Modelling and Simulation of Dynamic System

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Lecture – 26
Block Diagram Algebra

I welcome you all in this lecture on block diagram algebra which is part of modeling and simulation of dynamic system course which you are going through previous few classes. We have seen how to find out the transfer function for a save first order system. Then transfer function for the second order system. The block diagram algebra basically helps us in finding out the transfer function of a complicated system.

That is system involving many, many components. If that is there then with the help of block diagram algebra, we can find out the transfer function for that so here in this lecture. I will be giving you initially some introduction on the block diagram. Then we will be seeing how can we draw the block diagram of a physical system and after that we will be looking at the various ways of reducing of the block diagram okay.

So, this is how I intend to go here, so what is a block diagram? A block diagram of a system is actually the pictorial representation of the functions performed by each component and flow of signals okay. So it essentially pictorially represents okay. The flow of signals and the function performed by each component block diagram also represents the interrelationship that exists among the various components okay.

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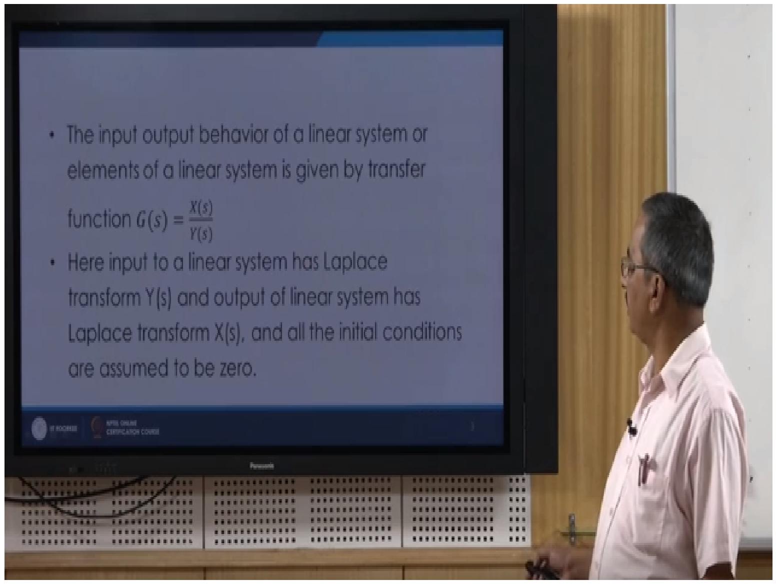

It indicates more realistically the signal flow in a system. So suppose the input output behavior of a linear system is given by a transfer function see gs=Laplace transform of the output/the Laplace transform of the input, here as I said input to a linear system as Laplace transform ys and the output of linear system has Laplace transform xs okay.

Of course, when I have been telling you that when we are finding out the transfer function we assume that all the initial conditions are 0, so here suppose above we had a block and block is representing a transfer function say gs and say the input to the block is say is ys and the output is xs of course various and xs are the Laplace transform of the input and output signal okay.

Then, here we define the gs as xs/ys that is the ratio of output by input signal Laplace transform of output signal/Laplace transform of the input signal it is a very convenient graphical representation of course the one which we are talking about block diagram and here as I said the signal into the block which we call it as the input signal is YS and signal output of the block is represented by output xs okay.

The block itself represents the transfer function now we need to remember certain points about the block diagrams the flow of information is unidirectional from input to output okay. That is the flow of information goes from input to output in the, that is the unidirectional okay. This is not the bi-directional way as we have seen in case of bond graph where we have the effort and flow both communicating.

If one end has got the effort information then other end has got the flow information. In the case of bond graph okay, but here as I said we have the flow information is unidirectional from input to output. If you remember in case of bond graph, we have seen that if your bond is like this and if your causal structure is like this then we say that this end is receiving flow whereas this end is receiving effort okay.

So, here we have the both way information now a complex system with several non loading element is represented by the interconnection of block for the individual element okay. The blocks are connected by lines with arrows indicating the unidirectional flow information from output of one block to the input of the other block. So, this is what I mean by it that is if say you have a block like this you have certain input to this block then output from this block.

