Modelling and Simulation of Dynamic System
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Lecture – 23
Performance Measure for Second Order System

I welcome you all in this lecture on performance measures for second order systems, which is a sub model for modeling and simulation of dynamic systems course which you are going through we have seen in previous lectures the modeling of the second order systems okay. Now in this lecture what we will see is that how the system we have how can we measure the performance of the system okay.

These performances can be measured in terms of certain quantitative parameters so we will be seeing all those quantitative parameters and then I will take an example to illustrate how we can evaluate these quantitative parameters. So, there are some parameters by which we can specify the performance of an under damn second order system to a step input okay.

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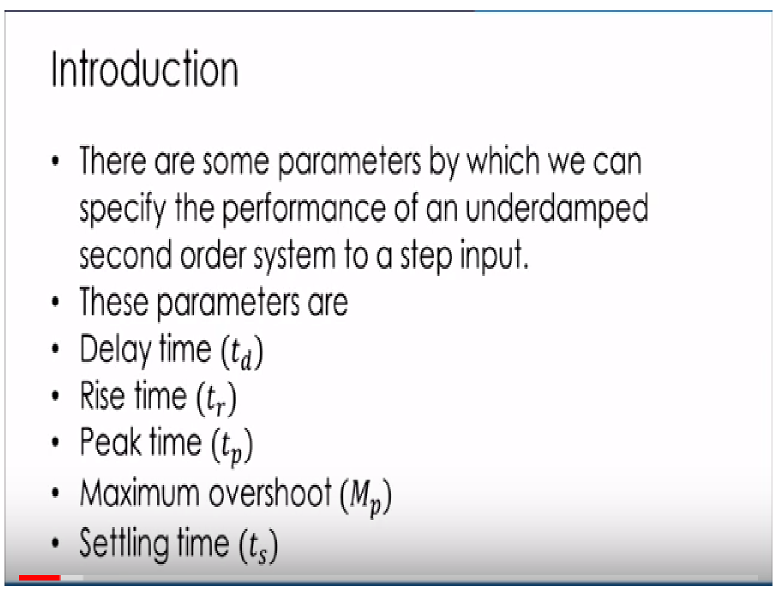

These parameters are say delay time rise time peak time maximum overshoot and the settling time, okay. And these parameters are yes these are by going through these parameters we can say about the behavior of the system and not only that if we know all these parameters for a system even we can create we can create the system behavior okay with the help of these parameters, so let us take the first one the delay time.

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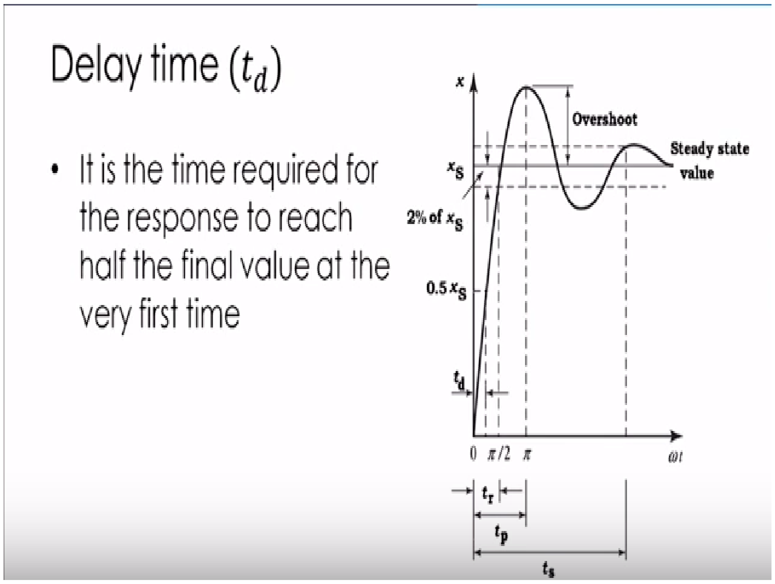

Which is represented here by okay, but the subscript basically tells for the delay and of course t indicates the time that delay time now you can see that here in this plot I have plotted or the output x with respect to the Omega t okay, were omega is the angular frequency fine and this is the typical behavior of a second-order undamped system for a second order system under damn system and when it is subjected to a step input okay, as we have seen.

