Modelling and Simulation of Dynamic Systems

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Lecture No - 20
System Models of Robots

In this lecture on system model of robots which is a part of modeling in simulation of dynamic system course. Here in this lecture next thirty minutes we will be looking at how we can model a robot. As all of you know that a robot is an example of the mechatronic system. Here you will find the arms of the robot they have got their own dynamics and you can that these are there in the part of the mechanical system. Now there are actuators which actuate these joints.

And this actuation is usually the motors which we consider to be in the electrical domain apart from that there are going to be sensors and coder's potentiometers vision sensors there can be many more peripheral things are going to be there. So it is a true example of the mechatronic system now in this lectures on ah robots essentially I would like to take you up to the development of the dynamic equation of the manipulator. But before that I need to give a little

Background of the robots and its kinematics and them we will move on to the dynamics because without kinematics we cannot go into the dynamics of the manipulator. So remember aware an aim here is to develop the dynamic equation for the manipulator.

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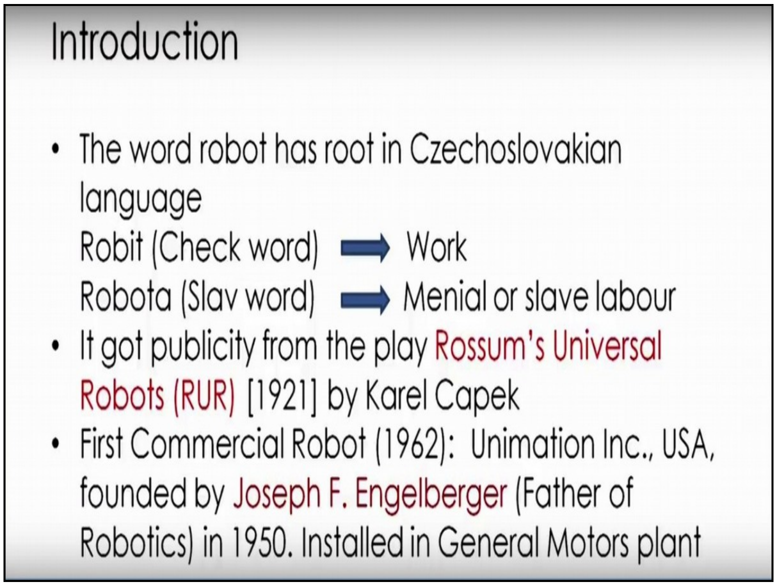

So the word robot has root in Czechoslovakian language. Where the robot is the check word means work and robot which is a slave word it means that menial or slave level. It got publicity from the play resumes universal. Robots in 1921 by karelcapek and first commercial robot came into 1962 and that was by the animation founded by Joseph f Eagleburger who is called the father of robotics in 1950. And it was installed in the general motor plant.

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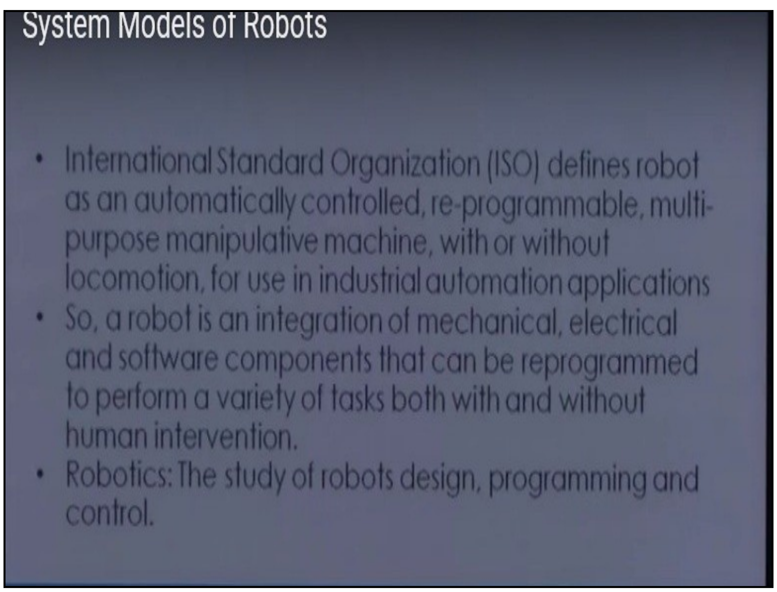

There are many definitions for the robot but I would like to just go through the international standard organization is definitions which defines robot as automatically controlled re programmable multipurpose manipulative machine with or without locomotion for using industrial automation application. Now here the world re programmable is basically the world which distinguish the robots from the fix automation here this device is reprogrammable where as in fixed automation the programming is difficult.