Then, you have another block like this and another block like this okay. So you can have this type of cascading. Now in a block diagram summing and differential of signal is represented as here. In fact I am showing you are the differencing of the signal okay. That is if a signal a is coming say from here and a signal b is coming from here, here this is+and this-sign basically indicates that this signal will be a–b.

Of course, if we had the symbol here plus then it will be a summing signal okay. So we have the summing and differential signal can be represented here. Likewise the take-off point of a signal is represented as here, so from here we can take the same signal value here okay. So this is how the take-off point is represented then let us see the block diagram of a closed loop system okay. So suppose are here there is a input to the system which is represented by Ys okay.

There is an output of the system represented by Xs the transfer function here is say Gs okay. Then what we do it actually is that through this branching point I take our take the same signal Xs from here and through a summing point here I subtract this output signal from the input signal so here what I get error signal ES which is nothing but Ys-X s so the Es there is basically Ys- Xs okay and okay.

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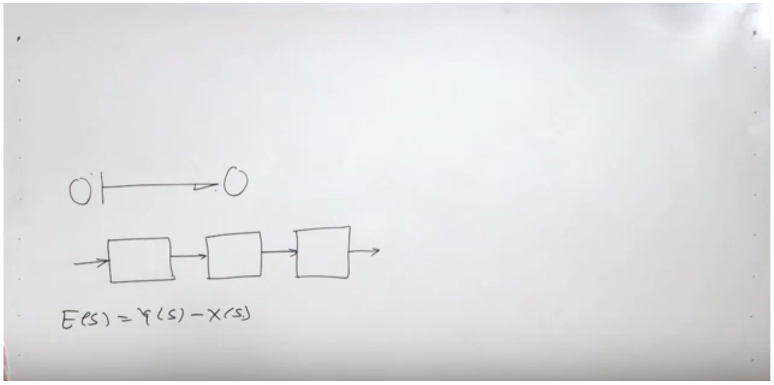

So and here you can see that the output the output of this block diagram that is access is being fed here okay. To the summer to find out the error, so this type of block diagram is what is called as the closed loop system okay. You may ask a, you may ask some practical example or why we are going for this closed loop system or what is the practical example of this closed loop system? so let us look at this figure.

Now you see that here the output is Xs and then this output basically Xs is multiplied by this Hs okay. Certain gain okay it is multiplied with certain gain and whatever we get it is Bs and then this Bs is taken to the Ys is subtracted from Ys. Now this type of system if I say take the example of say temperature control of a room fine then suppose the signal here Xs which I am getting okay.

I am getting a say the temperature this Xs signal in the form of a say temperature okay. Now the thing is that suppose I want to feed it here with input, input setting usually we do with the same with the help of some parts okay. So some who are input signal is say some voltage, then this temperature signal can be converted into a voltage signal.

So, basically this Xs does the role of conversion of this temperature signal into a voltage signal and then of course this can be compared error can be found out okay. This type of signal is what is called as the closed loop system. Closed loop system means that where the closed-loop systems are usually the error driven system or you can say that where the output has got the influence of influence on what is going on inside the block.

That is what is we mean by the closed loop system, so let us define some transfer functions and say in this case the open-loop transfer function is defined as the ratio between the Bs and Es here. So you can see that the Bs is a input to the summer here and Es is the error signal that is Ys–Bs okay. So this is what is called as the open-loop transfer function and here you can see that the Bs/Es is given by multiplication of Gs and Hs.

So this is basically GsHs or you can see that this Bs is basically HsxGsxEs alright. So that we can find out so this is what we call it as the open-loop transfer function okay I can also define the feed-forward transfer function and this feed forward transfer function is Xs ratio of Xs/Es okay. That is output by this error signal and this Xs/Es is what is called Gs okay.

