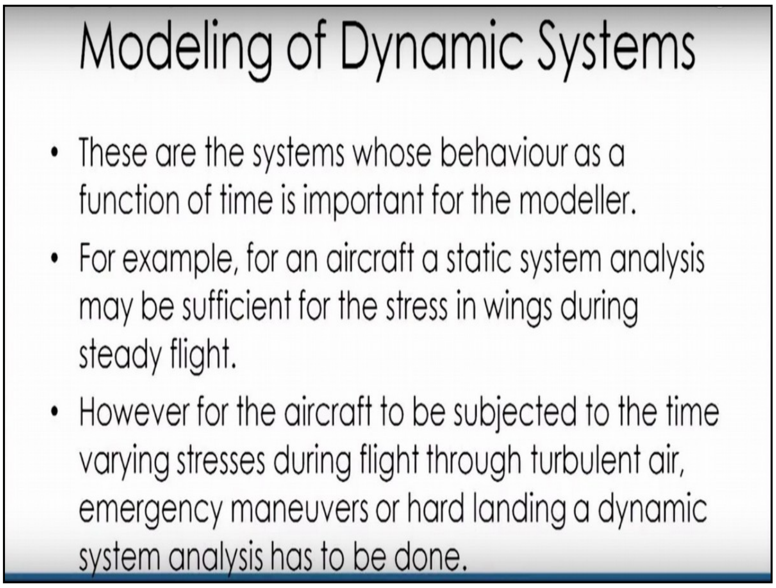
Modelling and Simulation of Dynamic Systems
Dr. Pushparaj Mani Pathak
Department of Mechanical and Industrial Engineering
Indian Institute of Technology - Roorkee

Lecture - 3
Modelling of Dynamic Systems

In this lecture on modeling of dynamic systems which is a sub module of modeling of course on modeling and simulation of dynamic systems. So let's go through about the modeling of dynamic systems basically.

(Refer Slide Time: 00:49)

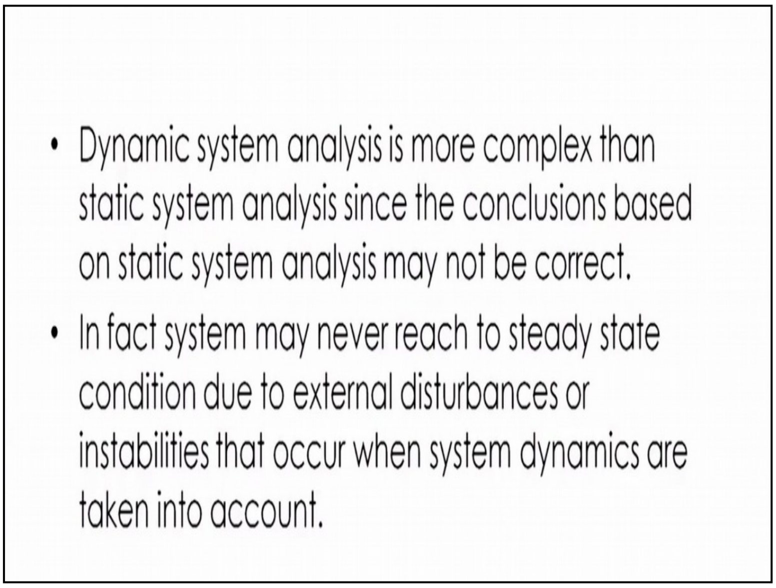


The dynamic systems are the systems whose behavior depends on the time. So, this is the systems whose behavior and function of time is very important for the modular. We can take some example of the dynamic system say be a moving car; or say a moving aircraft.

Now if I just take the example of an aircraft now, in this aircraft there would be analysis based on the static one or based on the dynamic one. For example, if say I the aircraft is moving at a steady speed then if, we are interested in finding out the stresses on the wings we may go for a static analysis. But, the results of this static analysis may not be accurate if you want to predict what happens to the aircraft in case of the dynamic situations such as say wing, the air craft is passing through a turbulent joint are the caption has to do certain emergency menu over on the aircraft when they are pilot dash an emergency landing.

