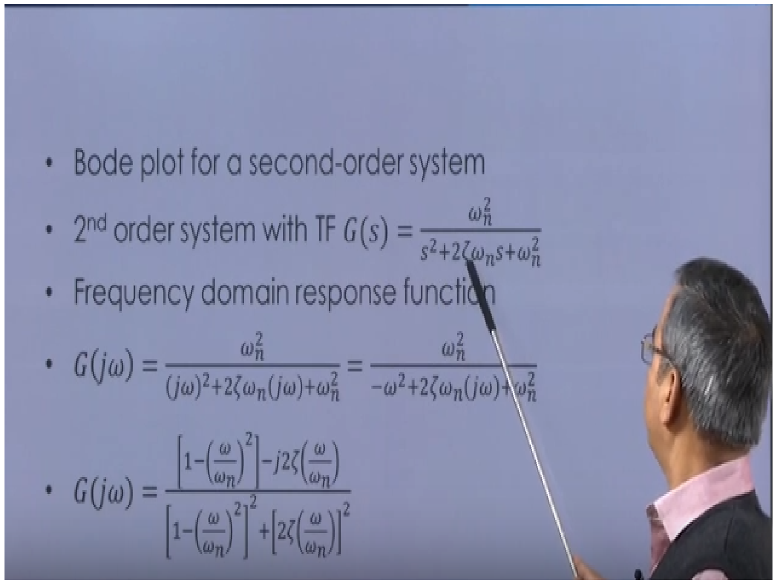

Now, the frequency domain response function can be given for the second-order system by replacing S/j Omega okay, so if I do that I replace S/j Omega, this is what I get Gj Omega=Omega n square by here j Omega square+2 Zeta Omega n in place of s, it will be j Omega+Omega n square okay or here I can write this as Omega N square, here it will be actually J square, Omega square and j square is -1.

So, we have minus Omega square+2 Zeta Omega and j Omega+Omega n square okay and again I can write in the conventional form of x+jy by multiplying numerator and denominator by say Omega N square-Omega square-2 Zeta Omega and J Omega in the numerator and similarly in the denominator. So, if I do that this is what I get Gj Omega as 1–Omega/Omega n square-j2 Zeta Omega/Omega m okay.

This is what the denominator, which I am going to have. so this is my J Omega value. Now if I am asked to find out the magnitude for this, again the magnitude will be given by under root x square plus y square, so if I do that that is what is x component will be this divided by the whole and similarly the y component will be this one divided by the whole okay, so if I do this, this is what I am going to get 1/under root, 1-Omega/Omega N square whole square+2 Zeta Omega/Omega n square here okay.

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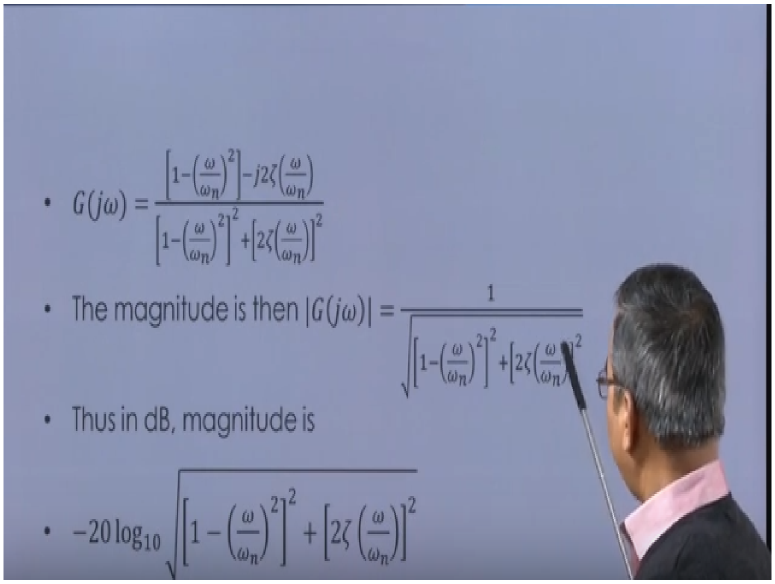

So, this is there thus in db, this magnitude can be written as -20 log 10 under root this whole factor that is 1-Omega/Omega n square, whole square+2 Zeta Omega/Omega N whole square. Now let us see the behavior for this that is the plot of this magnitude with Omega/Omega and so here we have in x-xs Omega/Omega and in y-xs magnitude is there plotted now here you can see that for Omega/Omega n very much less than 1.