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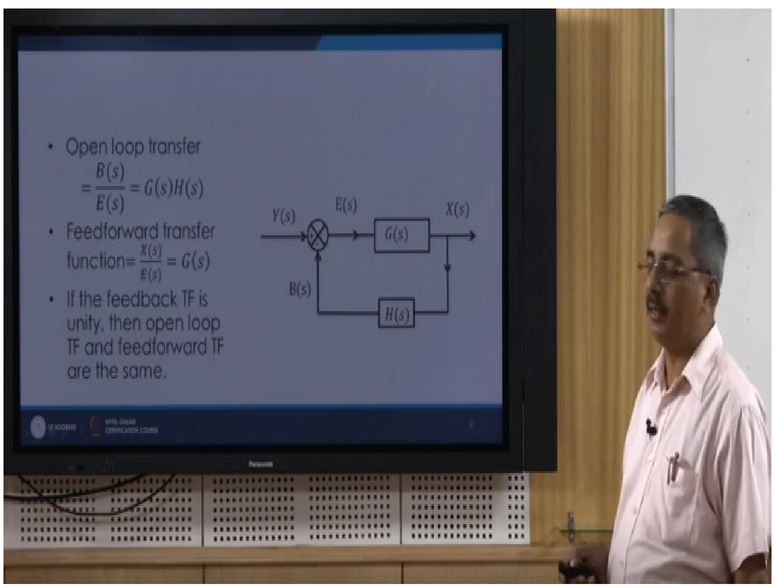

If the feedback transfer function is unit, then open loop transfer function and feed forward transfer functions are same. So you can see that if I put this as unity, unity then whatever are we get here this and this both are going to be the same value okay. Next is the closed loop transfer function now this closed loop transfer function is basically defined as the ratio of Xs/Ys okay, so to get the derivation for the closed loop transfer function okay.

We can find out the input to the forward path so this is by the forward path so what is the input to the forward path this input you can find out this is Ys–Bs okay and what is this Bs this Bs is nothing but HsxXs okay so the input to the forward path is various–HsxXs okay and what is the output of the forward path output of the forward path is Xs okay.

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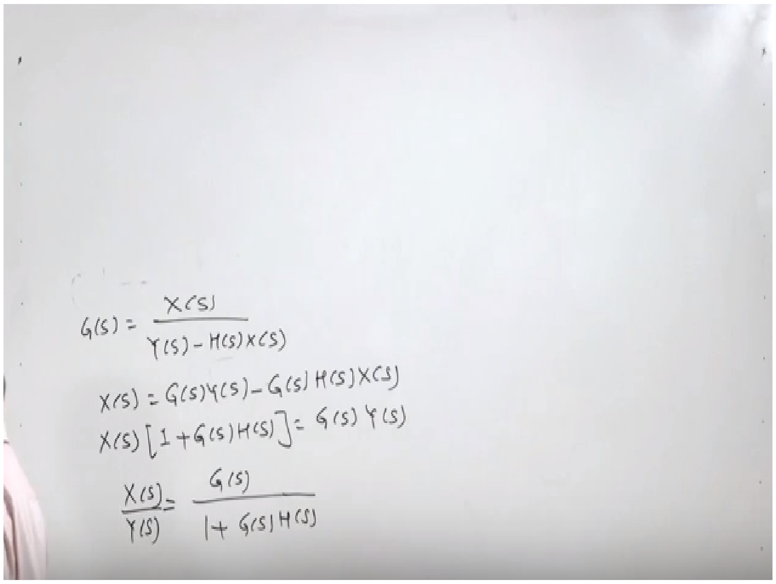

So, I can define Gs as Xs upon Ys–HsXs okay So here I can define as you can see the Gs=Xs, which is output divided by the input here and that input is Ys–HsXs okay so from here I can get Xs as GsYsGsHsXs. So I can take XsX one side so here what I get is 1+GsHs=GsYs okay. So what is my ratio of our Xs/Ys that is the output by input so this is my Gs upon 1+GsHs. So this is what we get here so Xs upon Ys is Gs upon 1+GsHs.

So, this way we can find out that closed loop transfer function so here you can see the output of this system is X s and the input to the system is Ys okay So from here I can find out Xs as Gs upon 1+GsHsxYs and what does this means this means that the output of closed loop system depends on both the closed loop transfer function and the nature of the input.

We can take an example to understand this further say if the forward path transfer function of a system is given by this one and there is a negative feedback is 5s okay then the find that a transfer function of the complete system okay so here basically the overall transfer function will be Gs upon 1+GsHs. So Gs is given as 2/s+1 and 1+GGs again 2/s+1 and Hs is given as 5s. So we can simplify this and this way we get the overall system of the transfer function for the system.

