Modelling and Simulation of Dynamic System
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Lecture – 27
Signal Flow Graph

I welcome you all in this lecture on signal flow graphs, which is a part of the modeling and simulation of dynamic system course in our previous class we have seen the block diagram method of finding out that transfer function of a system okay. There are certain problems associated with the block diagram that is if the system becomes large then, finding out of the transfer function is difficult okay.

Because one need to resort to the reduction of the block diagram and then in subsequent steps and then find out the transfer function, so the signal flow graph overcomes this drawback of the block diagram because here we have got a formula for finding out the transfer function okay. So let us begin signal flow graph is a graphical representation of the relationship between the variables of a set of linear algebraic equations okay.

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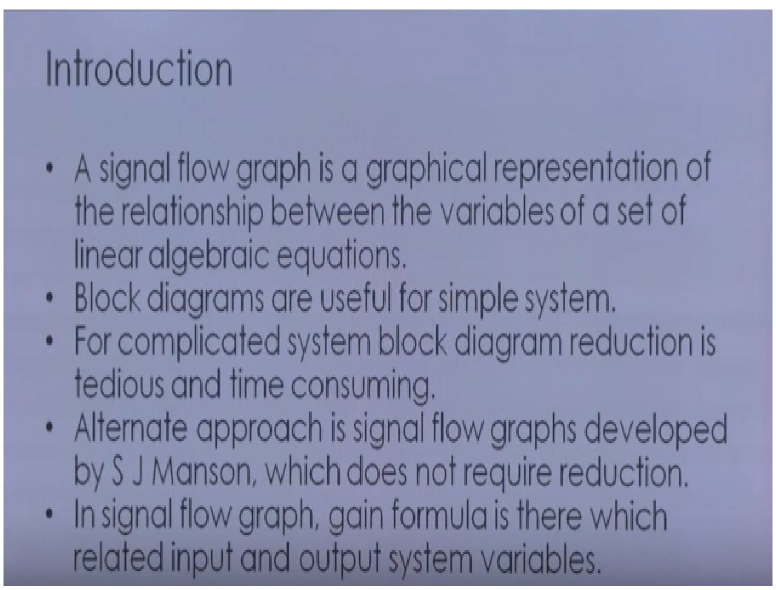

As I said block diagrams are useful for simple system and for complicated systems block diagram reduction is tedious and time-consuming, so the alternate approach as given by S. J. Manson is that of the signal flow graph okay. This was developed by Manson and this does not require the reduction so that is the advantage of it. Now in signal flow graph gain formula is there and which relates the input and output system variable.

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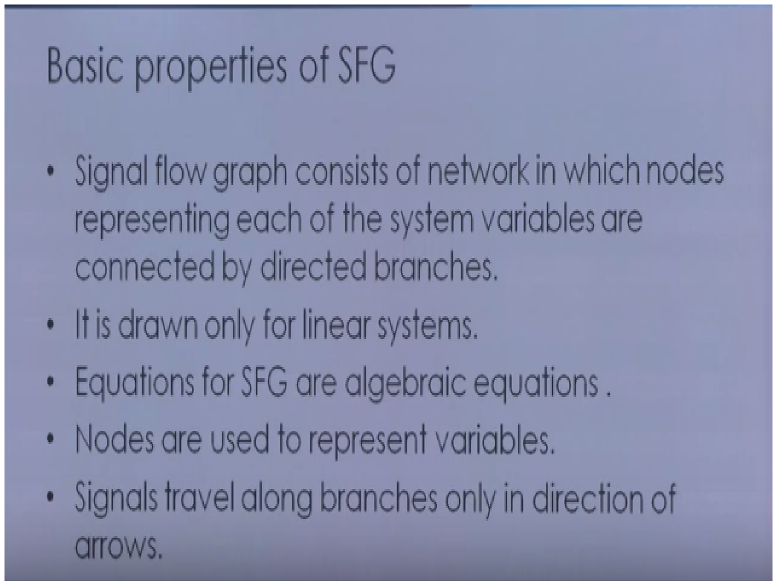

Let us see certain basic properties of the signal flow graph okay. The signal flow graph consists of actually network in which nodes representing each of the system variables are connected I directed branches okay, it is drawn only for the linear systems and equations for signal flow graph our algebraic equation nodes are used to represent the variables and signal travel along branches only in the direction of the arrows there are certain terms.

Here are in the signal flow graph say the first and most important one is the node it actually represents a system variable which is equal to sum of all incoming signals at the nodes here you can see say we have node excite node xj here and this is what we call as the branch which connects the node xi with node xj okay.