So if this value is very much less than, I can neglect this, I can neglect this, so what remains with me is 1 and that is log 1 is going to be 0, so this is what I am going to get this line okay, when Omega/Omega n is very much less than 1. Similarly if your Omega/Omega n is very much >1, then magnitude approximates to -20 log 10 omega/omega n square okay, so this is what we are going to have because in that case I can neglect this term.

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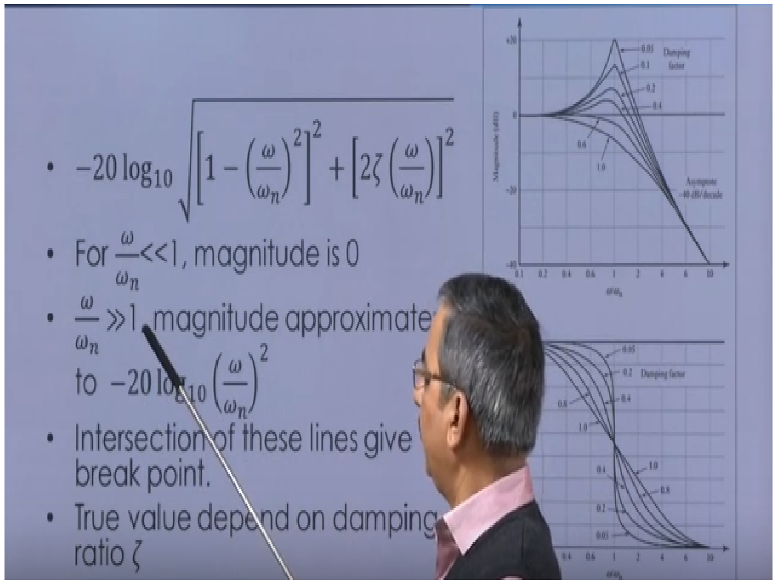

I can neglect this one okay, so this is what I am going to have -20 log 10 and omega/omega n square okay. So, this is what I am going to have now this intersection of this one and this one actually gives us a straight line approximate straight line okay and this both the intersection actually will give you the break point. Now the true value actually the true value here depends on what the damping ratios are here okay, so for the different damping ratio, this has been plotted.

So, this was our Gj Omega function okay and in this case the phase coming to the phase, this phase is given by tan Phi=y/x, so this portion is y with a minus sign divided by x is this one, now if I am interested in plotting say the phase with Omega/Omega and so let us say that initially Omega/Omega n is very less than 1, say 0.2 and in this case if this is very much less than 1, I can neglect this okay, and so this is going to be approximately a very small.

So, Phi value I can take as 0, so here for Omega/Omega N less than 1, you have this value almost 0 and if this omega/omega n is very much >1 okay then say this value is Phi and so this value is since going to be high, we can take tan Phi is infinity and so Phi=180 okay and so this value will be 180 and when omega=omega n, this value becomes infinity and so phi=-90 degree okay.

So here you can say this value corresponds to phi=-90 degree, now when we are building up a Bode plot okay, we can do so for a system, which is composed of many subsystems okay and say we have a system that has got transfer function Gs=G1s, G2s, G3s and so on, so the frequency response function for 2 element system will be given by I can replace this S by say GJ Omega is equal to this is G1j Omega x G2j Omega.