So where I was telling you a robot is an integration of mechanical electrical and software components mechanical I j said the arms of the manipulator electrical means the actuators which actuate the joint of the manipulator and software which is essentially different codes intended for the control of the manipulator.

That can be program to perform a verity of task both with and without human intervention. And robotics is basically the study of these robotics design programming and control so before as I said moving onto the dynamics I would like to give you a little background so first let us see the mechanics and control of mechanical manipulator.

So here as you can see that a manipulator has been shown and there are certain objects which are kept in the involvement of the manipulator. Now in robotics we are mostly concern with the location of the object in the three dimensional is space because robot needs to manipulate that object. Now this object which is there in three dimensional is space that can be describe by two attributes and these attributes are the position and the orientation.

So the position of the object the three x y z coordinates of the object and three orientations of the object.

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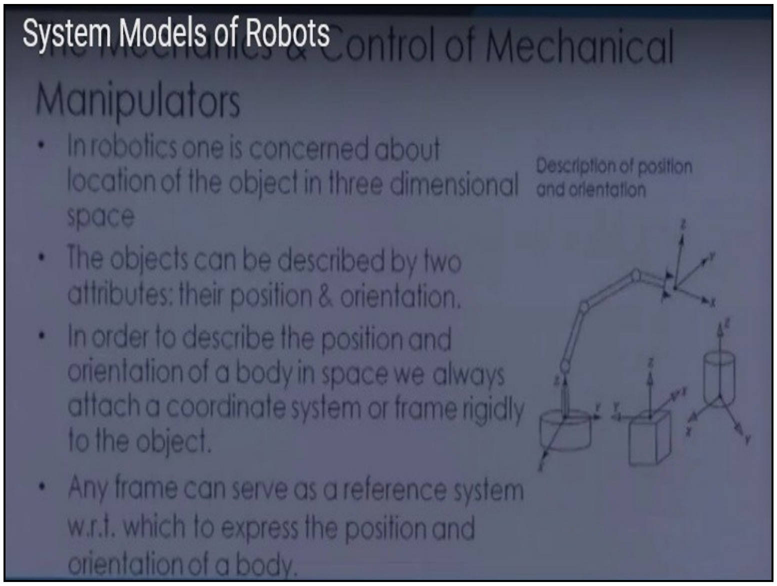

Now in order to describe the position and orientation of a body in a in a space we always attach a coordinate system to the body to that particular object and then we describe that co ordinate system.

And in this process any frame can serve as reference system with respect to ways that expression for the position and orientation of a body can be described. So here you can see that we are fixing a frame at the base of the robot we are fixing a frame at the end effectors of the robot and we are fixing different frames to the object which are there in the surrounding of the robot.

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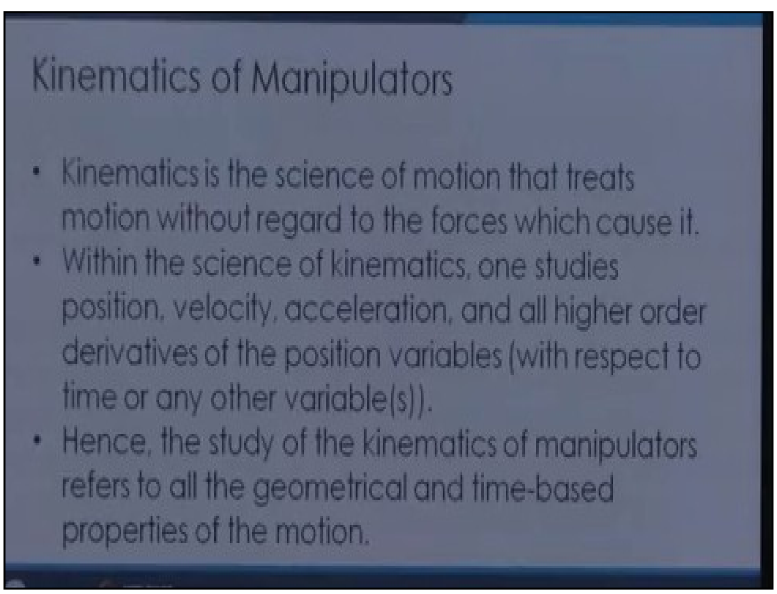

Next important thing is kinematics of manipulator. Now as all of you know that Kinematics is science of motion that treat motion without regards to the forces with cause it so here with the within the science of kinematics one studies position and its derivatives.