So actually, this is the dynamic situations dynamic situations and to predict the behavior of the system; the system has to be model taking care of the dynamics of the system into consideration. Now, as all of us know the dynamic system analysis is more complex than the static system analysis since the conclusion based on that static system analysis may not be correct always.

(Refer Slide Time: 02:57)



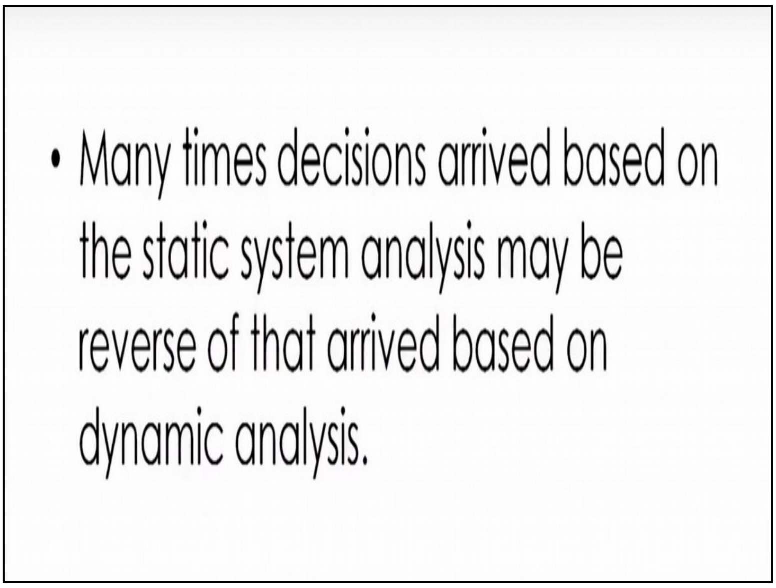
All of us know the case of the simple extension of rod if, we consider the static consideration then the states which are developed in a rod are going to be equal, two is say b by a where b is the apply load and a is the area cross section of rod.

But, if I take the case of an impact which is a type of dynamic loading then all of us know that, the load which is going to come on the rod is going to be two times of this one; two times of that of the static case. Likewise, when certain vehicle is passing through say the bridge then what ever loads or coming on the bridge that load going to be different then when a vehicle is part is stationary on the bridge.

So this is what is the difference in the modeling that is static modeling and the dynamic modeling is the results which, we get from the static and dynamic modeling. In fact, when we do certain static analysis the considerations is that the system has reached the steady state. For example, suppose we want to predict that what is the fuel consumption when the vehicle is moving in a steady state? Now, all of us know that the fuel consumption is more when we start the vehicle.

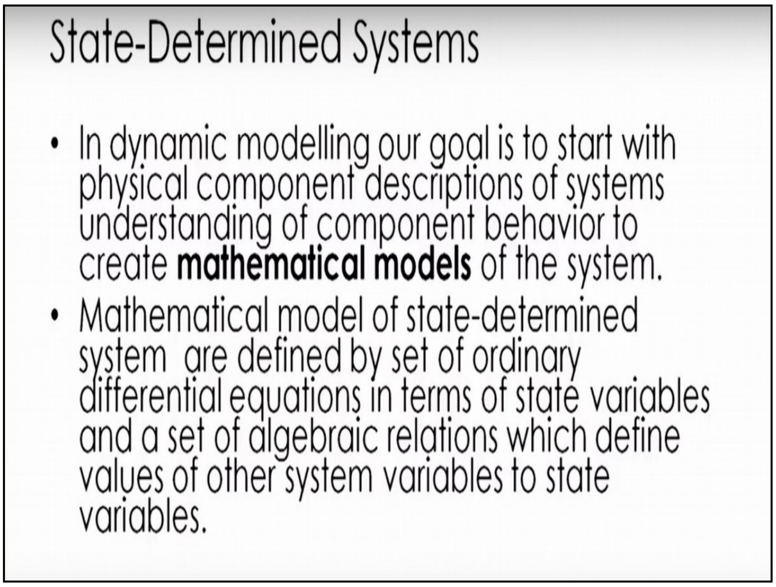
So the thing is that, to be very frank and practically speaking; we may never have the steady state condition and this steady state condition is not possible because of many reasons; may be the disturbances from the environment or some disturbances from the system within. So, we need to consider the dynamic of the model in other to predict the correct behavior of the system.