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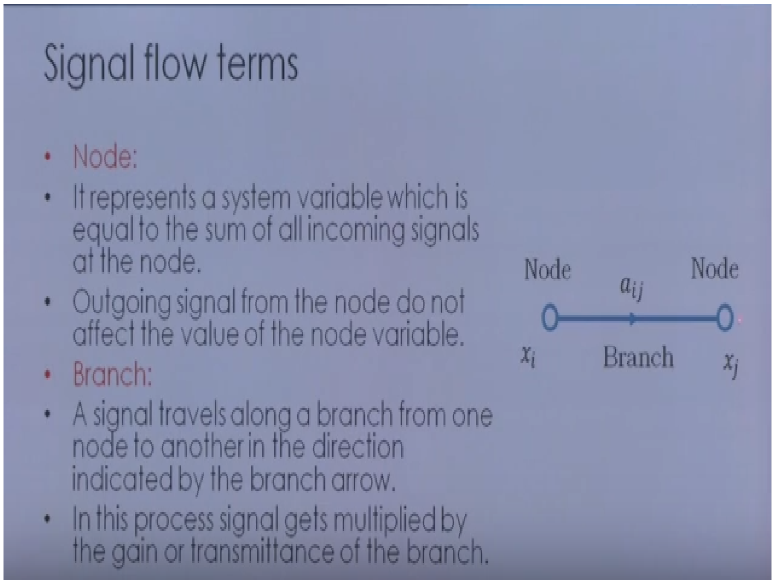

So it represents a system variable which is equal to sum of all the incoming signals at the node and outgoing signal from the node do not affect the values of the node variable. So that is there and branch as I said a signal travels along the branch from one node to another in the direction of the arrow indicated in the branch okay and in this process basically the signal gets multiplied by the gain or the transmittance of the branch.

So for example in this example xj=aijxxi, so this is how this is done then there are certain notations that is the input node or source a node that only has the outgoing branches that is called the input node or we call it also the source. Likewise the output node or sink is defined as a node that has only incoming branches okay.

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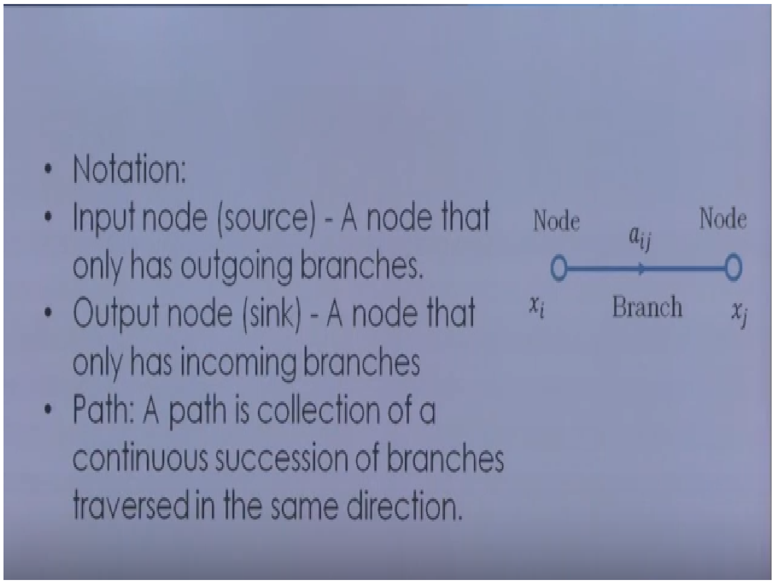

So that is called the output node or sink path is actually a collection of a continuous succession of the branches traversed in the same direction okay. So this is what is called path now there are further we can classify the path as the forward path okay the forward path is a path basically that starts at an input node and ends at the output node and along with no notice Traverse more than once, so that is what is called as the forward path.

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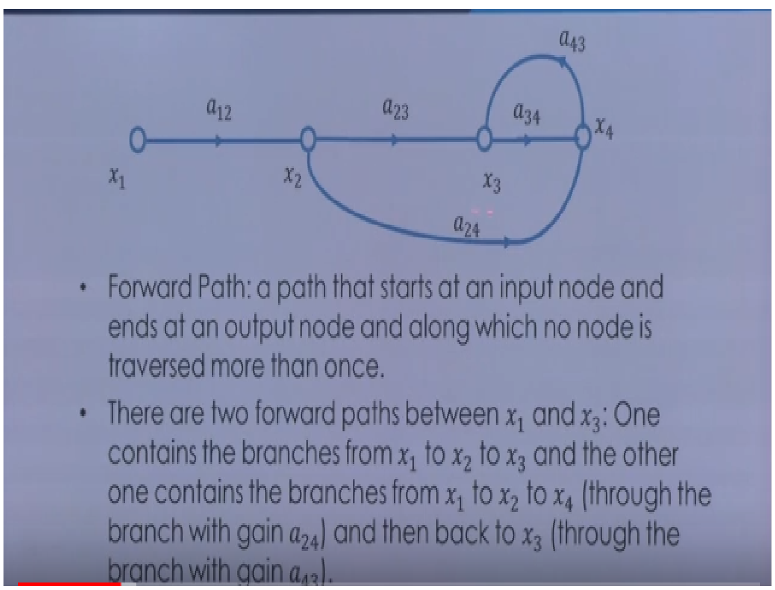

Now if we look at say this signal flow graph with nodes at say x1,x2,x3 and x4, so here say if I take the case of path between node x1 and x3, so here you can see that between node x1 and x3 there are two paths one path is basically this one x1to x2 and x1 to x3 then the another path is from x1 to x2 then it goes from x2 to x4 and from x4,x3. So this is how our there are 2 forward path between x1 and x3 here 1 contain the branches from x1 to x2 to x3 and the other contains the branches from x1 to x2 and then to x4 okay.

