Modelling and Simulation of Dynamic System
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Lecture – 25
Transfer Function of First and Second Order System

I welcome you all in this lecture on transfer function of second-order system, which is a cool which is a sub model for the course on modeling and simulation of dynamic systems. So the previous lecture we have seen how can our how do you derive or how a first order system transfer function can be worked out.

What is the form of that first order system transfer function in this lecture we will see that derivation of the transfer function of a second order system. Then we will see that what information we can extract from this transfer function.

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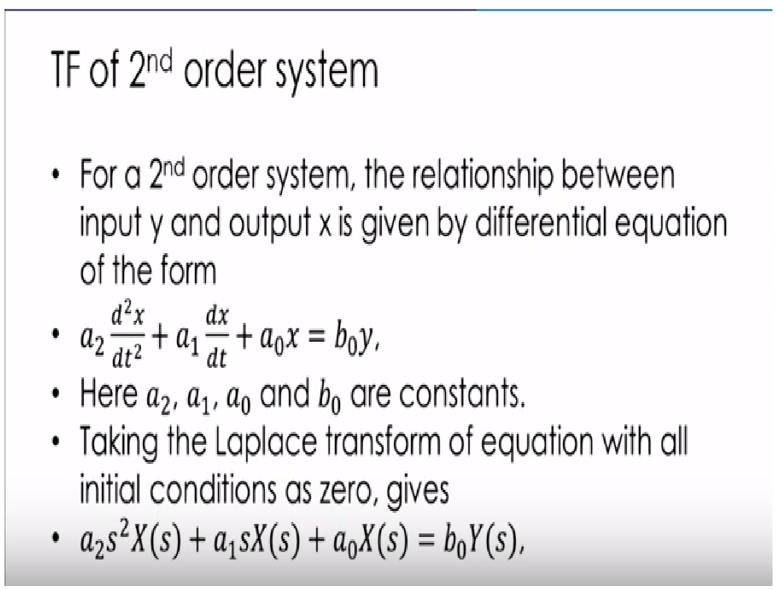

So a second order system we have seen a spring mass damper system to be a very good example of the second order system okay. So this spring mass damper we have seen earlier so if we are derive the equation for this system.

This is my x and suppose there is certain force F acting on it then we know that the system equation for this system is going to be md square x/dt square d+cdx/dt+kx=ft okay. So this equation we call it as of the second order or system equation fine. So this is a very good example of that and of itself electrical counterpart also we have seen that is we have say a voltage source.

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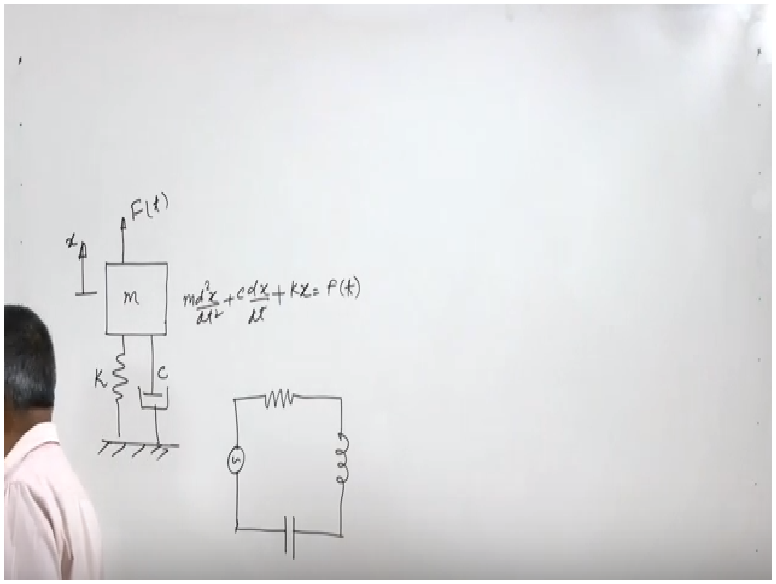

There is a register there is an inductor and say there is a capacitor, so for this system also we are going to have the expression of the same form. So what I intend to say is that for a second order system the relationship between the input Y and output X is given by differential equation of this form okay.