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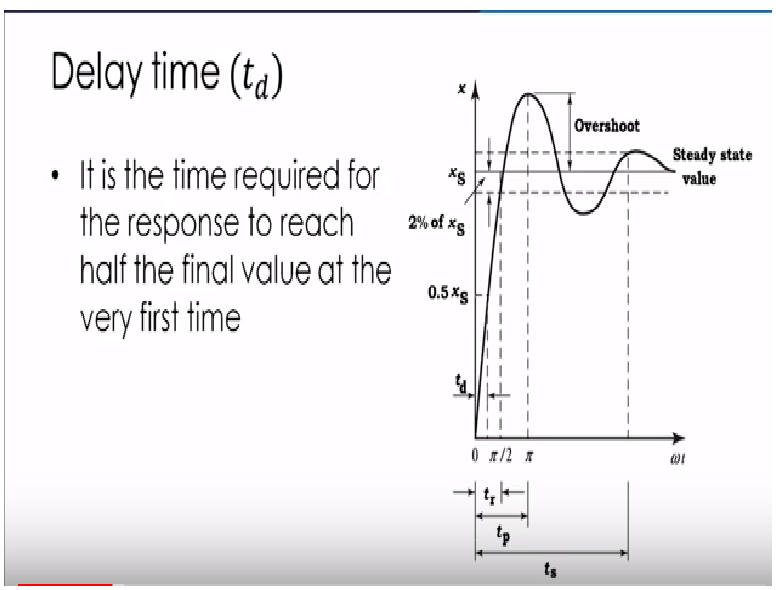

So, second order under damped system subjected to step input so in this behavior we can see that there are various parameters as I talk to you has been defined the delay time tt is basically this that is this parameter that is the td and it is the time required for the response to reach half of the final value at very first time okay so half of the final value final value means basically we are talking about the steady state value okay.

So, if the steady state value of the system is accessed, so the delay time is basically the time required for the system to go or reach up to the point five times of the steady state value very first time okay.

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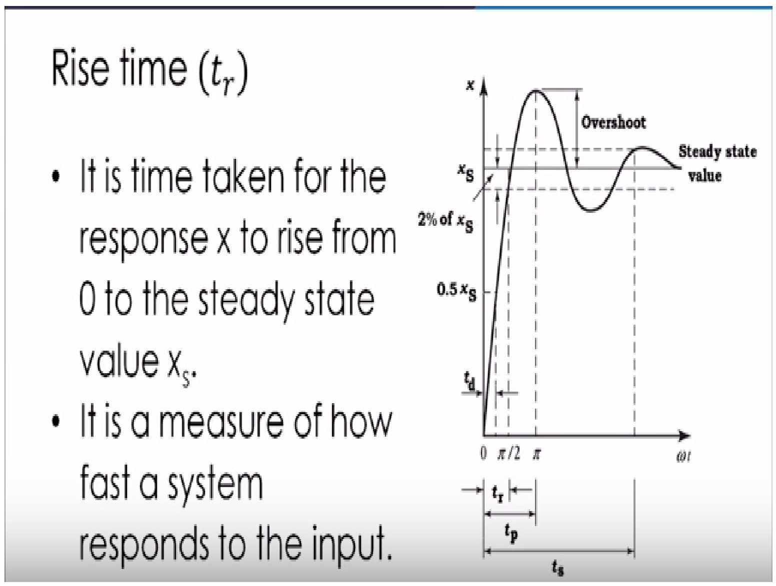

So, this is how we define the delay time next time is the rise time okay and you can see in this figure that this is that vice time represented by tr and this is basically the time required for the system to reach up to the steady state value okay.

So, as you can see it is the time taken for the response to rise from zero to the steady state value access okay and it is actually it actually indicates or it is a measure of how fast the your system responds to the input signal okay, so with the rise time we can judge about the system behavior that is how quickly the system response to the given input okay. It is the time for a oscillator response to complete a quarter of a cycle.

Basically are that is the pi/2 value okay, so from here we can see that this Omega tr= pi/2 value, and this is how the rise time is defined okay. For overtime system tr is considered as the value for rise of response from some percentage of steady state value okay. That is we can say that say from 10% to 90% whatever time is required okay for the output response stories and that is defined as the rise time for the overtime system.