(Refer Slide Time: 06:00)



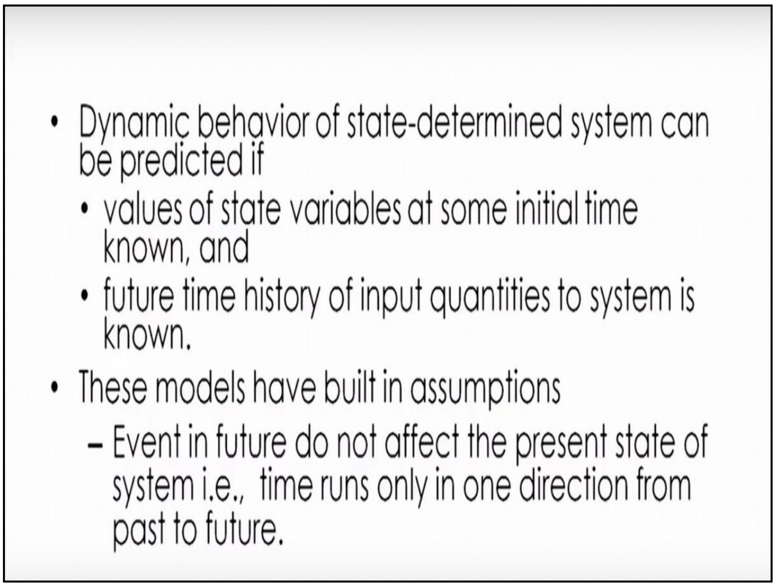
Many times the decisions which are arrived based on the static system analysis may be reverse of that which has been arrived based on the dynamic analysis and this further messages for the dynamic analysis and do not go by the intuition. Now, when we are talking about the dynamic modeling now in this dynamic modeling, our principle goal is to start with the physical component descriptions as a talk to you in my first lecture describing the system and understanding of the component behavior in order to create mathematical models of the system.

(Refer Slide Time: 06:35)



Now, this mathematical model of state determined systems are defined by set of ordinary differential equations in terms of state variables and a set of algebraic relations which define values of other system variables, to state variables. So this is how state space equation is represented.

(Refer Slide Time: 07:37)



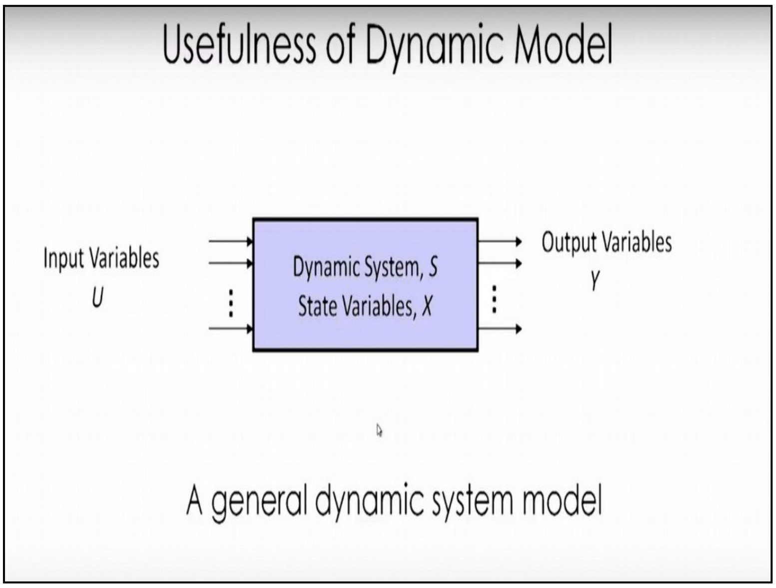
Now, the dynamic behavior of state determined system can be predicted if these two things that are the values of is state variables at some initial time. So, if the known values of the state variable of some initial time and also future time history of input quantities to systems is known, we have these two things, and then we can predict the dynamic behavior of the state determined systems.