So, if we talk about the mechanical system which I just described here then here this Y is basically the input that is the force here and X which we are talking about that is the output here okay. So this is the general expression for any second order system and here in this expression we have b0,a0,a1 and a2 these are the constants.

Now if I want to take the transfer function for this one what we have to find out the transfer function for this one which is again the ratio of the Laplace transform of output device and the Laplace transform of input so if I take the Laplace transform of this equation with of course all initial conditions as 0 which is one of the essential conditions for finding out of the transfer function.

You can see that this expression becomes a2x square xsa1sxs+a0xs=b0ys okay. So if you take Laplace transform for this I can write it in this form here. Since this is a second derivative, so we have the s square comes coming here. For first derivative we have the s term coming over here. So this expression I can write in the form of output by input, so output is my access and input is my ys if I do that what ratio I get this is the transfer function Gs okay.

So, this can be written as b0/a2s square+a1s+a0 okay. Then there is alternate way of writing of this transfer function for a second order system okay and that alternate way is in terms of the natural angular natural frequency Omega and the damping ratio Zeta okay. So for the same system, which are I just described the second order system Omega n square is defined as ratio of say a0/a2+zi square as ratio of a1 square/4a2a0 okay.

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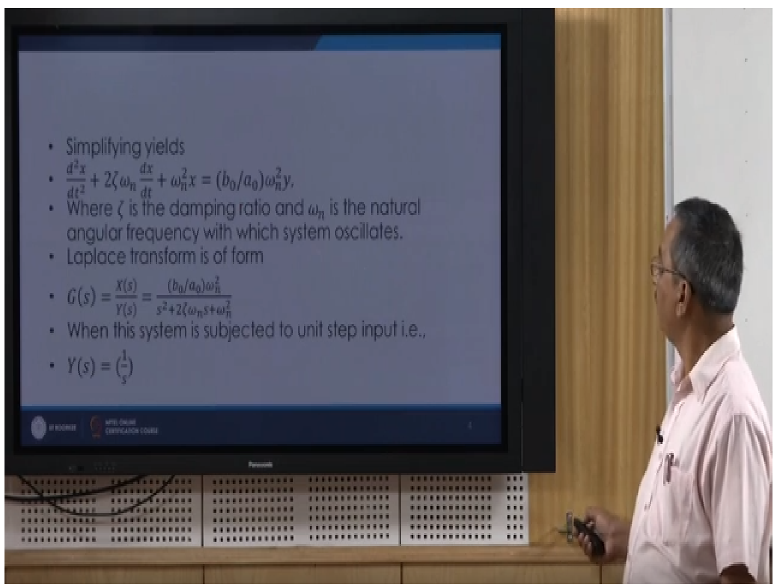

So, if we do this then what we get is this one okay, so if I simplify that okay. Then this is what we get fine. Now here you can see that this expression I am writing in terms of Omega n and zeta okay so here as I said Zeta is the damping ratio and Omega n is the natural angular frequency with which the system oscillate okay now if I find out I take the Laplace transform of left hand side as well as right hand side with initial condition as 0.

This is what I get that is the ratio of excess xs/ysGS will be b0a0 Omega n square/here will have the s square term for this plus 2 zeta omega+ns+omega n square okay. So now you see that this is output by input again this is output by input. Now when this system is subjected to say a unit step input in that case for the unit step input the ys that is input can be written as 1/s okay.

So far unit step input the Laplace transform is 1/s so I can substitute this 1/s here and I can get the output okay. Which is our aim of getting it so here xs I have substituted for yf as 1/s here, So this is my xs okay. Now you can see that this is the form of it are some constant here divided by S Plus here. It is a basically the quadratic equation you can see that okay.

So here I can further simplify this I write the same constant here there is a s term and this quadratic equation I can always write s+p1xs+p2 okay. Where this p1 and p2 are basically the roots of this quadratic equation okay, so I can write this in this form okay, then in order to find out the value of p1 and p2 what I can do is that I equate this term quadratic term equal to 0.

I write the root of P as a say the –b+- under root b squared -4ac/2a okay This actually corresponds to the way of solving a quadratic equation which we have seen okay that is if my equation say x square +bx+c=0 the root x will be –b+-under root b square-4ac upon to a alright so from employing that I am writing here the root P as are basically here this will be value s and we are calling that s as the root.