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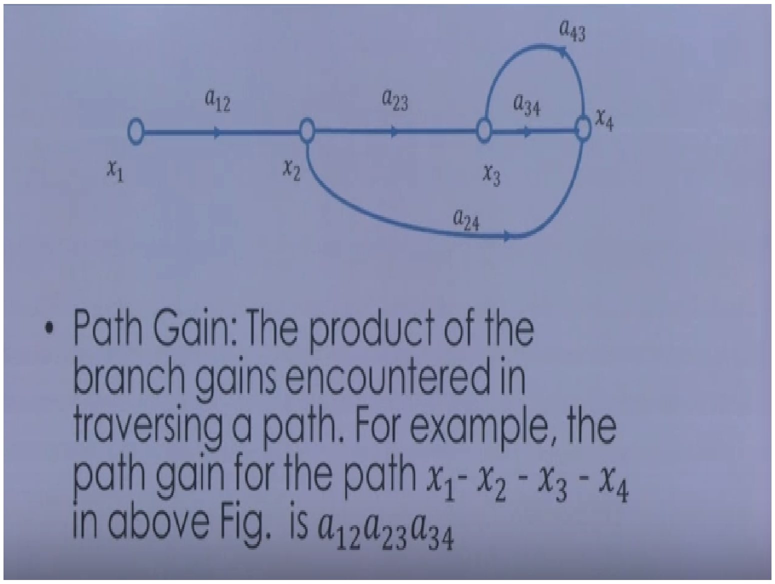

Through the branch gain a24 here and then back to x3 through the branch gain a43 this one as I explained then the path gain this is actually defined as the product of the branch against encountered in the traversing of the path okay. So for example here the path gain for path say x1 to x2 to x3 to x4 is basically the product of these gains that is a12 to a23 and a34. So this is what we call as the path game a12xa23xa34.

Then the loop actually a path loop is a path that starts and terminates at the same node okay. And along which no other node is encountered more than once okay. So this is the definition of the loop and that is the one which starts and terminates a path that starts and terminates at the same node and here you can see that this part. So if you begin from x4 and then we go to x3 and then again we come back to x4 so this is actually one loop okay.

So this is how the loop is defined that is a path that starts and terminates at the same node and along with no other nodes is encountered more than once then the forward path gain ah is the path gain of the forward path as we have seen and the loop gain is the path gain of a loop, then there is another terminology which we come across that is the non-touching loops okay.

So these are defined as actually say if I say that the two parts of a and of a signal flow graph are non touching if they do not share a common node okay so if there is no common node between the two paths then those two paths are said to be the non touching now for example are in this signal flow graph we can see the loop x2x3x2 that is this loop x2x3x2 okay.

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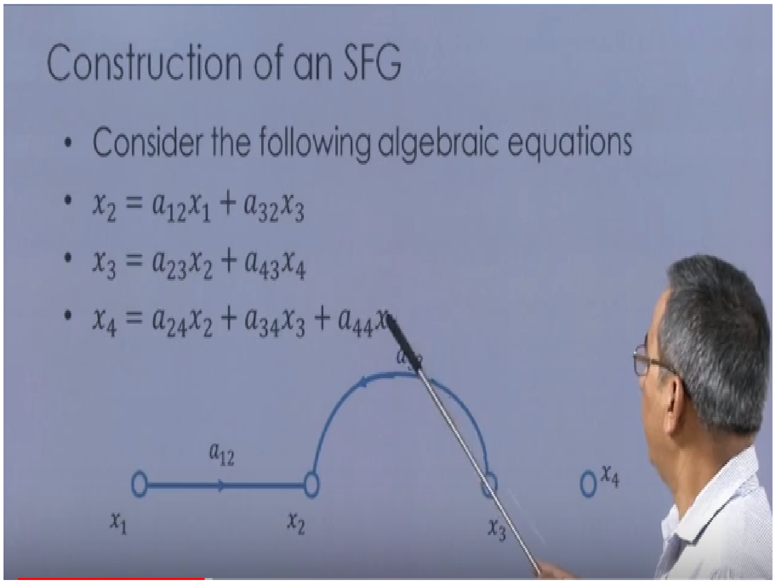

The another loop x4 sorry x4 to x4 theses are the non touching loop because they do not share a common node okay, so this way we define the non-touching loop. Now if we are asked to construct a signal flow graph from say set of algebraic equations that we can do so let us take an example to illustrate this concept say we have these three sets of algebraic equations given here x2=a12x1+a32x3x3=ax3x2+a43x4 and say x4=a24x2+a34x3+a44x4 okay.