Of course, as I said here basically we need to know what are the inputs which are going to come in the systems time history of the inputs and, the values of state variables at a certain initial time now this models that is the state determined system models have built in assumptions and this built in assumptions that the event which are going to occur in future they are not going to affect the current that is the events in future do not affect the present state of the system.

This is assumption in build in assumptions in these models. Or, we can say in otherwise, that time runs only in one direction from pass to future and does not a comes from the future to past. Now, what are the useful of dynamic model, we will be taking up various dynamic models which are residing in different energy domain in my coming lecture such us electrical energy domain mechanical energy domain hydraulic pneumatic and so on apart from that, will be taking up the systems which are integrated.

Integrated in the sense that, the multi energy domains; sub systems constitute a bigger system. So, that is what is going to be our aim. Now, the dynamic model which we get what is the usefulness of this dynamic models, what all we can do with the help of these dynamic models let us see so in this figure explains you a general dynamic system model.

(Refer Slide Time: 10:26)

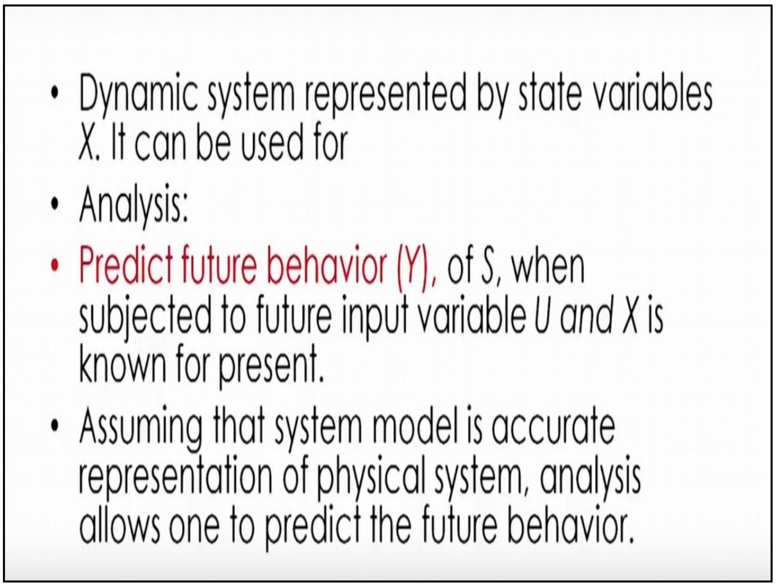


So, this general dynamic system model is represented by say s and this model is in terms of state variables x. So, here what I am doing is I am writing the dynamic system model which am calling it by x and this model is written in terms of state variable x. Now this dynamic system models x is subjected to say the input variables u. So there are set of input variables which are being sent into the dynamic system. Now, when this input variable are pass two dynamic system say we get the output variables y. Now, when this input variable are pass two dynamic system say we get the output variables y.

This constitute a general dynamic system sub model, were we have a dynamic system as return in terms of state variable x and this is subjected to a certain input variable and the output of this dynamic system when it is subjected to the input variables u is the output y. Now, what all we can do with this dynamic models, as I said the dynamic system represented by state variables x, and this can be used for the analysis.

The four most important usefulness of the dynamic model is in fact in the analysis.