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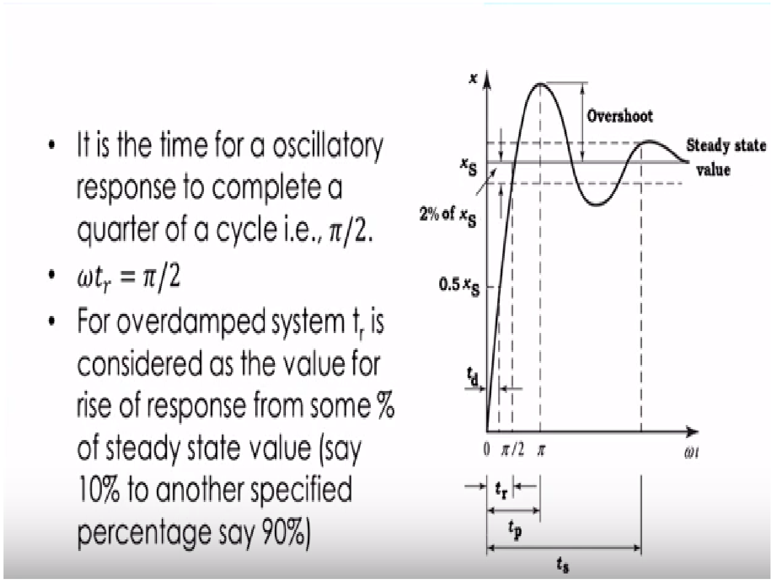

But as I said for the under damped system we define it as the time taken for the system to for the output to go from 0 to the steady state value. Next is the peak time now if you look at this plot you can see that this is the maximum value of the response x which the system could reach okay, so actually the peak time corresponds to the time okay by which the system attends the peak value okay.

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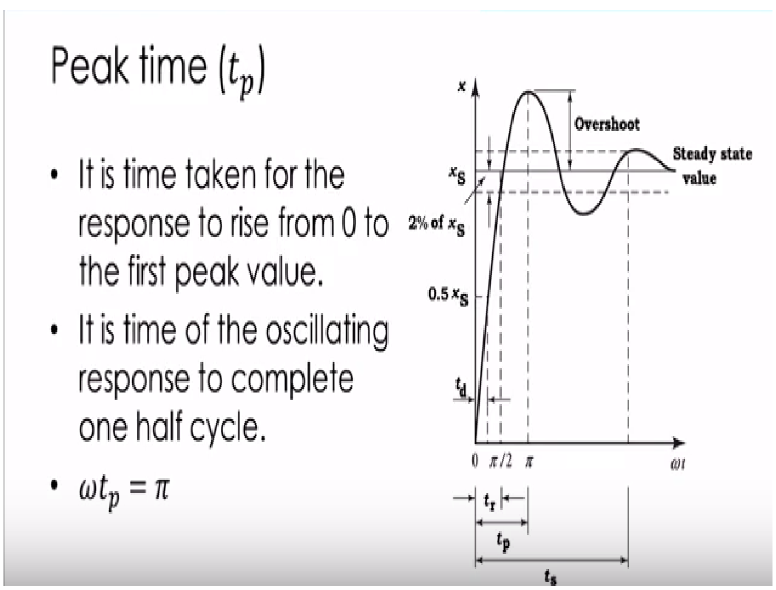

As I said it is the time taken for the response to rise from 0 to the first peak value and this is how it is defined and it is the time of the oscillating response to complete one half of the cycle so we write here Omega pp = pi and from here we can find out the pp value ok that is pi/Omega then next parameter is maximum overshoot mp okay which is represented in short form by mp.

So, here you can see that the value above the steady state value this is what is or we call it as the overshoot or this is also called as the maximum overshoot here in this case okay, so it is the maximum amount by which the response exceeds the steady state value okay and it is the amplitude of the first peak basically okay.