So by seeing these set of equations we can easily identify that there are 4 nodes here okay. These nodes are say x1,x2,x3 and x4. So what we do is that we mark the nodes as you can see in this figure, so there are 4 nodes so say x1 then we have x2 and we have x3 and we have x4. So we have the four nodes here now my first equation is x2=a12x1+a32x3 okay.

So here we can see that x2 is basically this x2 is going to be a12xx1. So what is the coefficient of x1 here that is a12. So this is going to be the gain for this branch okay. Likewise + we have a32x3, so x3 is here so it is x2 a contributing to x2, so it will be this one and this is going to be a32x3 okay.

Now in order to show it we can show it this way or we can show it another way also as it is shown in the previous graph say this one that is it is a32 okay. Now after drawing this we can take up the next equation okay and our next equation is say x3=Aa23x2+a43x4 okay. So here basically this graph we already have and what we do is we superimpose on this the graph are because of the other equation.

So, I have this one already here fine now you see x3 is x2 times a23 okay. So it is this is x2, so this gain will be basically what a23 okay, so this = this x2 times this 1+x4 times a43 okay.

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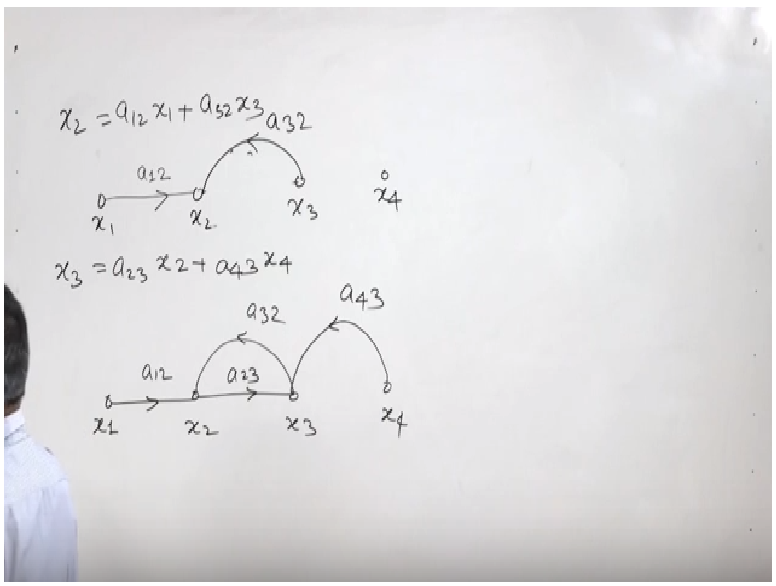

So, a43 we take it up this way, so this is again a43 and it is into x4. We can see that x3 there are the 2 input branches here that is a through gain a23 and through gain here a43. So this way we get the second equation okay, likewise we can plot for the graph for the last equation say x4=a24x2+a34x3+a44x4 again we have to superimpose on this one.

So I will just try to draw this one quickly, so these are our nodes x1,x2,x3,x4 and here this is gain a12 here we have a32 here we have a23 and here we have a43 Now you see here x4=a24 times x2 that is our x2 times a24 so x2 we can take as the input, input node and x4 will be the output node. So I can connect x2 to x4 here okay, and with the gain as a24 okay then we have a+a3 for x3 okay.

So a3 to 4x3 so I can connect this path with this as the gain and the last one is a44x4 okay. So x4 is this one and this will be basically constituting a loop something like this and this gain will be a44. So this way we can draw the signal flow graph for these algebraic equations okay. So this is the second step which I have shown here okay.

So that is there alright and then we have the third step which I have shown here so this is how we create the signal for graph from for these algebraic equations okay. Now let us see the signal flow graph algebra okay this signal flow graph algebra actually helps us in simplifying the signal flow graph okay.