(Refer Slide Time: 11:59)



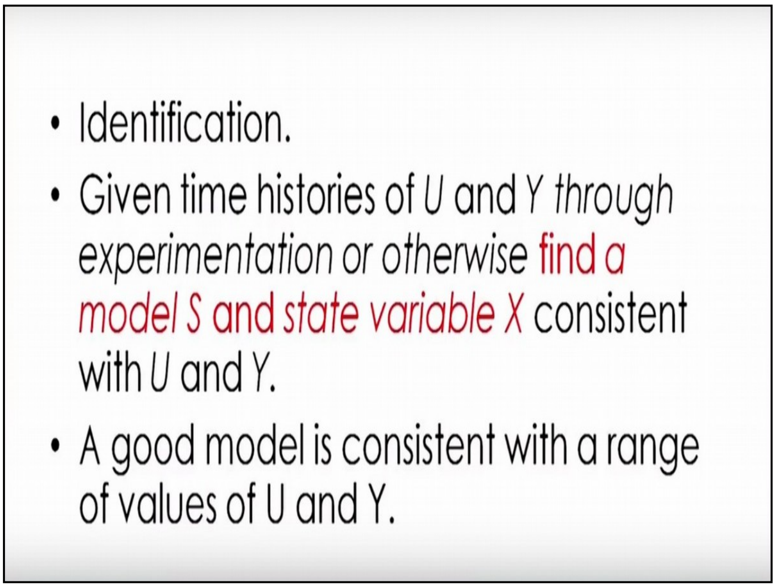
What does it means by analysis, here we predict the future behavior y of the system as and when subjected to input variable u and x is known for the present that, is we know the state variable x and we know the present value of the state variable x, we know the system x then, for the given value of the input variable u what is my output variable y and this is what we call it as the predicting the behavior of the system.

This is one of the foremost usefulness of the dynamic model that is with help of the dynamic model we can predict the future behavior of the system. And of course this prediction is going to be true based on the assumption that, whatever model we have created that model is so correct enough. So this system model accurate representation of the physical system analysis allows one to predict the future behavior.

So, how accurately we have modernized our system based on that the accuracy of the prediction is going to be. So, if you have very rightly modelled then your system behavior is going to be very accurate, and as I told in my first lecture when we go for the modeling of a physical system there are lot more assumptions built into and this assumption again are the factors which we consider in the modeling, those factors are important from the model point of view.

I give you a simple example if, suppose in some situations we want to say test aircraft model say in a internal, then we are not bothered about to say the air conditioning system of the aircraft because, if I am interested in studying about dynamic behavior, am just interested in what loads, what stresses are and what loads are coming on the wings of the aircraft. So this so how accurately you have modelled your system based on that the accuracy of the output depends and this is what we called as the analysis.

(Refer Slide Time: 15:45)



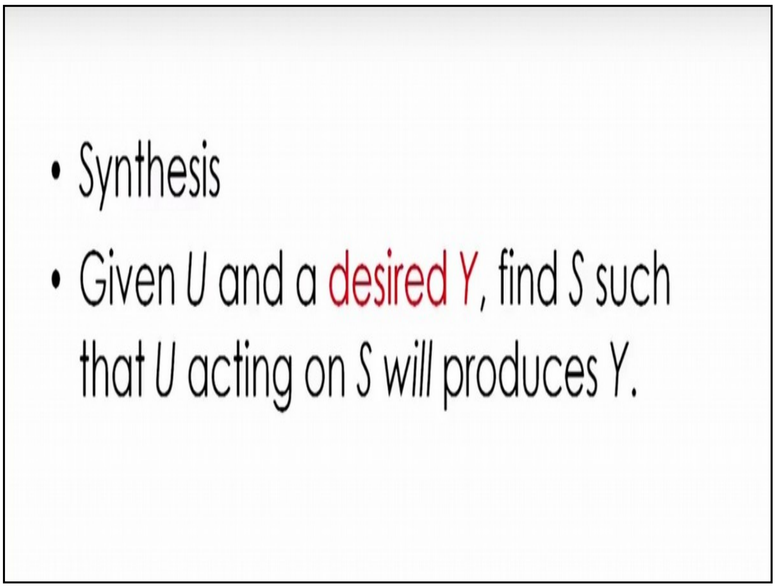
The next usefulness of the dynamic model is the identification. The situation is like this many time what happens, we have certain input to your system and we get certain output of the system but, we don't know what is happening inside of the system, that is if I talk about in terms of the parameters which I just discussed here so for the given time history of u and y that is I know the u and y through experimentation.