Its higher order derivatives so position velocity acceleration and all other higher order derivative of the position variable and it could be with respect to time or with respect to any other variable. Hence as I said the study of the kinematics of manipulator refers to all the Geometrical and time based properties of the motion.

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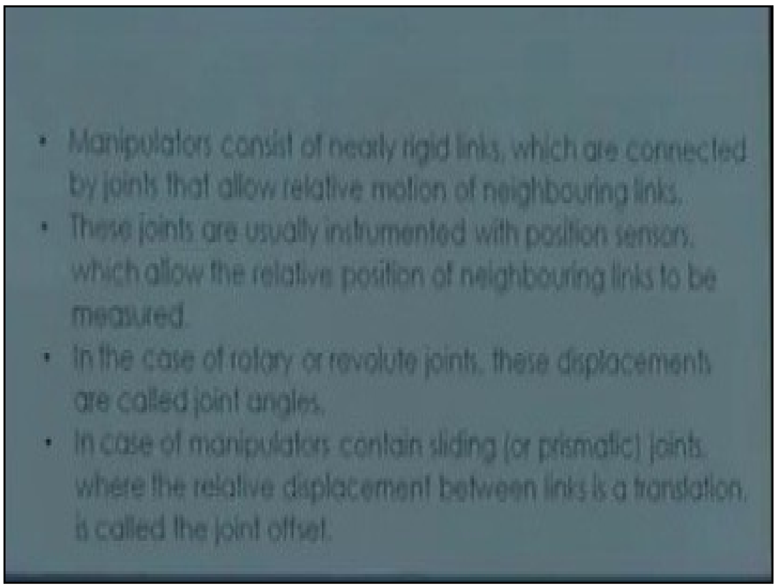

Now manipulator which I am going to discuss here we assume that they are rigid link. Because if manipulator links are not rigid there are flexible there is different treatment which is to be needs to be given in order to take care of the flexibility of the manipulator that I am not going to discuss with you today.

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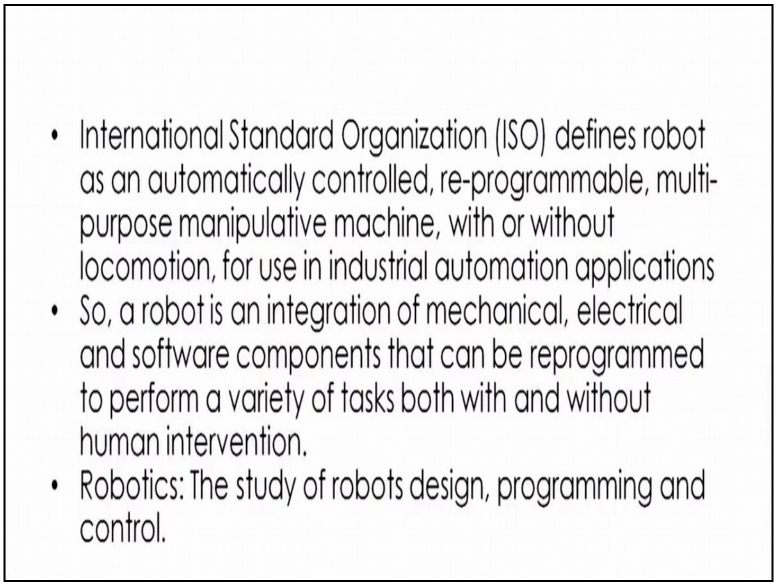

Here so here the manipulator consist of nearly rigid links which are connected by joints that allows relative motion of the in every links. So this is what assumptions we are going to make in our course of the study. Now these joints as I said are usually instrumented with positions sensors which allow the relative position of the neighbor link to be measured.

Now these sensors could be as I said it could be encoder it could be potentiometer. Now in case of rotary or revolute joints these displacement what we call it as the joint angle and in case of manipulators if it contains some sliding joint which we also called as the prismatic joint the relative displacement between links is a translation and that we call it as the joint offset. So first of all let us see what do you meant by the forward kinematics of manipulators.