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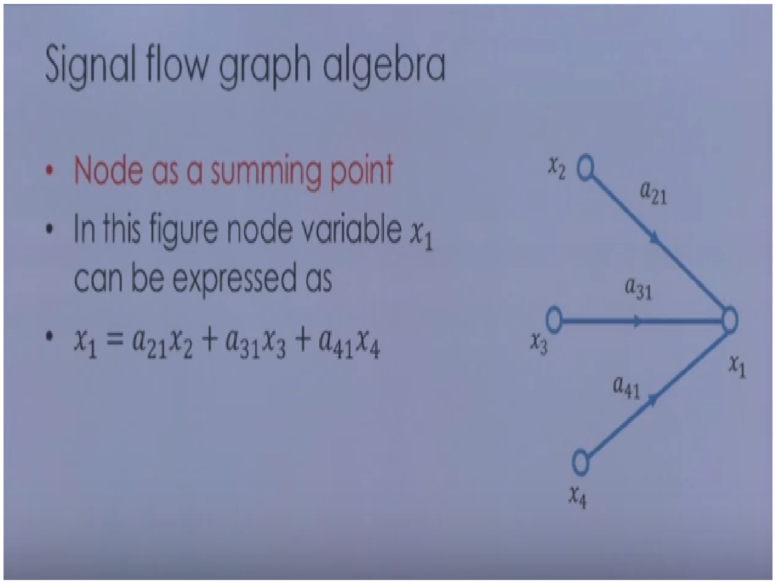

So, the nodes as a summing point. Now here you see that this node x1 in which there are branches from where the signals are coming in all are coming in, so this node x as a summing point okay. So in this figure node variable x1 you can see that it could be expressed as x2xa21+x31xx3+a41xx4 okay.

So, this way this node x as a summing point okay, then node as a transmitting point, so from a node if these signals are going out then we can consider that node as a transmitting point okay, So a node variable is transmitted through all branches outgoing from that node, so suppose we have this node x1 this is x6 and this is x5 and say signals are going out from x1 then here if this gain is a15 and this is a16 then you see we can write x5 as a15 times x1 here.

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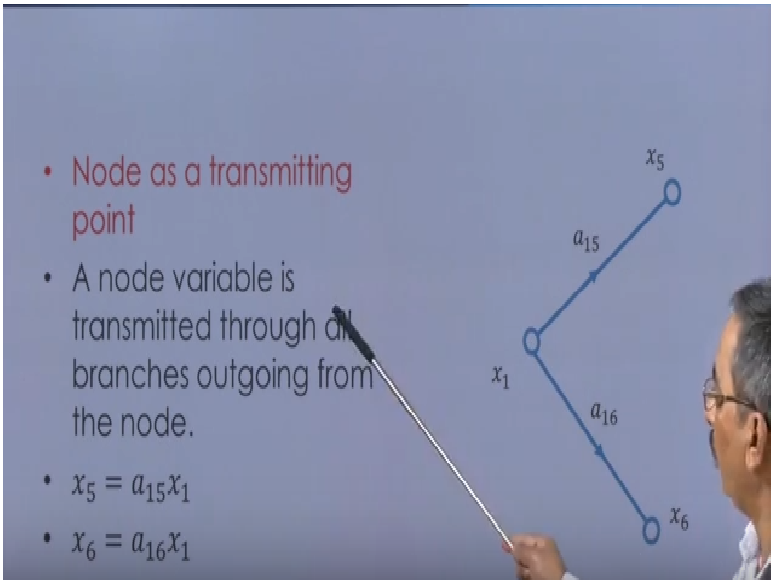

We can write x6 as a16 times x1 okay. So here you can see that this node x1x as a transmitting point then are parallel branches in same direction connecting 2 nodes are equivalent to a single branch with the gain equal to some of the gains of the branches okay. So here you see that in this signal flow graph between x1 and x2 there are 2 parallel branches one has gone got a gain of a1 and the other another branch has go the gain a2 okay.

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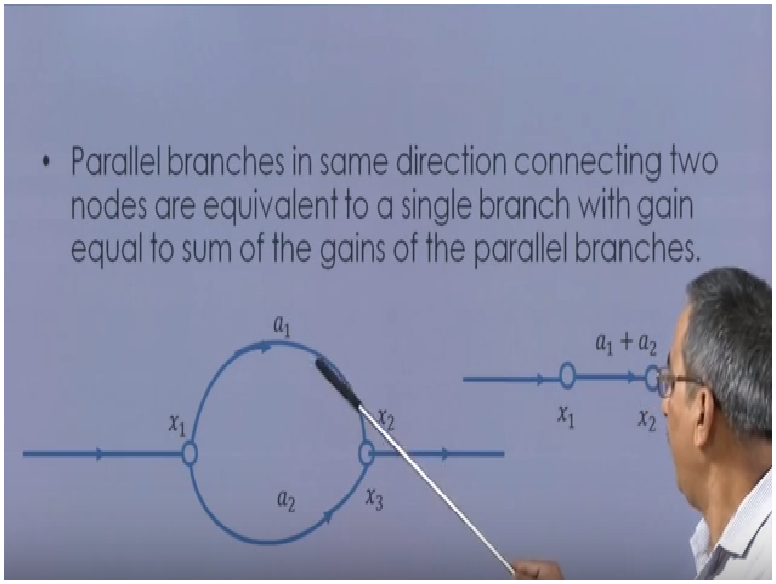

So this signal flow graph could be simplified okay, and it is equivalent graph could be like this that is between node x1 and x2 we have a signal a single branch with the gain as a1+a2 okay. So here these parallel branch gains can be replaced by an equivalent branch with gain equal to the summation of these 2 gains then if we have a series connection of unidirectional branches.

