Modelling And Simulation of Dynamic Systems
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Lecture – 21
Dynamic Response of the First Order System

I welcome you all on this lecture on dynamic response of the first order system, so till now we have seen or the basic building blocks for various types of systems say mechanical system, electrical system, hydraulic, pneumatic, thermal and then we have seen the combinations of the system also such as electromechanical system, hydro-mechanical system.

So on now in this lecture, we will be seeing the dynamic response of the first order system or dynamic response for the systems whose dynamic behavior can be described with the help of first order system, so we will see that how these systems we have or how we can find out the behavior of the system.

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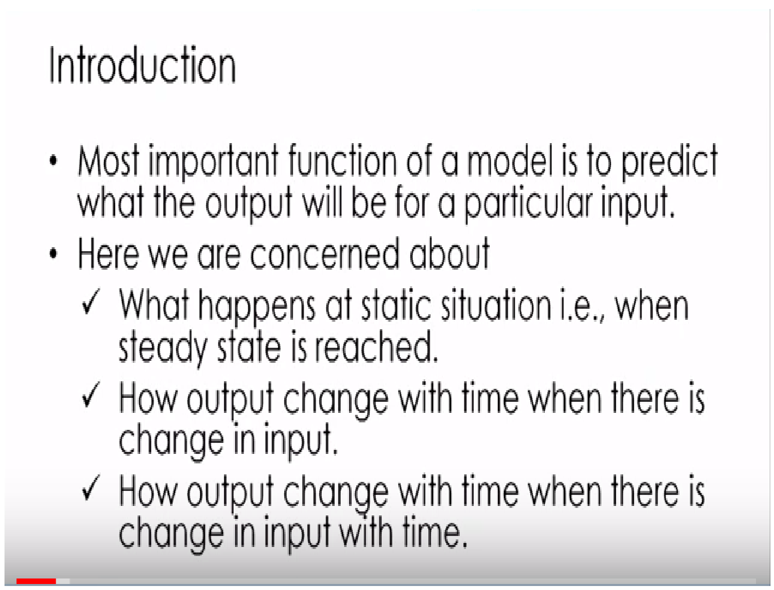

So as I was telling you, I have been talking to you throughout this lecture the most important function of the model is to predict what the output will be for a given particular input. In fact, that is the most important function. Our most important job of any dynamic system or job of any modeling exercise that is we want to know the output of the system for a given particular type of input.

Now we are concerned about these 3 aspects mainly, that is the first one is what happens at static condition. All say that is when steady state is reached what happens what is the behavior, what is the magnitude okay. How the output changes with time, when there is change in input okay, so if input is changing, how output changes with time and how output changes with time. When there is change in input with time okay, so these are the 3 cases, which are particularly important in a simulation study of the system.

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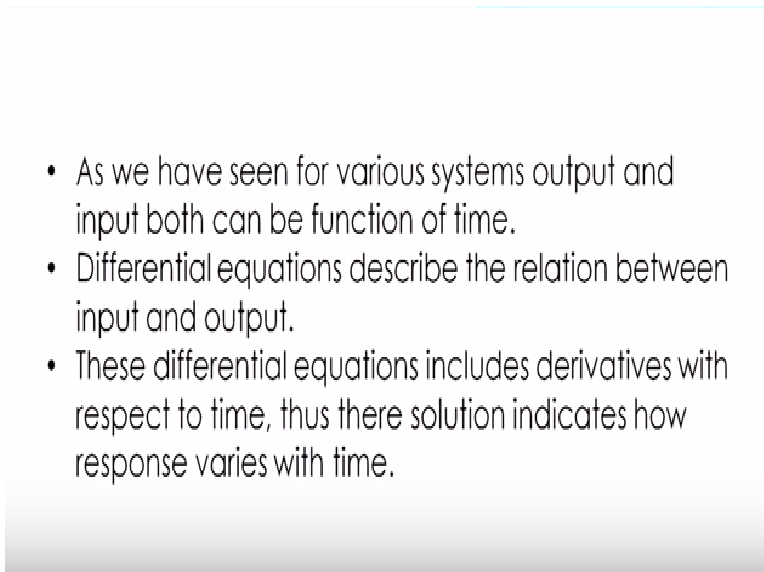

Now, as we have seen for various system output and input both can be function of time and the differential equations, describe the relation between the input and output these differential equations includes derivative with respect to time. Thus, their solution indicates how response will vary with time.