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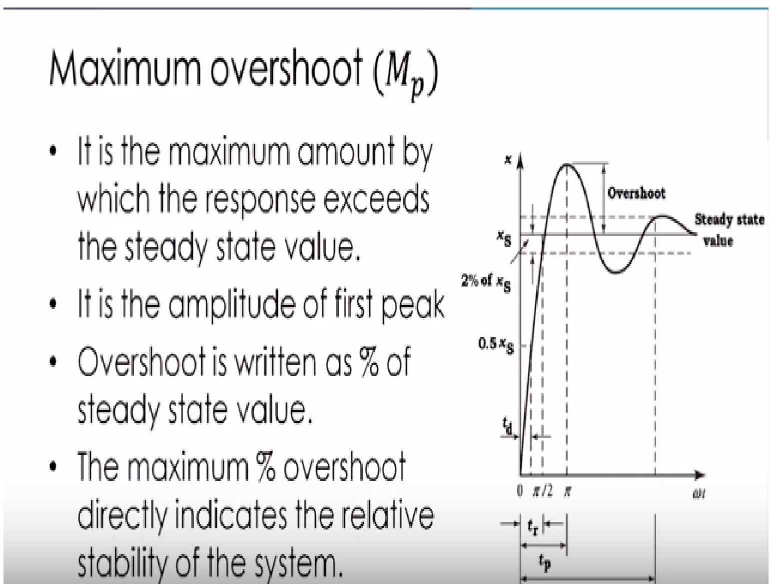

That is what you call as the maximum overshoot now overshoot is written as percentage of the steady state value and the maximum percentage over shoot directly indicates the relative stability of the system okay, how your system is stable or not aura() that indication is given by the maximum percentage overshoot okay, so we can find out how can we determine this overshoot or the maximum percentage overshoot okay.

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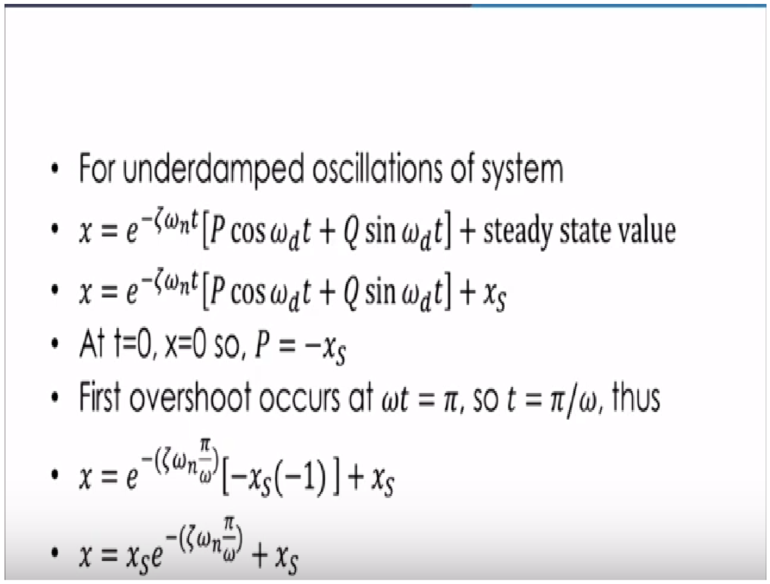

So, for under damped oscillations of the system we have seen in our last class that is last lecture that we can give the under damped oscillation of the system the amplitude x output is given by this relationship okay +the steady state value, so but basically this portion corresponds to that transient behavior add this is the steady state behavior okay our value corresponding to the steady-state.

Now as we have been noting steady-state if I represent it by access it is this equation. Basically okay, but you see that at t is equal to 0 x is equal to 0 this is the response as we have seen so if i substitute this here this 0 and here this value will become 1 and this value will be become 0 this weather will become one so what I will get x essentially is that p=-xs okay.

First overshoot occurs at omega t=pi this is how we have defined the peak time also again so here I can just write expression for t as pi/omega and if I substitute this value of t in this expression this is what I get -eta omega+ pi/omega and here this P is minor success and this value will become -1 and of course this value will become 0+xs that is what is going to be there and here I am writing this omega d basically is omega okay.

So, if I do that these minus will get multiplied, so we will have x here so x=xse to the power -eta zeta omega n+pi/omega+xs okay, so this is what expression we had I can take this access in the left hand side and this is x-xs is basically the overshoot value okay and so this overshoot value is given by this expression so this overshoot will be xs into exponential of this particular okay.