Now this G1 J Omega I can write as magnitude of G1j Omega x cos Phi 1+j sine Phi1, where Phi1 is the phase angle. Similarly, I can write G2j Omega as G2j Omega x cos Phi2+J sine Phi2, where this Phi2 is the phase angle, so the product of these 2, G1j Omega and G2j Omega is given by this and we can substitute these 2 expression back here, so for j Omega, I have G1J Omega cos Phi 1+j sine Phi 1 and for G2j Omega, it is G2j Omega magnitude into cos Phi 2+j sine Phi2.

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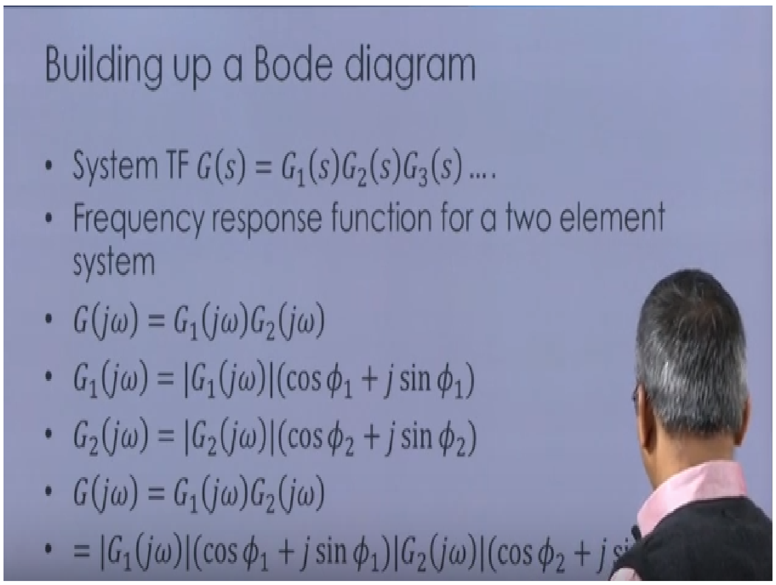

So, this GJ Omega is this one, the 2 magnitudes we can keep together and here are the cos phi 1+j sine phi 1 x cos phi 2+j sine phi 2 okay, so now if I split up this bracket, so what I will be getting cos Phi1 x cos Phi2 and what I will have is here, I will multiply this with this one, so I will have -sine phi 1 sine phi 2, which will be basically cos phi1+phi2.

Likewise, I will have the multiplication of cos phi1 with sine phi2 and cos phi2 with j sine phi1 will be giving me j sine phi 1+phi 2 okay. So, for Bode plot this is about the combined magnitude, so in the log scale I can take this log 10 GJ Omega and since these are the product its logarithmic is going to be sum, so we have log 10, a magnitude of G j omega+log10 magnitude of G2j Omega.

So for this type of system we can just add the Bode plot for the individual subsystems in order to get the Bode plot for the whole system. So let us see an example of building up a Bode diagram, so the bode plot of Gs=10/2s+1, if I want to do okay. I can basically see here that I can split up this whole transfer function into 2 transfer function that is 2 transfer function for 2 subsystems, one given by G1s=10 and the other given by G2s=1/2s+1.

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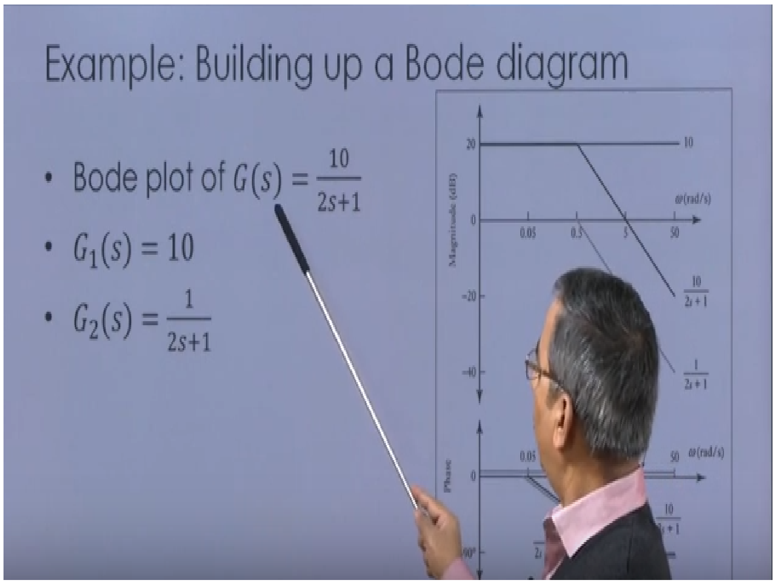