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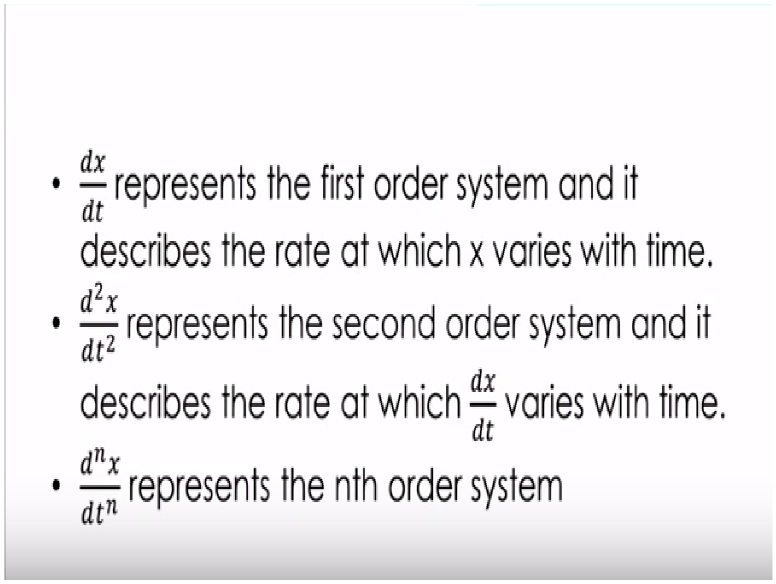

Now as all of you know dx/dt represents the first order system and it is described the rate at which X varies with time. Similarly d square x/dt square represents the second order system and it describes. How dx/dt varies with time and similarly, we describe the dnx/dtn and for the nth order system. So here in this lecture will be discussing about a system involving dx/dt term which we call it as the first order system now, the first order system examples are many in fact so let us take this example that is the water coming out from a tank okay.

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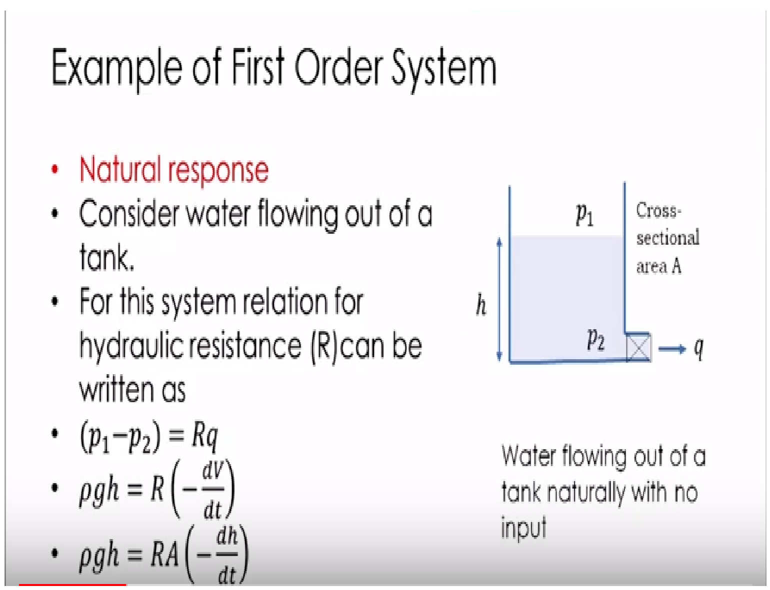

Suppose first we consider the natural response. Natural response means that here there is no input okay, so there is no path filling this tank okay. So in that case let us see how this h varies okay. So for the natural response okay, as I said consider the water flowing out of a tank and for this system the hydraulic resistance R can be written as p1-p2 =Rq. Basically the equation very similar to that of the electrical resistance that is V=IxR okay. We are here, q corresponds to I and we are p1-p2 corresponds to V and of course R represents here the hydraulic resistance.

Now in this case, we know p1-p2 as Rho gh and the q will be basically dV/dt and I am using a - sign just to indicate that this rate of flow is negative that is it is decreasing okay, so when I put it here then what I get Rho gh= RA-dh/dt.