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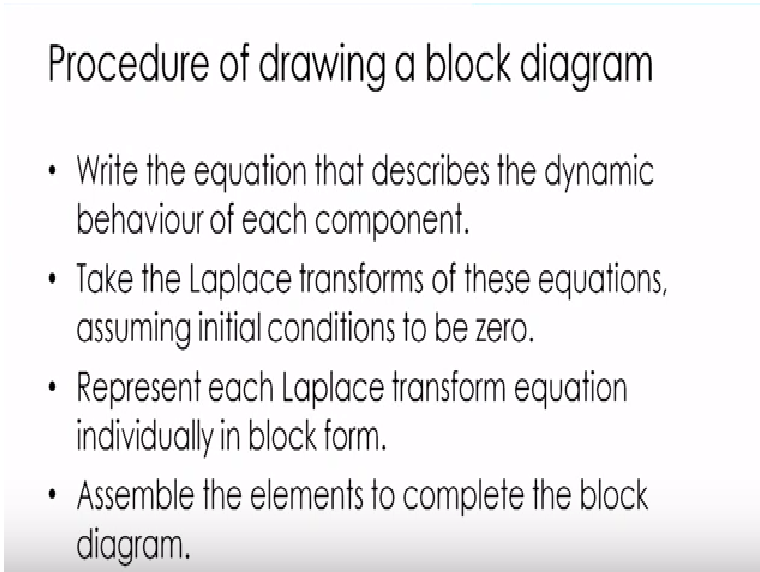

Now the question is what should be the procedure to draw the block diagram. How can we draw the block diagram for any physical system okay so to do that here are some guidelines. What we can do is that we can write the equation that describes the dynamic behavior of each component okay. So this we can do then we take the Laplace transforms of these equations assuming initial conditions to be 0.

We represent each Laplace transform equation individually in the block form and finally we can assemble the elements to complete the block diagram okay. So first we write the equation which describes the dynamic behavior of each component then we take the Laplace transform assuming initial condition 0 then we represent that Laplace transform as a individual block form and of course then we can assemble all the blocks okay.

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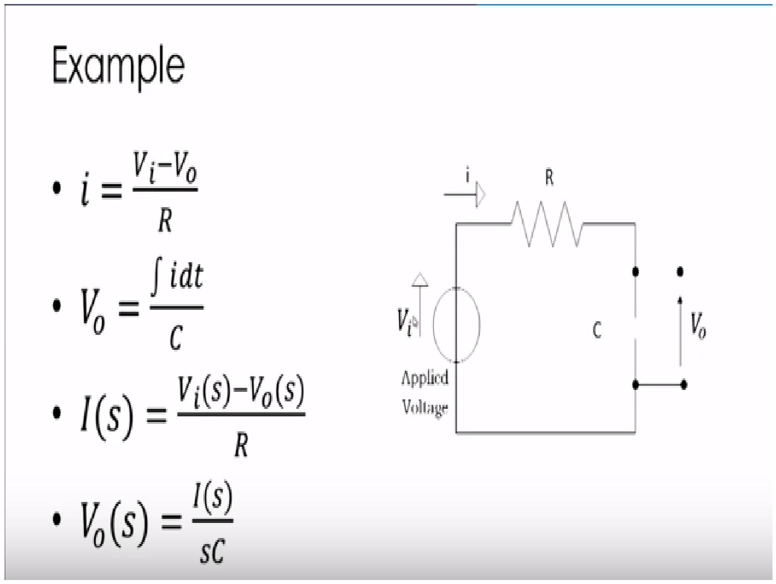

In order to complete the block diagram for the whole system. Let us take an example suppose this is a RC circuit okay and to this RC circuit there is you say applied voltage Vi and say I am interested in finding out the observing the voltage across the capacitor say Vo, so first step that we write the dynamic equation for the system.

So here you can see that I can write expression for the current i=Vi-V over here is Vi voltage here. It is Vo so Vi–Vo/R okay. And the Vo I can write integral of idt/C same as that is the charge across the capacitor plates / the capacitance now in the next step what we do is that we did the Laplace transform of both these equations okay so the first equation if I take the Laplace transform so this is Is=VisVos/R.