So, this is very much similar to what we have seen that is Gs=K and this is what we have seen in the previous slides of Gs=1/tau s+1 okay, so I can replace this s/j Omega and then I can draw the Bode plot. So for the first one, you can see that this is constant so for the 10 this portion is going to be the Bode plot for the first one and similarly for the second one this is the bode plot 1/2s+1.

I have shown you in the previous slide for the combined that is a transfer function G1sx G2s, we can get the Bode plot by simply adding up those, so I add up this with this one, so this is what I get so this represents the magnitude in the Bode plot for the transfer function 10/2s+1. Similarly for the phase in case of the first one okay, we had phase as 0 and for the second one this is the phase okay.

For the combined one, we will be having this that is the darkened one as the phase 1. So for higher values, this becomes -90 degree, lower value this is 0 and in between value is given as a straight line and this and 0 summed up will be giving you the same thing. Now coming to the performance specification okay, so if we want to indicate the performance of the system, there are 2 parameters.

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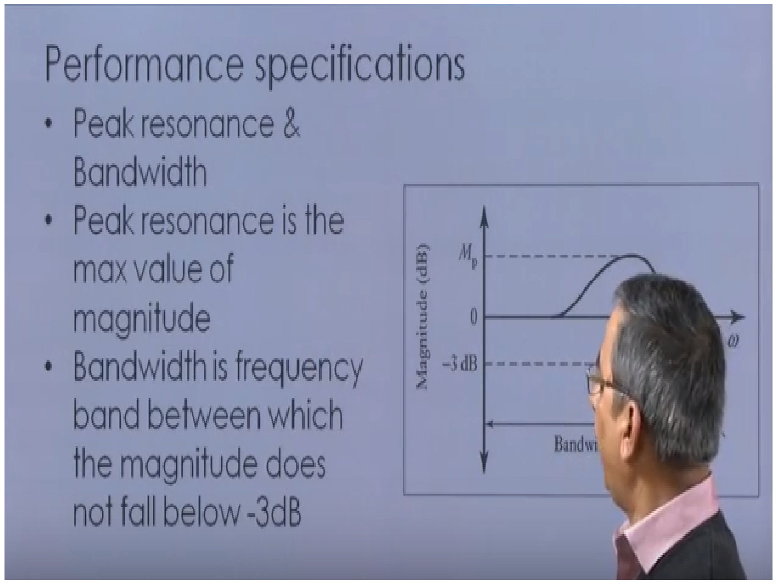

With the help of which we can indicate the performance for the frequency response okay and these are the peak resonance and the bandwidth okay, the peak resonance basically is the maximum value of the magnitude, so as you can see that in this plot we are plotting the magnitude in decibel with respect to Omega and say this is the system response.

So here the peak value is actually the peak resonance is the maximum value of the magnitude that is what is called the peak resonance and the bandwidth is basically defined as the frequency band between, which the magnitude does not fall below -3db, so say if this is our 0 value here, so say after this, the magnitude falls below 3 db-3 db, then this is going to be the bandwidth for the system.

So, with the help of these 2 parameters that is the peak resonance and the bandwidth, we can specify the performance of the system okay. So this is what I wanted to tell you in this subtopic of further reading, you can refer a very nice book by professor Bolton that is Mechatronics by Pearson Education. Thank you.