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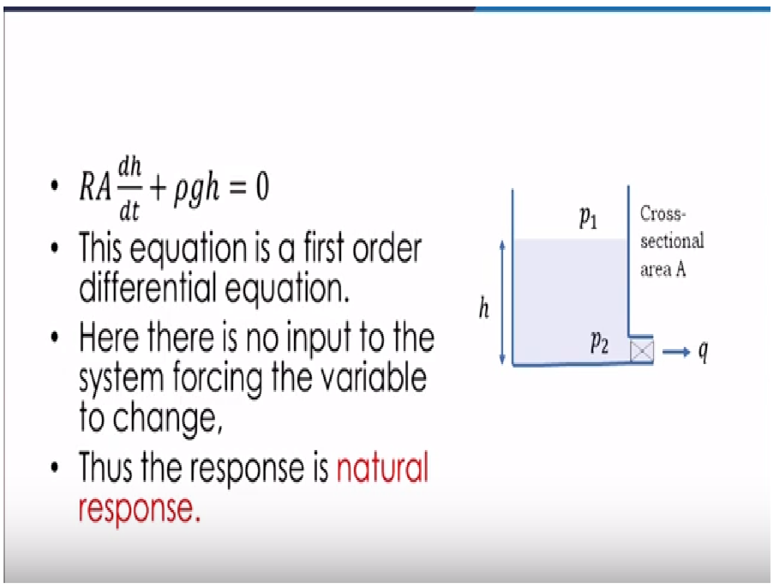

This is how it look like now we call this equation is their first order differential equation because here you can see that our h is the output variable that is our aim is to find out how h varies with the time. This is natural response because here there is no forcing function okay. Nothing we are we have assumed that there is no other tap which is going to fill up the water okay. So this is 0, so this equation is first order differential equation.

And as I said here there is no input role system forcing the variable to change and thus the response of the system is called the natural response of the system.

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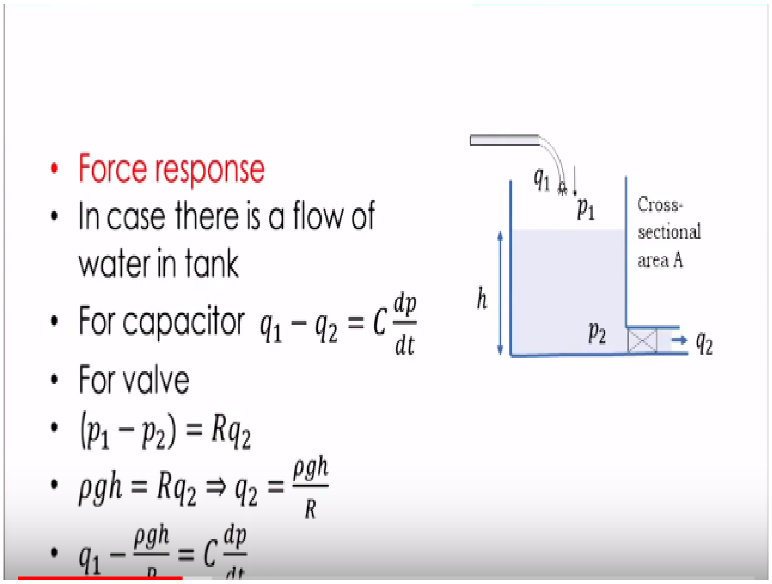

Then we could have a condition, where there is a tap with certain water coming from the tank and into the tank okay and then the response of the system whatever will be there that we call it as the force response. Fine, in this case there is a flow of water in that tank and we have already seen this derivation when we were discussing with the hydraulic system. I just want to take you to the first order differential equation in case of force response okay.

That is in case the flow of water is taking place into the tank okay. Then how the system behaves so far device are defining the hydraulic capacitors. I can write q1-q2=Cdp/dt here and then here for the world. I can write p1-p2=Rq2 and this is basically my Rho gh so.Rq2, so I find out q2 I substitute this q2 value here. So this is what I get q1q1-Rhogh/R=Cdp/dt. Next I can substitute the value of C which is A/Rho g rest of the things remain same okay.