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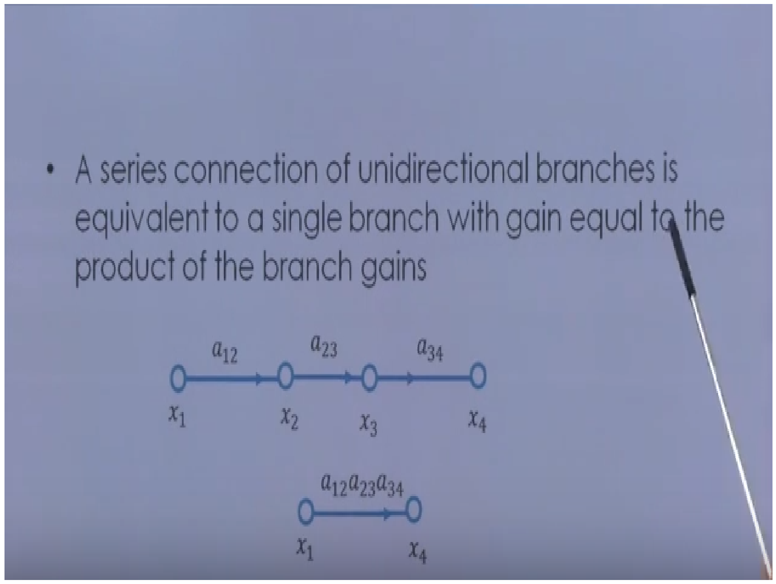

Here you can see that there is a series connection of unidirectional branches okay, then this signal flow graph can be replaced by one simplified between of course here say the same input x1 and output x4 with the gain equal to the multiplication of the gains of the individual branches okay, of a series connection of unidirectional branches is equivalent to a single branch with gain equal to the product of the branch gains.

So here the gain is a12xa23xa34 so this way we can replace this signal flow graph with this equivalent signal flow graph okay fine next we will be taking this equivalent signal flow graph okay fine next we will be taking up the signal flow graph of a feedback control system so we will consider a signal flow graph of a feedback control system and then we try to find out the transfer function for it okay.

So, suppose I have got a node say xs and then I have here say the node as es and here is say another node sorry as per our convention let us take this as ys the input one and say the xs as the output one and here also say this is xs and with the gain as unity here and the gain as unity here okay. Let the gain between xs and es we say gs okay and say there is a feedback here with the gain as -hs okay.

So here say I am finding out I am interested in finding out the transfer function for this case that is my interest is to find out what is ratio of xs/ys okay. So this is what are we have to do, so here in this case if we go by the signal flow graph algebra which we have seen in previous slides so I can write here the expression for xs as it is gsxes okay.

So here this xs will be gsxys okay, so this is a what we can write and what is my eses, i can write as here this es will be what are my inputs to this particular node, so this is ys and it will be - this one hsxxs okay and now let us substitute this is from say equation 2 to equation 1 what we get as xs=gsx say ys–hsxs or this i can further simplify as gsys–gshsxs.

Now, I can take this xs one side, so what I have is 1+ this is gshs and this =gsxys, so my interest is to find out xs upon ys. so this will be gs upon 1+gshs. So this way we can find out the transfer function for this feedback control system okay, so here you can see that what we have done basically in this feedback control system xs the feedback signal of xs taken and it was multiplied by again hs.

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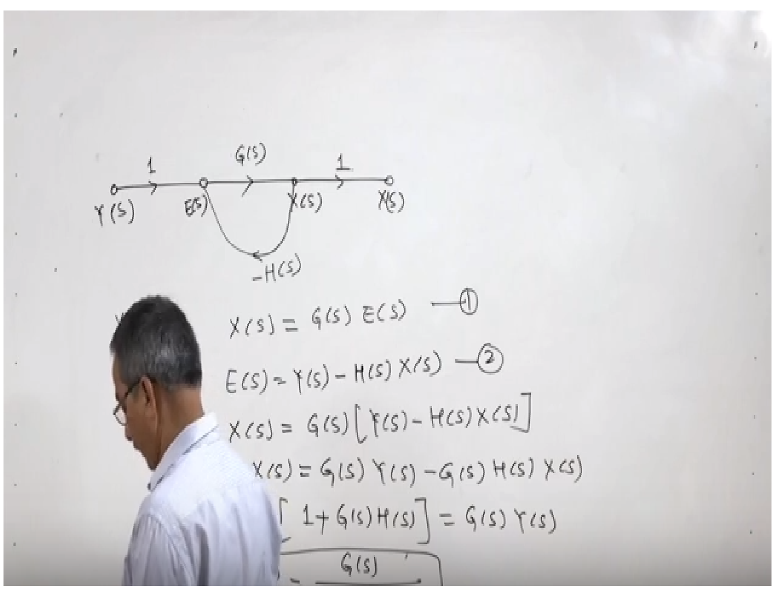

Basically, this signal has been subtracted from the ys and that makes us our es signal okay so this way we can find it out next let us see the block diagram and their signal flow graph equivalent so in our previous lecture we have seen the block diagram and their reduction various ways of reducing the block diagram that we have seen now let us see how can we this equivalent block diagram okay.