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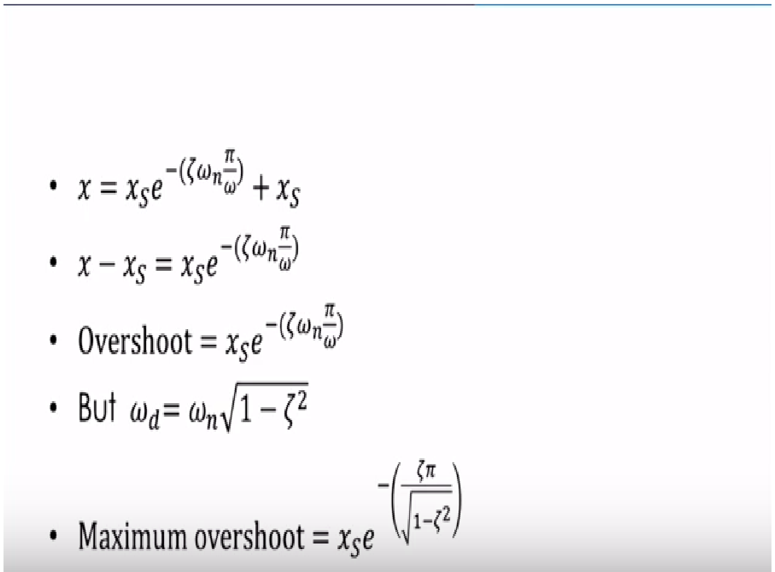

But you see I have been telling you that this omega is nothing, but our omega d value that is the damped natural frequency. So what I can do that we have derived it omega d in the last lecture as omega n under root 1-eta square okay. So here, I can substitute for this is basically your damping ratio.

So, I can substitute for this value here so I get the maximum overshoot as xsxe to the power this particular parameter where I have substituted for here omega d and so this omega n and this omega n will be getting cancelled and this is what I get so this is maximum overshoot okay and I can find out the maximum percentage overshoot just by dividing this by xs here this is what I get the maximum percentage overshoot and this is in terms of the damping ratio okay.

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- % Maximum Overshoot = $e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$
- Percentage peak overshoot

Damping ratio	Percentage overshoot
0.2	52.7
0.4	25.4
0.6	9.5
0.8	1.5

So, as you can see that as the damping ratio changes your percentage peak overshoot changes okay. So here I can substitute the different values of damping ratio and I can find out what is my % overshoot, so say a damping ratio of point to the % overshoot turns out to be damping ratio increases the % overshoot decreases okay and so this is how the system behaves with the change in the value of damping ratio okay.

Then, we can define the subsidence ratio or the decrement okay this actually provides the information about how fast the oscillations decays okay how for the oscillations dks and the system rates to the steady state okay. So if you want to find out that value we can find out this decrement and this is defined as the ratio of second overshoot to the first overshoot okay, so the first overshoot occurs at omega t=pi.

The second overshoot occurs as omega t=2pi. So the first overshoot here I can write here this is in fact the expression which we have derived earlier okay and for the second over should I can replace this pi value with that 2pi here okay, so this is the second overshoot I can find out by replacing pi/2pi here and then I can find out the decrement that is the ratio of second overshoot by first overshoot

So, this is the ratio which I get. So this way we can define the decrement and as I said it essentially tells you that how quickly your system is reaching to a steady state condition. Next important factor is settling time okay because we are interested in knowing that how the system changes from the transient behavior are some performance changes from transient behavior to the steady-state behavior.

So what happens that these oscillations will keep on doing for a very large amount of time Okay. So to get the settling time what is usually done is that we allow certain percentage of the steady state value as the variation and we find out that amount of time after which that particular value has reached okay, so for example here in this figure as you can see that we have allowed this variation to be 2% of xs okay it is 25% of xs.

That is taken as the your that variation has been taken for finding out the settling time so it measures the time taken for the oscillations to die away okay and it is measured by time taken by response to fall and remain within some specified % of the steady state value okay, so to fall and remain within that specified % of the steady state value we can talk about say 2 % value or we can talk about 5% value okay.

So, if you are considering say that 2% value so it means that the amplitude of the oscillation should be less than 2% of the axis okay. So that is what is called this time t as corresponding to that where the amplitude of the oscillations remains within 2% of the xs that is what is called as the settling time. So we can find out what will be the expression for the settling time again we go for the behavior of a second order under damped system.