For the second equation if I take the Laplace transform it is Vos=Is Laplace transform for this is I/L into this see okay. Now let us come to the third step that we represent each equation in the form of block diagram so the first expression you can see Is is Vi–Vo/R. So you can see that here we are taking the subtracting the 2 voltages, so subtraction I can do across affirming element okay.

So, my Vi is positive here and Vo is negative here. So I use a summer here which differentiates this voltage Vi with Vo okay so here and then this Vi–Vos/R. So I use a block diagram here a block who is this transfer function is say 1/R and what I get is Is. So this is my Is okay. So this part of the block diagram represents the first equation okay.

The second part of the block diagram I can represent here Vo is Is/Sc. so here Vo is my output and say Is is my input and 1/Sc is that transfer function okay. So this block diagram represents this equation whereas this block diagram represents this equation. Now I combine these 2 blocks okay. So how can I combine, I can combine here that see the output from this block is Is and this output is same as the input for this block okay.

So, I combined here, so this is this is I s same as this one and here the output of this block is Vos and remember. Here I am subtracting Vos so I just take a signal from here and subtract it through Vos okay. So this way I can combine both of these block diagrams in order to make a single block diagram and of course this single block diagram represents the block diagram for the whole system okay.

Now there could be various ways or there could be various ways in which we can simplify the block diagram in order to find out the overall transfer function okay. So here are certain equivalent original block diagram and certain equivalent block diagrams are given, so here you see that there is a first block with transfer function G1s and say input to this is X1 and output 2 this is X2, similarly there is another block whose output is X3 and the input to this is X2 okay.

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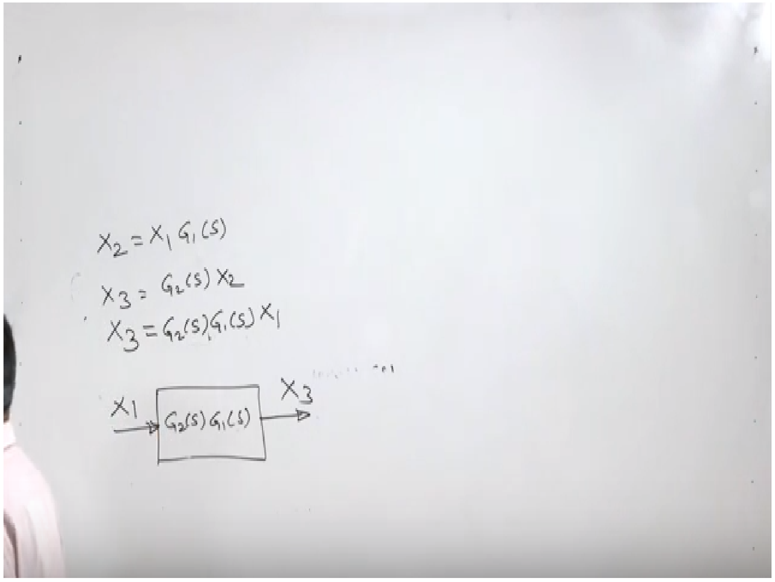

So this whole thing can be represented like this, why we can represent it like this because we can simplify these equations say for the first block I can write X2=okay X1xG1s and for the second block I can write X3=G2Sxx2 okay. And then I can substitute for X2 here. So this is G2sxG1XxX1 so this is X3 okay.

So, what does this means that both the blocks can be combined and the trans combined transfer function is g1G2sxG1s and output is X3 and input here is X1 okay. So this is how it is represented here in the first figure okay. So we can write it this way or we can write it other way also. Now let us look at the second figure now here say there is input X1 here we are there is a summer okay.

With sign+ or -X2 is there Gs here transfer function and here say X3 is here. Now what I am doing here is basically I am taking this summer ahead of this block okay. So if I write if I do this then what we have to do is that here I have to create one additional block with the transfer function G okay. And we can see the equivalences of both the blocks okay that is the original block and the reduced block.

So if you look at the first one if you look at the first one for the first case X3=it is Gx what we have the error one okay. There are input to the G that is X1 then what we have is+-X2 okay. And if you look at the second one here then in the case of second what we get X3=this is GxX1 so GxX1 and here we will have +- what GxX2GxX2.