This is how this is what is the reduced equation and this is how I can write the equation. Now, here you see what we have we have the output variable h here and there is a first derivative of the h here okay. And we have the input variable q1 here and this is basically the forcing function okay so the response of the system are say response of this differential equation.

The first order differential equation is what we call the response in presence of the forcing function okay. I hope the concept is clear.

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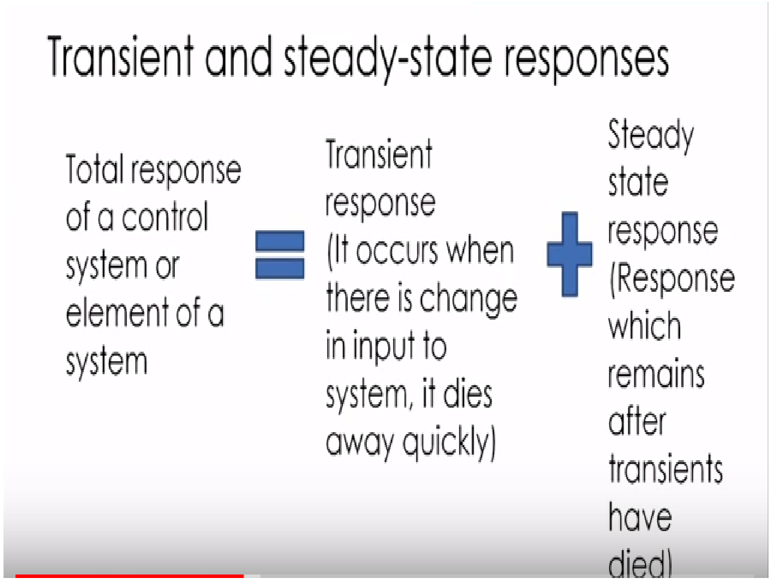

Then let us see the total response of the system. Now if I talk about the total response of the system it could be basically could be divided into divided as the summation of the 2 response okay and these summation are first one is the transient response and the steady-state response. Now transient response is basically it occurs. when there is change in input to the system okay, as the name indicates the transient response died out after some time okay.

And the steady-state response is the response which remains after transient have died and when we sum of both these type of response what we call it as the total response of a control system or it is a total response of the element of a system.

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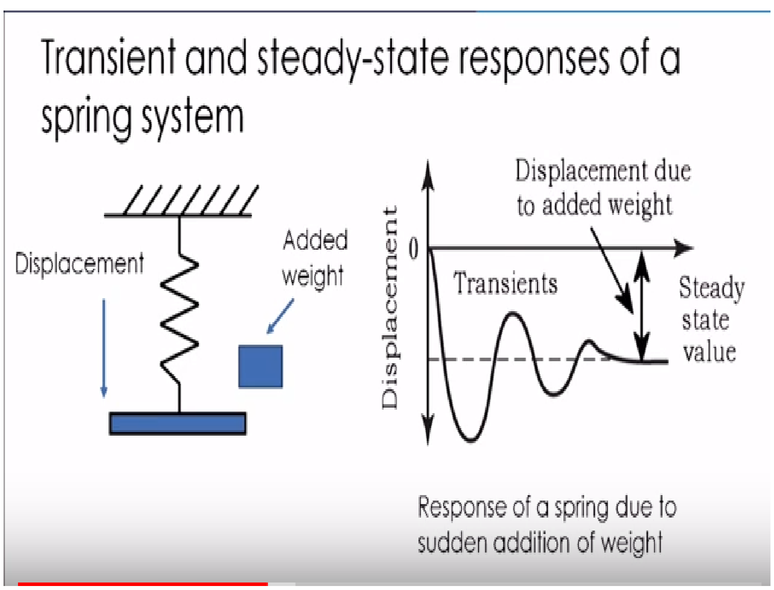

Let us take an example suppose, I have got a spring here and say there is some hanger here where I can aid up a weight okay. I say about the displacement of spring takes place in this direction. Now what happens initially there is no weight on the hanger which is attached to the spring and after some time I add a way to it. So what I am doing I am changing the input initially the input is 0 and then the input is that of equal to the weight.

And now as all of practically feel that or what practically, we have observed that after adding the weight, what will happen the spring will oscillate a little and then it will settle at a permanent displacement at a displacement specific displacement of the spring that specific displacement value will depend on how much weight has been added and what is the stiffness of the spring okay. So here if we look at the response of the system this is what happens you can see that initially there is variation in the displacement okay.