I know u and y through experimentation I give certain inputs and I get certain outputs and I measure that outputs of course through experiments. So, for a given time history of u and y through experimentation or any otherwise or through some other means the task in identification is to find the model and the state variables x consistent with u and y.

So here in identification we try to identify the state variable x of the model and the dynamic model itself, for rather you can say that we try to find out the dynamic model for a given input and output. So given time history of input and output we try to find out the dynamic model as and the state variable x. This is what is called as the system identification.

A good model is consistent with a range of values of u and y it is not that the model which you have developed it, as it is valid only a certain limited values of u and y know as if it is there then definitely it means that the dynamic model is not correct. So, in identification this is what we do. Next important usefulness of the dynamic model is on synthesis. Now in synthesis what is done is that, you are given a u and you have desired value of the y.

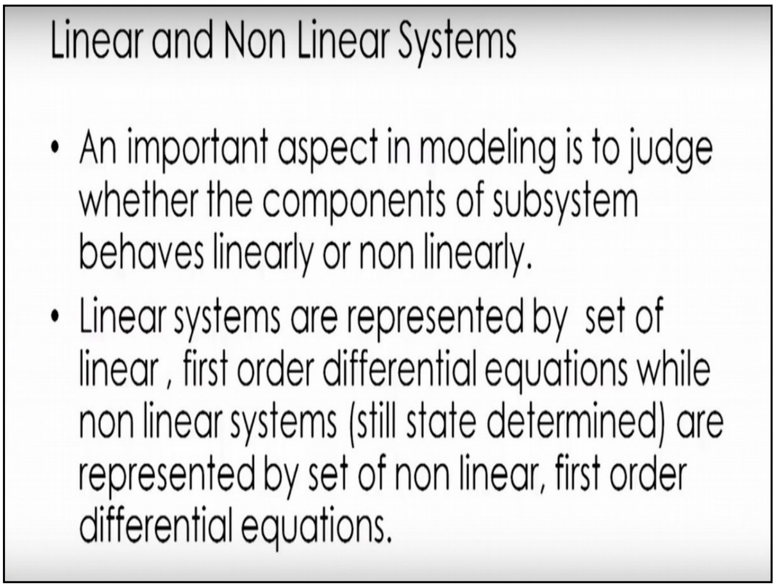
(Refer Slide Time: 18:13)



Now, you know the input and you know the desired output. What output you want from the system now, for this condition we can find out as that is the system such that when the input acting on the system it will produce the desired output and this is what is called as the system synthesis.

So, these are the three principle applications usefulness of the dynamic model of the systems; one is the analysis where essentially we want to predict the system behavior then, the next one is the identification where I have given input and output, we try to indentify the system and the state variable itself, and the third is the synthesis we are given input and a desired output we try to find out what the system is as. Now, before we proceed further let's look at the linear and non linear systems.