So it is a general expression for p-b4-2Zeta Omega n+-this is 4zeta square Omega n square that is b square-4a is 1 and c is omega n square/2a so the 2 here. Now I can simplify this equation as-zeta omega+-under root zeta omega n square-Omega n square. Now I can further simplify I just take this Omega n square terms outside here, so this is what I get-zeta omega n+-omega n under root zeta square-1okay.

So here you can see that we have the plus minus sign which means that the one root is say p1 that is having the plus or term here and the another route is having the minus term over here okay. Now depending on as we have seen earlier also depending on the value of Zeta we are going to have the three different behaviors ok, so when zeta is going to be more than one will be having the over damped case.

When zeta=1 will be having the critical damping case and when this zeta value is going to be less than 1 we are going to have the under damped case okay. So let us see one by one for the old damned case that is when zeta is more than 1 access as I said I can write as B0 Omega n square/ss+p1xs+p2 here it will b0/a0a0 okay. So that will be there and this is there then by applying the inverse Laplace transform to the partial fraction.

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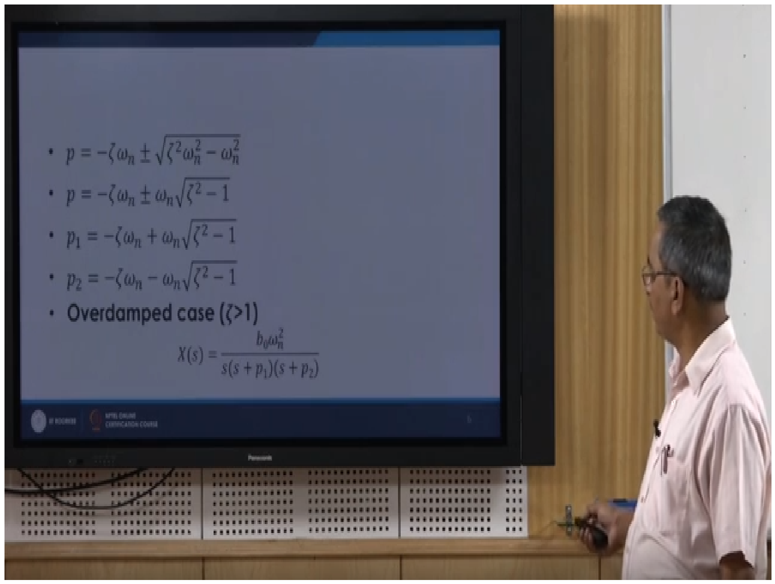

Now if we want to have that time response of it what we do is that we do the partial fraction for this term okay. Then we apply the inverse Laplace transform and this is what we get so we get the time response as this one so b0/a0 Omega n square/p1p2x these terms all right. So this is how we get the time response further over damped case for the critical time case when zeta=1.

If I substitute zeta=1 here this term becomes 0 and here we get–omega n that is both the roots are going to be equal okay. So p1=p2=Omega n and so with this the xs. I can write as this is my in the same term earlier term and s into this I will have s+Omega nxs+Omega n okay or b0/a0 Omega n square/ss+Omega n square.

Now again, I can do the partial fraction for this and I can take the inverse Laplace transform in order to find out the time response of the system okay. So I can expand by partial fraction if I do that so this is what I get b0/a0 Omega n square 1/s-1/s+Omega n - Omega n/S+Omega n square. Now I take the inverse Laplace transform for this and this is what I get the time response okay.

So, this is b0x is 0 Omega n square 1-this will be e2 to the power-Omega nt and here since this is square here we will have-Omega and value because of the square we will have a t term xe to the power omega nt will be there okay. Likewise for the under damped case under damped case that is for Zeta are less than 1 the time response will be given by this expression okay.

So, this is how we are going to get it we can take an example to tell you that what information we get from the transfer function of say the second-order system. So suppose there is a transfer function given by this 1x1/s squared+8x+16 and by seeing this transfer function we need to tell the state of damping of the system what is there and say this system is subjected to unity step input function okay.