And then this displacement settles for a particular value so this Joan where there is a variation in displacement. What we call it as the transients or this portion of response is what is called as the transient response and this portion of response what we call it as the steady state value okay and of course this value of the displacement as I said is the displacement due to the added weight and of course this depends on what is the weight and what is the stiffness of the spring.

So you can observe here the response of the system to be consisting of the transient response, the steady-state response okay.

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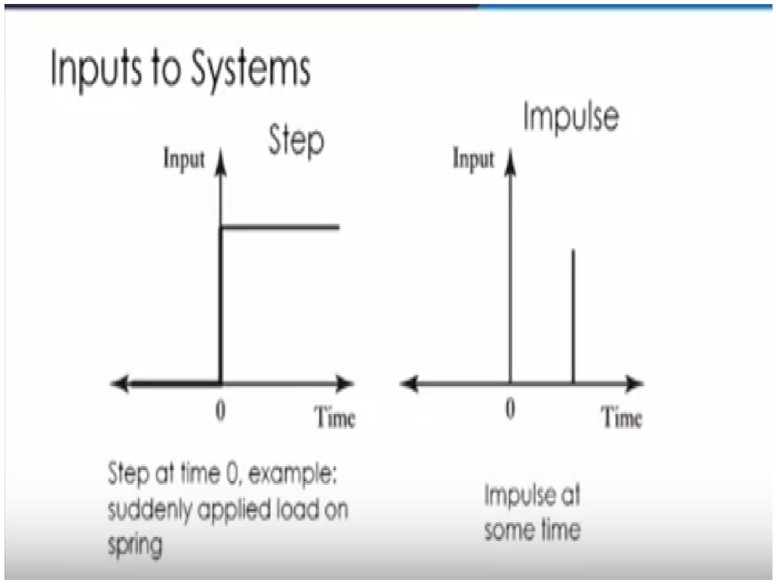

Now the previous example, which have seen that is adding up the weight on the hanger what it is basically it is a input to the system okay. It is the input to the system alright and that type of input. What we call it as the step input that is initially the value is 0. And then a definite value is given so this type of input is what we call it as the step input okay. We need to know the various types of inputs for which the system is subjected to in order to know the behavior of the system okay.

Then there could be another type of input what we call it as the impulse input okay. It is basically impulse at some time here at certain time value some time value you can have some value of the input and type of input what we call it as the impulse input. Then we can have another type of input a ramp input, ramp input means the input which varies with time okay. Linearly basically here if you say that the input is y then y=kt okay.

This type of input is what is called a ramp input, other type of input could be the sinusoidal input. You can see here the curve for variation of input with time is sinusoidal in nature and here. This input could be written has say k sine Omega t where k is a constant and Omega is the angular frequency which we write as 2piF and where f is called as the frequency. So your system could be subjected to any of these 4 types of inputs that is step, impulse, rhyme or sinusoidal.

Now, let us see the first order system okay. So, in a block diagram mode this is how we represent our system and say this system is subjected to certain input yt. We get an output say xt now I can write the expression for this first order system something like this okay. As we have seen that in case of water tank okay. The expression is something very similar to like this that is we have a constant then dx/dt that is the derivative first derivative of the output plus some constant into the output is equal to some constant into the input okay.

And here you can see that the highest order for here derivative is 1 here so this is what we call it the first system now actually have seen there could be 2 types of responses for the system one is the natural response and another is the first response. So let us see the natural response of the system. Now if natural response we are talking about naturally then the input has to be made 0.

As in that case of the water tank example which we have taken previously there was no flow from the tank okay so let us take here this input is 0. Now when this input is 0 this is the system equation fine. Now let us suppose the solution for this expression is x=Axe to the power st where A and s are constant then what we can do we can determine the value of this constant say s here by substituting this solution into this equation.

Now, if I substitute the solution into this equation this is what I am going to get a1Ase to the power st+a0Aeto the power st=0 and from here, I can find out the value of s=-a0/a1 okay. Now so by expression becomes x=Axe to the power st okay and when I substitute the value for s this is -a0/a1xt this -a0/a1, so this solution becomes like this. Now I can determine this constant A from the initial condition okay.