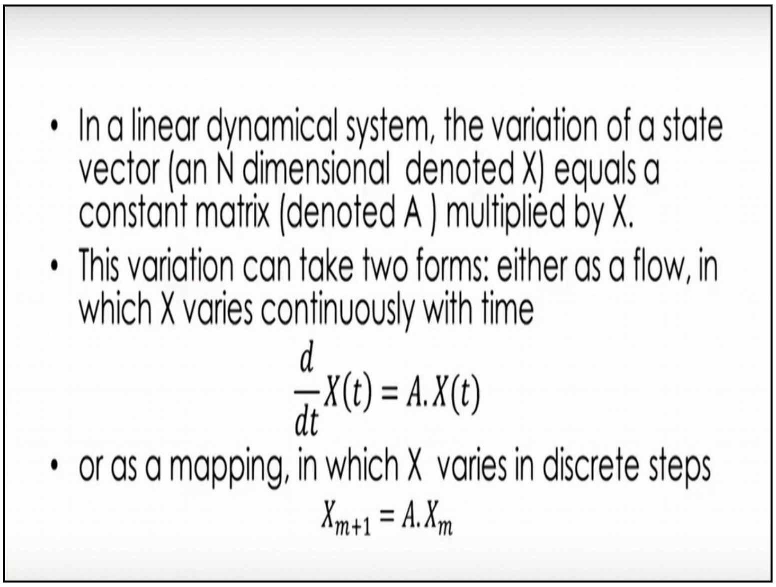
(Refer Slide Time: 19:38)



An important aspect in modeling is to judge whether the components of a system behaves linearly or it behaves non linearly, to be very frank whatever system does exist in nature it is non linear and to make our life easier many times we make certain assumptions to make the systems linear and then we carry out our analysis.

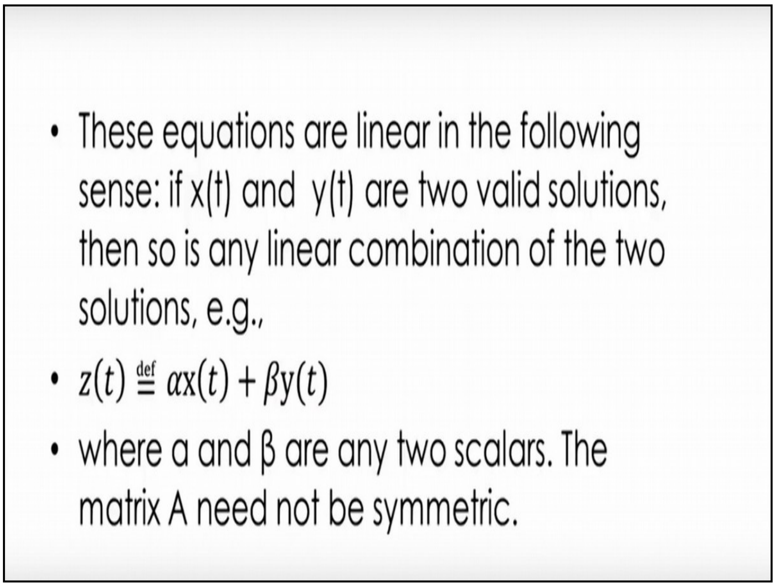
Now this linear systems are represented by set of linear first order differential equations; were as the non linear systems of course which are still state determined are represented by set of non linear first order differential equations this is the linear and non linear systems as represented mathematically.

(Refer Slide Time: 20:53)



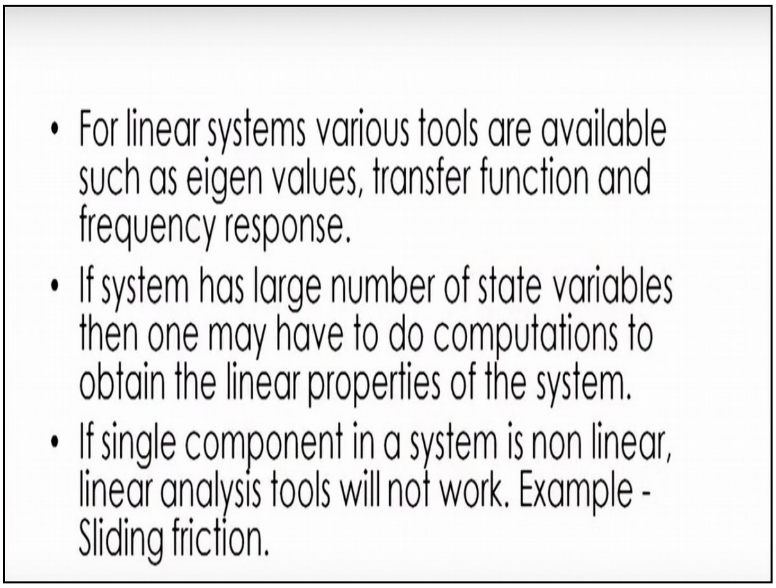
Now in a linear dynamic system the variation of a state vector of course e n dimensional denoted by a, equals to a constant matrix denoted a multiplied by x this is what there and this variation can take two forms either as a flow in which x varies continuously with time that is the equation represented as d by dt of x t is equal to a xt or it could be as a mapping in which x varies in discrete steps that is a in to x m equal to xm+1. So this is how the state is space equation look like.