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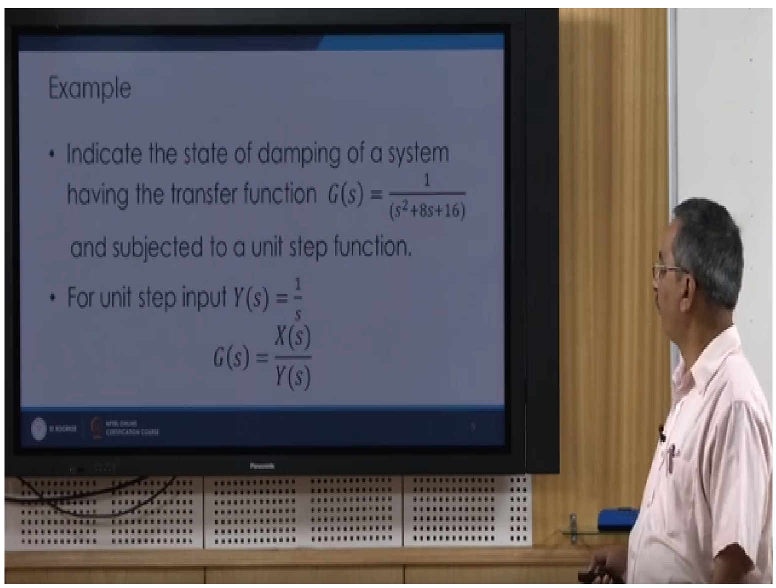

So, we know that for unity step input YS=1/s and GS=XS upon YS okay. So I can write expression for XS as 1/s and here this I can write s+4xS+4 all right. So from here you can see that the roots of this equations are p1=-4 and PT=4 so you can see that the roots are equal roots okay.

So, when the roots are equal here real they are real and equal we can immediately conclude that the system is going to be critically damped okay. So you can see that without going into that time domain through the transfer functions we are able to predict the behavior of the system. So that is one of the greatest advantage of this transfer function approach now we can also tell about effect of pole location on the transient response okay.

So, let us take a first order system with transfer function I say 1/s+input. So the transfer function that is XS upon YS is 1/s+1. So from here I can write XS=1/s+1 okay because YS is going to be 1 here for the case of impulse all right. Now here you see that XS is 1/s+1.

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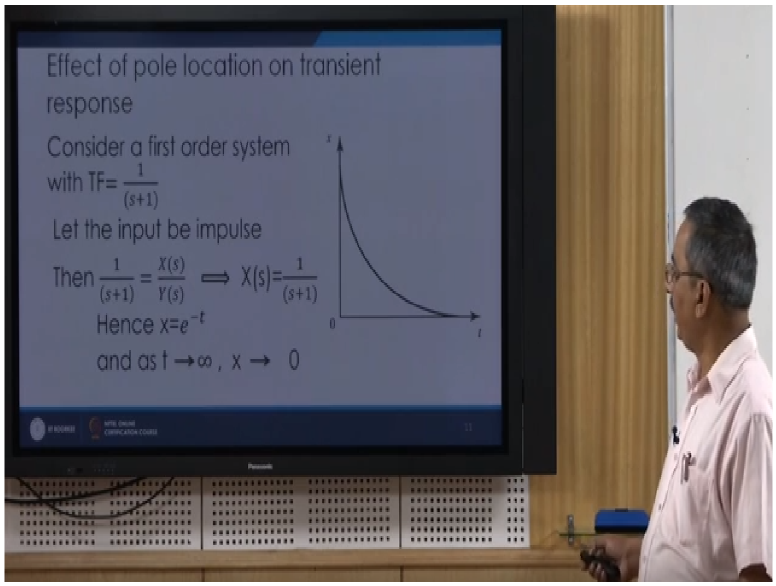

If I take the inverse Laplace transform for this is going to be simply e to the power-t as the inverse Laplace transform for s+ going to have the -t since here there is + sign really having the - sign here. Now from here you can see that as the value of t goes to infinity this term is going to XS going to be 0X will be tending towards going to get as the t value reaches infinity the value of x is going to be 0.