So I will just take one example, so suppose we have got a block diagram like this and as we have seen we have the output xs and input ys and say that here the gain is gs, then we have seen the xs upon ys is given by gs okay, so in signal flow graph the equivalent for this is basically say we have 2 nodes x1 and x2 and we can connect these 2 nodes through a branch and i can give a gain of gs for this branch.

So from here what we will have is basically x2=gsxx1, so this way we can write this one okay then we can also have the parallel feedback the one which we have seen so in the block diagram basically, we can have something like this say this is your ys and you have say the 2 branches here are 2 blocks in parallel with the gain as say g1s and g2s and of course these are summed up here in order to get the output, which we call as xs okay.

So if we are interested in drawing the signal flow equivalent for this one, then we can draw that, so we have a signal YS so first I can create some nodes x1 and x2 and to this one we have the signal ys and then we have the 2 parallel branches with the gains as g1s and the gain as g2s okay.
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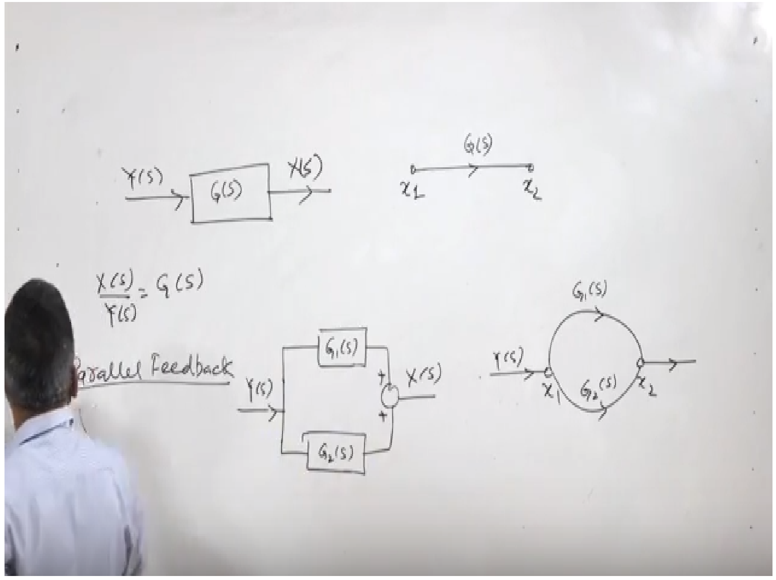

From here of course, we will have this signal x2 will be going so here whatever output you will be getting that will be g1 sx1+g2sx1 okay so this we can get, so this is equivalent for this one. Likewise, we can take the feedback example, which we have just seen for the signal flow okay so that example which we have seen just now what we had the ys and we had the forward path with gain as gs and from here.

we had xs as the output, we took the feedback and this was our hs let us give some name to this one, say bs say this is positive and this is negative, we write it for the negative feedback and I write some this one as a us all right, so the signal flow equivalent for this we have seen, so here is my ys, I take unit gain.

So I come here and from here, we draw the forward path with the gain as gs and from here we have xs with this one and we can take feedback here with gain as –hs okay and this will be my us okay so this way we create an equivalent for this one and the transfer function for this will be xs upon ys as we have seen, it is gs upon 1+ gsxhs okay.

Then we can take the very popular formula that is the Mason's gain formula through which we can find out the transfer function for a complicated system okay, that is the gain formula for the signal flow graph okay so for a signal flow graph with say n forward path and k loops, the gain between input node say x in and output node x out can be given as say m as x out/x in and this is given as sigma k=1 to n and mk del k/del okay.

Where as I said xn is our input variable okay input node variable, x out is the output node variable and m is basically gain between x in and x out and we have already seen n as the number of forward path and mk is basically gain of kth forward path okay of course between x in and x out okay and this del basically given by 1–sigma li1 where this i varies +sigma lj2 where j is variable–sigma lk3 where of course k is varying okay.

Here this left, m, r, terms which I am using basically it means that the gain product of the mth possible combinations of non-touching loops. What are non-touching loops that we have already seen where there is no common node and this r actually range of r is between 1 and k.