(Refer Slide Time: 21:50)



These equations are linear in the following sense, the general expression for the linear equation or the general property of linear equation you can say that, if xt and yt are two valid solutions then so is any linear combination of the two solutions is also going to be the solution of the system. That is if, xt and yt are to valid to solutions then say zt which is defined as sum alpha time xt plus beta time yt where alpha and beta are the two scalars that is also going to be solution and note that the matrix a which I defined earlier need not be symmetric. Now, the advantage with the linear system is that, there are various tools available.

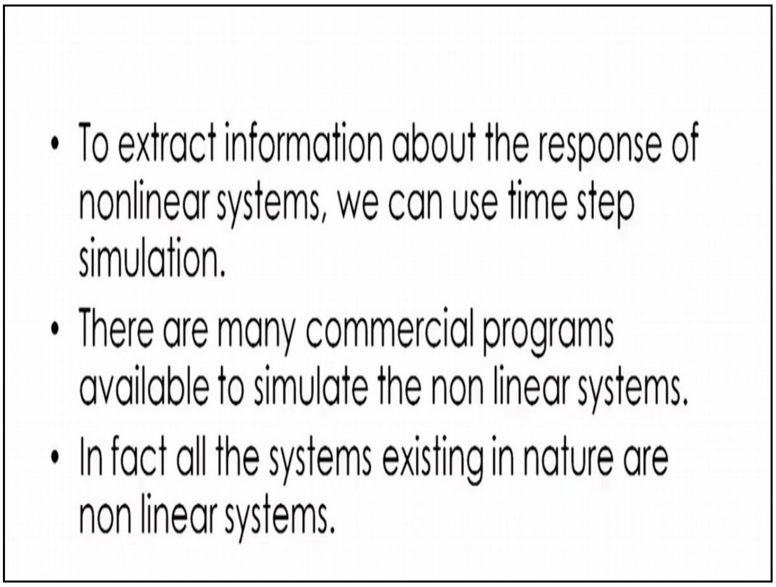
(Refer Slide Time: 22:58)



Which one we can use for the analysis, of various tools available. These tools are such as eigen values transfer function frequency response, so with the help of these tools we can analyze the linear system, and if system has large number of state variables then naturally the we can no longer go for a computation, we have to take help of some computational device to obtain the linear properties of the system.

During these modeling, we need to always keep in mind that, if even a single term of the equation, differential equation becomes non linear your system becomes non linear. So, if a single component in a system is non linear the linear analysis tools will not work. For example, if you try to model sliding friction naturally your system is going to be a non linear system.

(Refer Slide Time: 24:23)



The exact information about the response of the nonlinear systems. We can use the time step simulation to find out the exact response of the non linear system and if you are if you known the say operating point or you want to know the behavior of the non linear system at certain zones, certain location then about that location we can linearise that non linear system and predict the behavior. So, that is also a way of doing it that is converting the non linear system into a linear system and then predicting the behavior.

There are many commercial programs available to simulate the non linear systems in fact all the systems I said existing in nature are non linear systems. Thank you