So from here, basically I can take G out so this is X1+-X2 so we get the same expression okay. So this is why this is equivalent for this block. Similarly, here you see that in this block there is a point here we are taking this X2 signal are tapping this X2 signal from here okay. Now if I take this tapping point from here to say here I have to introduce one additional block here with the transfer function as G okay.

So, this is how we explain and again the output in both the cases that is the original block and the equivalent block is same. So in first case you can see that this X2 is basically GxX1 and we are getting the signal extra here so in this one you can see that this X2 will be GxX1 same as this one and we are tapping your X2 but here if we are typing it from this point it is X1 okay.

So, this X2 is basically has to be GxoX1 okay. This extra is 3 types of X1. So if I am writing in terms of X2. I have to put here Gx of X1. So this is how this is done okay. Similarly in the here suppose you have a point tapping point here say at before this block and if you want to take it after this block okay then again you have to introduce 1 block with transfer function 1/G here okay.

So, if you are taking it if you are taking this point back when the transfer function has to be G and if you are taking it ahead then the transfer function has to be 1/G. So here again we can see X2 value is going to be Gx1 and of course here X1 is already there so here this X2 is Gx of X1 and here this X1 will be 1/G okay. 1/GxX2 and X2 is again GX1. So here we will be getting X1 only okay.

So, this is how it is next suppose we have the original block like this okay, that is we have a summer here okay and say summer if we want to take a behind of this block okay G. Then this is what we have to do in this feedback signal here we have to divide this by 1/G okay And again we can establish the equivalence of both of these okay, so in first case it will be X3 will be what it will be Gx1+-X2 okay.

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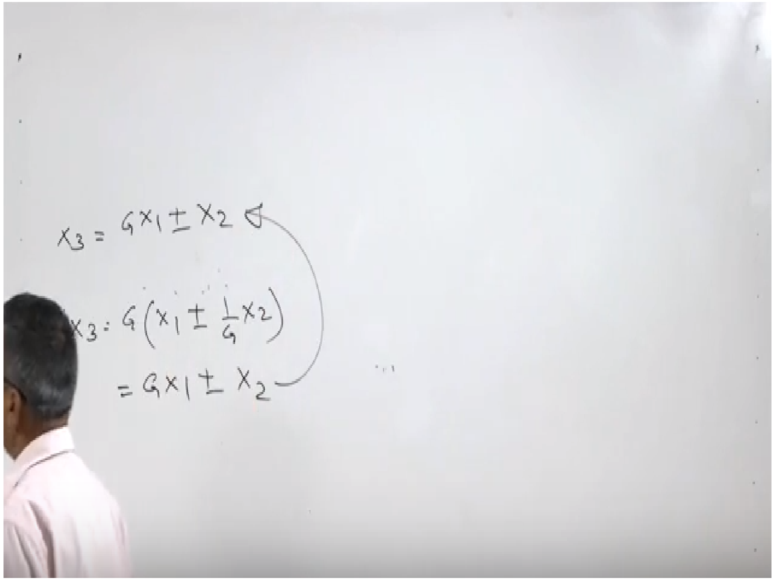

In the case or the second figure what we will have is X3=GX what we have X1+-okay, we will have 1/GX2 okay or this is a GX1+-X2. So this is same as this one that is why both these blocks are equivalent blocks okay and this one we have already seen that I am in case of a closed loop system okay, so in case of closed loop system the overall transfer function between the output and input is given by here X2=here X2/X1 is basically GH upon 1+GH okay.

Suppose, G upon 1+GH and of course this is valid if we have a - sign here so if you have a+sign then here the first sign will be linked at you and also for the second case that is the for the positive case you will be having that negative sign and for the negative sign negative case you will be having the positive sign so this is how this can be represented equivalently by this block okay.

We can follow all these procedures to find out that transfer function for a system something like this so for this system say Is is the output and say Rs is the input and in between we have many forward transfer functions a G1,G2,G3,G4 and there are feedback transfer function of function say H1,H2 and H3. So we can simplify this model as per the procedure which had just discussed and we can get the overall transfer function for this system.

So, I am not are going to solve this I am leaving this as an exercise to you so please try to solve this okay and this is the practical application of whatever we have talked in this lecture okay. If you want to further read please go through any of the book either Ogata or Bolton okay, thank you.