So, it can be found by initial condition say as time t=0,x=1 here, so when I substitute t=0 where x=1. I get A=1, so the output can be given as this one x=e to the power - a0/a1xt okay and at t=0 x value is 1 and then the rest of the behavior we can see here that is exponentially decaying behavior, so this is what is going to be the natural response of the first order system okay.

Now let us see the response of the first order system with a forcing function okay For example that water tank example which we have taken say there is a tap from which the water is coming into the tank alright, so if that is the equation say a1dx/dtxa0x=b0y let the solution. Now here you see that this part is not 0. Here this b0y basically represents that forcing function.

So, let the solution in this case be x=u+v and say u is the transient part and say v is the steady state part okay. Now let us substitute this x=u+v here okay. So when we put it here x in place of x, u+v. This is what we get and then I club that terms related with you here and that translated with v here and here is my forcing function. Now, suppose this part okay I take it as 0 that is if I make this part as 0.

Then naturally this part is going to be equal to b0y alright. Then the first equation is the differential equation with natural response, whereas the second is with the forcing function here. You can see the second equation is a first order differential equation with the forcing function and the first equation is a first order differential equation with the natural response here okay.

So, for the natural response we know the solution u=constant a, which we can determine from the initial condition xe to the power-a0/a1xt okay, fine now for the force response, output will depend on input that we know, so let input be a step function okay. For the force response output has to depend on the input for natural response there is no input so that value is 0 input value is 0 okay.

Now, suppose here let us take a step input in this case okay that is a t=0, so the step size of the step being k okay, so say my input is something like this that is initially it is 0 and then it has got a step size of k this is time okay and this is my say input now.

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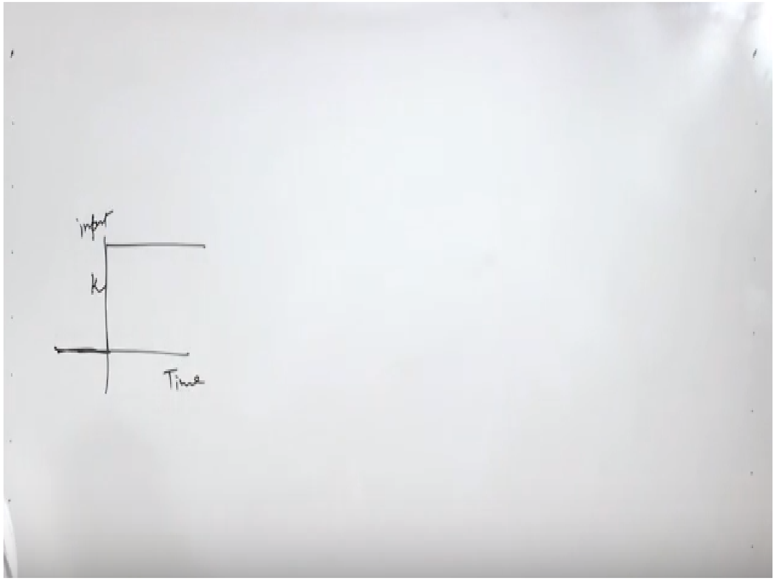

So what happens this was my equation for the force response part of the total differential equation okay.

Fine now let us assume the solution to be v=b a constant okay. Then what I do I substitute this solution into this equation this term will be becoming 0 and here what I will have a0b=bo and input assumed but k value okay the input is k here this value which I have taken, so I substitute it here and I find out the value of the constant this b that is b0/a0xk.

So the solution this v is going to be v0/a0xk and so the complete solution x will be u0 sorry u+v that is the natural response plus the force response part okay. When I substitute for you and we this is what equation I get so this is your natural response part and this is your force response part okay. So I put that equation here.

Now here you can see that there is A constant a all right I can find out this constant again using the initial condition say at t=0, x=0. Then if I put that I get well this is 0 this term will be 0, so I a will be –b0/a0xK and then I can substitute this A into this expression this is what I get okay, so this part is basically the natural response part and this part is the force response part okay so this is my general expression general solution, rather okay.

Now if you look at this equation you can see that at t tends to infinity this part is going to be 0 okay, so this part when becoming 0 whatever I am getting it is a constant term b0/a0xK and this term is what we call it as a steady state term or steady state response, so today state response x is b0/a0xK ok, so for this input and step input up size k okay.