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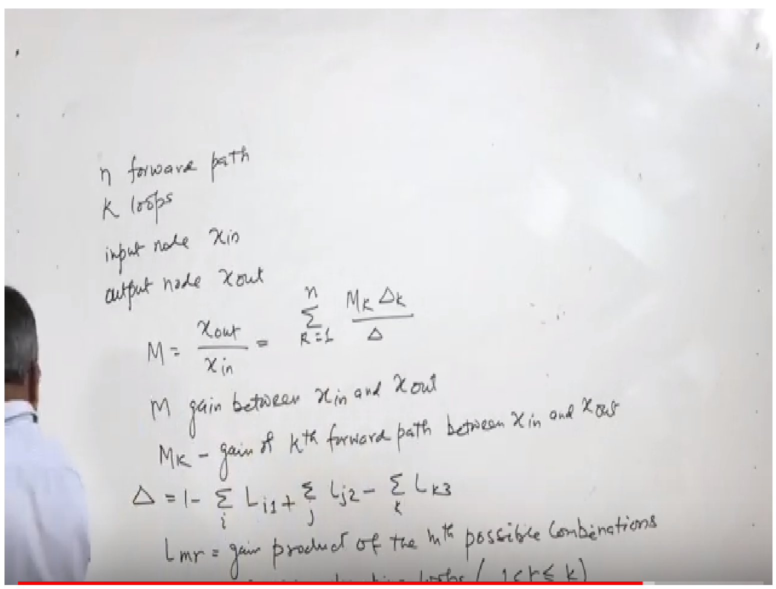

There is a simpler way of expressing this del basically and that way is that it is basically 1–sum of the gains of all individual loops + then what we have to do is that sum of the products of gains of all possible combinations of 2 non-touching loops and so on then we will have alternate plus and minus comes, then we will have minus then same sum of the products of gains of okay.

All possible combinations of 3 non-touching loops and it goes on okay so this way and the del k is basically this is the del part of the signal flow graph that is non-touching with kth forward path, so this is how del k is defined, now to explain the application of this formula we can take the same example for which we have found out the transfer function using the signal flow graph algebra, so I will take the same example and then we can demonstrate the application of the formula okay.

So let us take the same example here, say we have node YS, say this is es with the gain 1, here say the gain is gs and say the output is xs here, this is once and here so output is xs and so there is a feedback this way okay and the gain here is –hs, now here we can see that there is only one forward path between ys and xs okay.

So what we have is the forward path gain here is only gs and this is nothing but this is the m1, so one forward path so m1=gs and there is only one loop as you can see here so we have only one loop, so the loop gain l11=say what will be the loop gain here, so this is –gsxhs okay. and there are no non-touching loops, so what we will have is, we will have del as 1– then we will have this one, -gsxhs okay. so this is 1+gshs all right.

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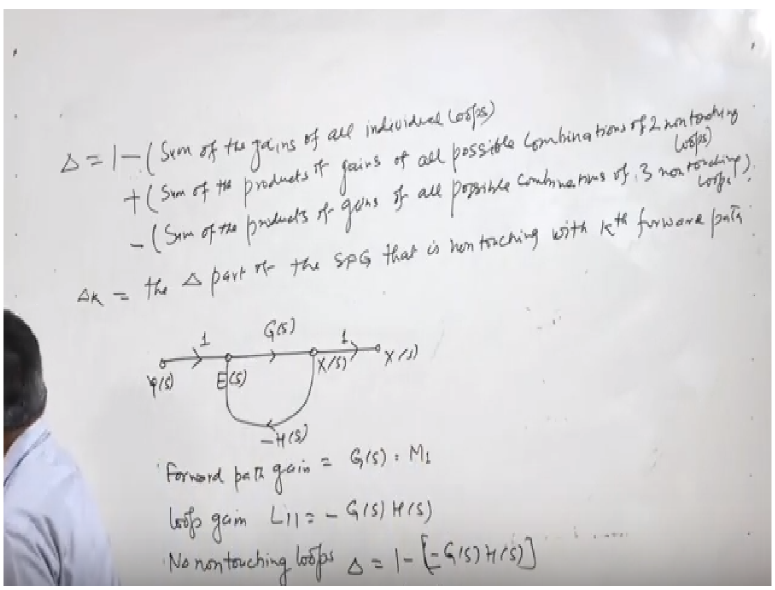

Then we can write the final expression so the del 1 here in this case is going to be =1 because these terms are going to be equal to 0 because the forward path is in touch with only one loop so this del 1 is going to be equal to be 1 and so finally, we can write xs upon ys and this is basically m1 del 1 upon del okay.

So m1 we have found out, this is gs and del 1 we have calculated as 1 and/del and that we have calculated as 1+gshs okay. So this way, we can find out the ratio of output/input or the transfer function from the signal flow graph using the Mason's gain formula and this is one of the biggest advantage of the signal flow graph over the block diagram approach okay because here we can apply a formula and we can get the transfer function, so this is all for this lecture, thank you.