We have seen the behavior given by this expression okay where xs you is the steady state value and other parameters we have already defined. Now here again as we have seen previously if I put at t=1 the x=0 that we know, so if I put it here I get the value of p=-xs okay, so I can put this value of p, so this x-xs are the overshoot value is this one okay. Now the maximum value will occur when Omega t is multiple of Pi this we have already seen okay.

When the Omega t is multiple of pi then we know that the fine Omega t terms will be 0 and cos omega t will be one always okay so in this condition suppose we are talking about say 2% of the settling time then the time when magnitude is 2% are factors that we can find out okay. So let us in this case what as I said we are talking about x-xs as 0 2 of the xs value okay x-xs is 0, 0 2 of xs value.

So, I can just say that 0 2 into xs equal to this expression xs to power-eta omega and ts okay, so this I can substitute it here and then I and this expression I basically got it from this term okay, because I am making this as one I am making this as 0 and I am interested in absolute value. So this is what I am getting it here okay and when I take the log of both the sides this is what I get or I can round it off and this ts I get as for by eta Omega n okay.

So this is my settling time corresponding to 0 2% of the value okay that is the 0 2% of the amplitude of the oscillation okay that is the steady state value fine. Now if the settling time corresponds to 0 5% up the variation in the steady state value. Then we can see that this expression turns out to be here numerator you are going to have the 3 and risk of the things remains the same okay and it is this ts value, which we are interested in okay.

This basically tells us that how quickly your system or transforms from or is behavior transforms from the transient behavior to the steady-state behavior. Then we may also be interested in knowing the number of oscillations okay. So if we want to find out the number of oscillations we can get it by the settling time by the periodic time okay, so if the settling time ts corresponds to say 0 2% % of the steady state value.

The number of oscillations will be this settling time/the periodic time which is 2pi/omega okay or I can just simplify this expression this omega goes up in the numerator so this is what I get and this omega is nothing but omega d, so I can substitute for this omega s omega n under root 1-dt square so this is what I get and if I simplify I can find out the number of oscillations and you can see that this expression is in terms of the damping ratio okay.

So this way, I can find out the number of oscillations which are going to be there okay before the system gets shattered all right. Now let us take an example to illustrate how can we find out these performance parameter okay, so I say the equation system equation for a step input is given as this one okay so this is a second order system as we can see d square x/dt square+5dx/dt+16x=10x.

I am interested in finding out for this expression the under damped angular frequency the damping factor say the damped angular frequency rise time peak time say maximum percentage maximum overshoot and say the 0 2% is of the settling time okay. So I can find out that so we know for a second order differential equation of this form the last soft model we have seen that if my differentiate general differential equation second order differential is of this form.

Say a2d square x/dt square+a1dx/dt+a0x=b0y for this system the natural frequency angular natural frequency and the damping ratio can be defined as this one these are in terms of the coefficients of the differential equation okay. So, for this problem which is under our consideration I can compare this equation with our general expression and I can we can see that a 2=1 year a1=5 and the az row is 16 okay.

So from here, I can evaluate this omega n square term 16/nr omega n is for Radian per second okay similarly I can find out the damping ratio from here by substituting the values of a1,a2and a0 here so this is 0.39 or the damping ratio will be 06 25. So, once I have got the natural frequency and I have got the damping ratio. I can find out the damned angular frequency okay.

So, the damned angular frequency I can find out omega n=omega, omega d=omega n under root 1-theta is Scott so by substituting here. I get this omega d as 3.1 double to say Radian per second. Now, we know that for the rise time omega tr=pi/2, so from here I can get tr value pi/2 Omega and this Omega is nothing but the damn natural frequency here okay.

Angular frequency, so I just put pi/2 Omega d or I substitute it here and I get the rise time as 0.52okay. Similarly I can get the peak time by putting Omega tp=pi and if I substitute value of Omega here, which is nothing, but Omega d I get the peak time okay and I can find out the maximum percentage overshoot because I have already determined eta value okay.

So, I can find out the maximum % overshoot here by substituting the value of eta here that is the damping ratio and I get this value as say 8% fine. Then, I can find out the 2% settling time and that is we have seen for by eta Omega n and this is say 1.6 second okay. So with this I would like to end this lecture these are the references which you can go through the Bolton a book by Bolton on Mechatronics Pearson and Ogata, Modern Control Engineering. Thank you.