The response that is output is this one and this steady state value which occurs at t tending to infinity will be b0/a0xk okay. So this is input and this is the resulting output of the first order system okay. So fine I have taken one example of the first order system that is that of the water tank okay.

Filling of a tank fine, we can have another example that is a DC motor now we have discussed this DC motor modeling in the electromagnetic section okay. modeling of electromagnetic systems okay, so here relation for an armature control motor this is what we had that is Va is the armature voltage Omega was the angular velocity of the motor La is the inductance of the motor ia is the current and Ra is the armature resistance of the motor okay.

So they are and the second equation for the system is this was that is acceleration of the load Id Omega/dt this was k4ia-RbOmega where Rb is that damping okay or the bearing resistance Now, here what we can do is that we can neglect save inductance part all right and we can substitute ia in the second equation okay.

From the first one to get the relation between input Va and the output omega, if you do that I am not explaining you, you can work it out if you do that this is the equation which you are going to get okay, so here you see you have input part Va and here you have output part Omega. So this equation is basically the first order differential equation okay and we can compare this first order differential equation with the general form of the equation which we have seen okay.

Now you can see that here x corresponds to the output here. So, here is omega here x so omega here input disturbs corresponds to y and of course a1 is a constant here a0 is 1 and b0 is 1/k3 like that okay so this way we can have one-to-one relationship okay and then we can see the response for this system similarly as we have seen for the general case okay so I leave it as an exercise to you just have a look at it.

Now, let us see that time constant okay. so our general expression general solution for the first order system has been this one okay and we know that this b0/a0xk was the steady state value so it is a steady state value x1-e to the power-a1/a0xt. Now, here if you look at this equation, if I put t=a1/a0 then here those a0a0a1a1 terms will get cancelled okay.

What you will be getting is that 0.63 evaluations of all these things okay so, what we can say that at t-a1/a0. The x value is point is 0.63 times is that of the steady state value okay and this time, tau of magnitude A1/A0 is what is called the time constant of the system okay.

So the response for first order system for a step input we can write in terms of the time constant as x= steady state value x1-e to the power-t upon time constant or-1/time constant xt.

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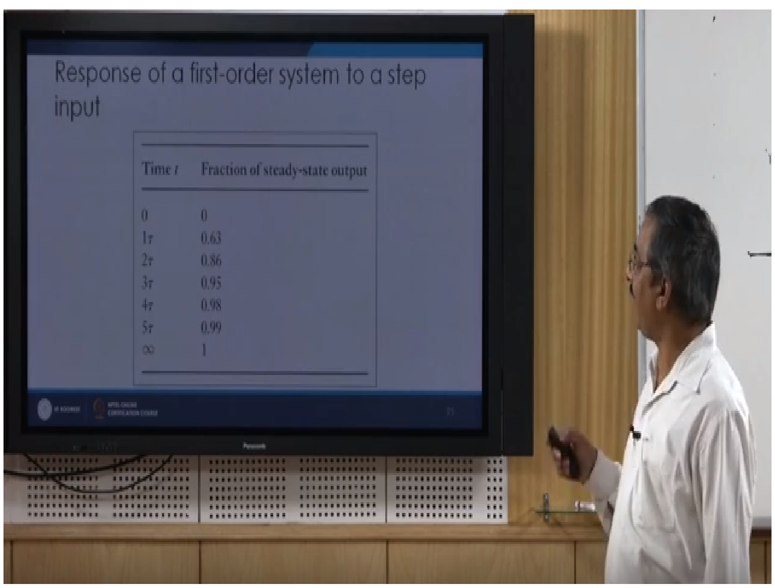

So, if you look at that that expression then as I said at one time constant the fraction of steady state output is going to be 0.63 okay. Likewise, we can look at 2 time constant 3 time constant and act infinity the value is going to be that of one that is which is that yeah steady state value okay. And this is how we can plot that okay. That is here at 0.63 and 0.86,0.95 and at 1 so you can see that at 1 we get the steady state value okay.

So these are the references if you want to read further about this the Bolton and our book has got some discussion on this okay, so I thank you and in next lecture we will be seeing the behavior of the second order system, thank you.