Similarly, suppose if my transfer function is 1/s-1 and it is subjected to unit impulse input okay, then here you see 1/s-1–XS upon YS and from here. I can write XS as 1/s-1 and when I take the inverse Laplace transform for this that is I find out XT it will be e to the power t okay and if it is e to the power you can see the behavior of it that is as t tends to infinity x is going to be infinity okay.

So, this system is unstable one whereas the previous one this system is going to be stable one okay. So are there for in general for a first order system with the transfer function as 1/S+P okay. System is stable if pole is positive okay. The system is unstable if the pole is negative because if the pole is negative if the pole is negative then this is what we are going to get okay.

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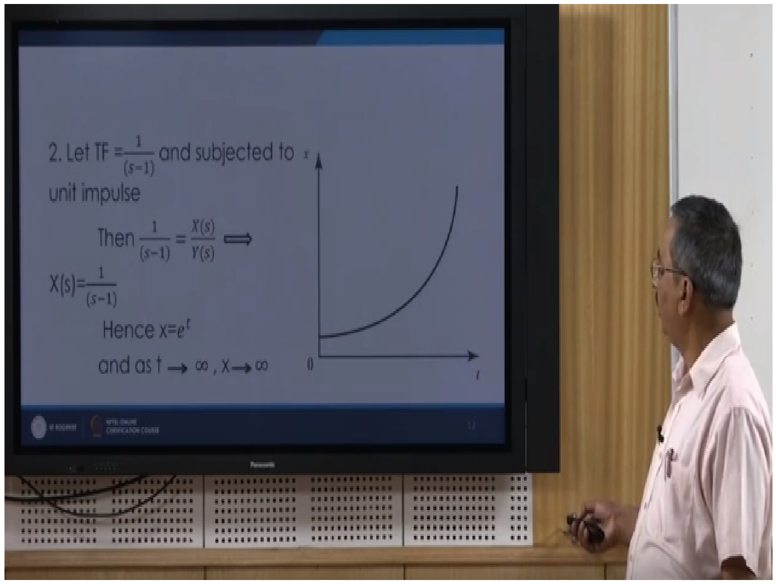

If the pole is positive this is going to be the behavior so this is what we have seen okay fine. Now for the second order system suppose the transfer function GS=b0/a0 Omega n square/s square+2Zeta Omega n+Omega n square and when subjected to an unit impulse input okay. The YS is going to be=1 so what will be my XS in that case the XS will be b0/a0 Omega n square/of the same term okay.

So this is what we are going to get and this is the same constant in the numerator and again this denominator I can write as s+p1xs+p2 okay. Where p1 and p2 are the roots of the characteristic equation this one now here again you see that as we have found out previously I can find out the root of this characteristic equation and this is what we are going to get okay.

So,-b+-under root b squared-4ac/2a or this is-Zeta Omega n+-zeta square Omega n square-Omega n square, now depending on the value of Zeta okay. p can be real or imaginary and imaginary terms involves the oscillations so that we have seen that if Zeta is going to be more than one we are going to have the real root and if Zeta is going to be less than 1.

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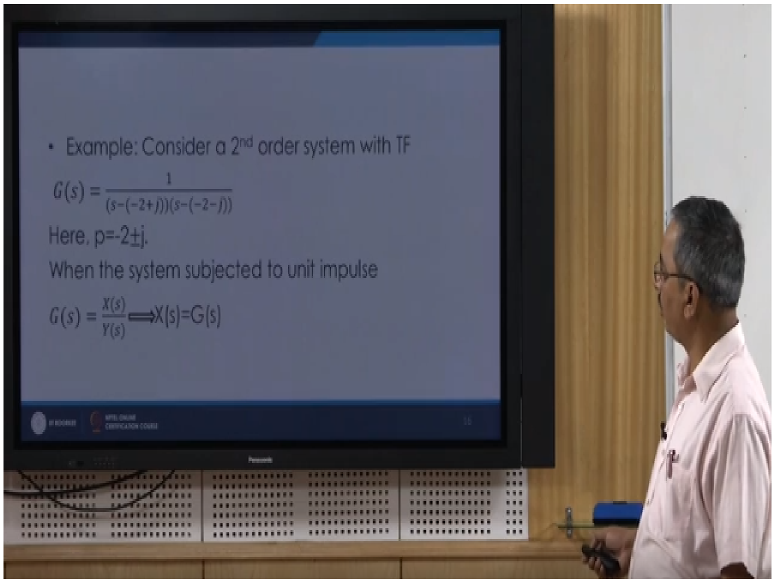

We are going to have the imaginary root and the imaginary terms involved oscillations okay so let us take an example so consider a second-order system with transfer function say this one okay. Now here you can see the root sr-+-J okay. When the system is subjected to unit impulse then your XS is going to be equal to GS okay and also XS, I can write as same as GS and if I take the Laplace transform for this inverse Laplace transform for this is what I am going to get e to the power minus 2tx sine t, okay.

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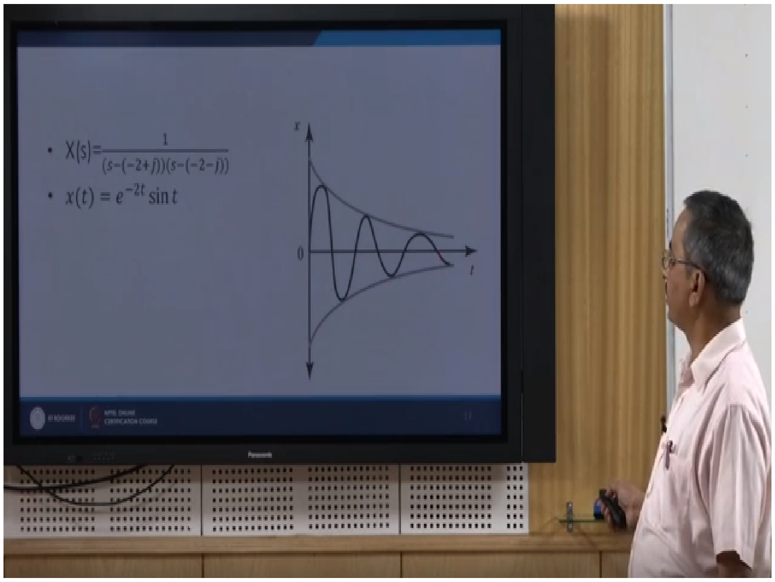

So, because of our here so basically what is there, there is a sine term and this sine term is being multiplied by an exponential decaying term okay so this exponential decaying term basically makes this envelop to be reducing in size okay. So this decaying term basically makes these to reduce size okay, similarly if I have say GS is equal to this, okay.

Then, the root is going to be A2-J value and when this system is subjected to unit impulse then again we have GS=XS upon YS our XS is going to be = GS okay. If I do that this is the term okay, of what we get a same as GS and if I take the inverse Laplace transform for this, this is what I am going to get e to the power 2T and sign team okay.

So with this what actually happens that there is an envelope for that is a sign term is being enveloped by the exponential term here and you can see that as T increases this value will be going on increasing and basically this makes your system an instable one okay so that is there so this way by looking at the transfer function that is from the transfer function we can derive the system behavior okay.

You can see that here we have not solved the differential equation for the system now in order to change the behavior of the system that is the previous two cases which we had seen one case was the stable one where the magnitude of the sine curve was goes on decreasing. That is a case of stable one whereas the other one where the magnitudes goes on increasing that is the case of the unstable one. Now suppose if we want to change that characteristic of the system okay.

The given input then we may play with the system okay and that is what is called the compensation okay, so the output from a system might be unstable or the response may be slow are there is too much of overshoot. So it can have any of these are characteristics, so the system responds to an input can be altered by including a compensator okay, so, that is the role of a compensator okay.

So, a compensator is a block, which is incorporated in a system so that it alters the overall transfer function of the system in order to get the desired characteristic okay. So this is the compensators are mostly used in systems in order to get the desired characteristic of the system and this characteristic could be that we do not want very much overshoot or we may want to change the response of the system may we want to make the response to be faster or we may be interested in making the system stable one okay.

We will discuss further about this in our next lecture on the block diagram algebra okay. So these are the references which you can look at Bolton Mechatronics, Ogata Modern Control Engineering, thank you.