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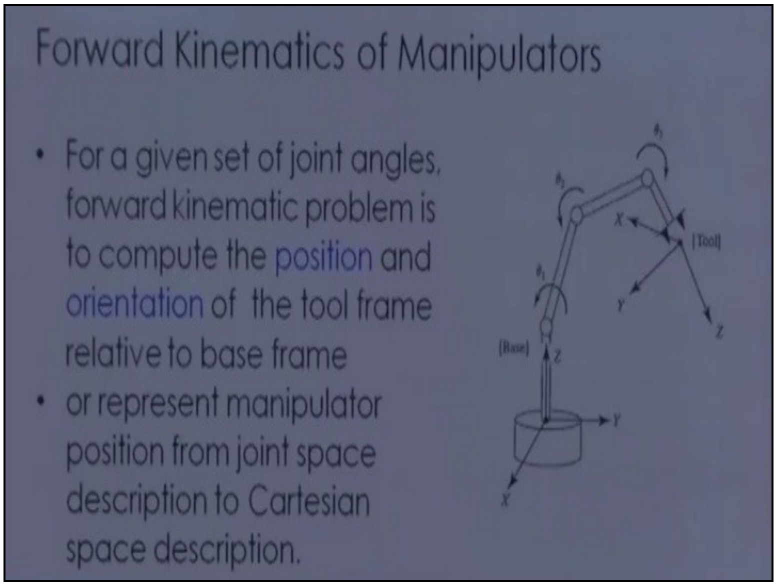

Here basically we will be seeing understanding what is the forward kinematics and what do you meant by the inverse kinematics of the manipulator. Now as you can see in this figure this is a manipulator and we have a co ordinate system fixed at the base and one co ordinate system fix at the end of vector which we also call it as the tool co ordinate system.

Now you can see that the are the three revolute joints and this joint rotations are represented by theta one theta two theta three. Fine so the forward kinematics basically means that for these given joint angles what are the Cartesian on ordinates of the tool of course which respect to a reference co ordinate system.

So as here the forward kinematics problem to compute the position and orientation of the tool frame relative to base frame this is what we mean the position and orientation computation of the tool frame which respect to base frame for given joint angles. This is what we also call it as the representation of manipulator position from joint space description to the Cartesian space description and this is what we meant by the forward Kinematics of manipulator.

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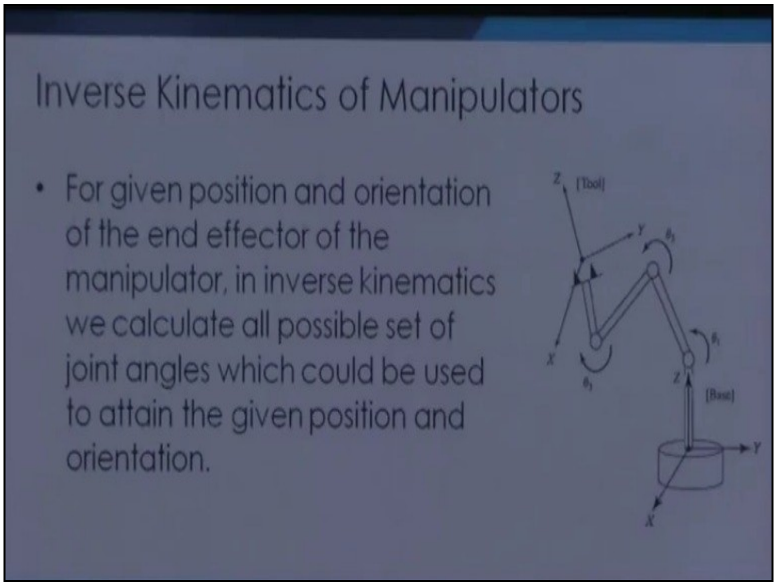

Now let us see what do you meant by the inverse kinematics of the manipulator now here as you can see in this figure. Again a manipulator is there we have the two frames that is the base frame and the tool frame. Now question is that if we have tool frame position and orientation if it is known then we are interested here in a new inverse kinematics finding out the different joint angles which will give us those position and orientation of the tool frame which respect to the base frame.

So here for given position and orientation of end effectors of the manipulator we find out the all possible set of joint angles which could be used to attain the given position and orientation. So this is about the forward and inverse kinematics of manipulator.

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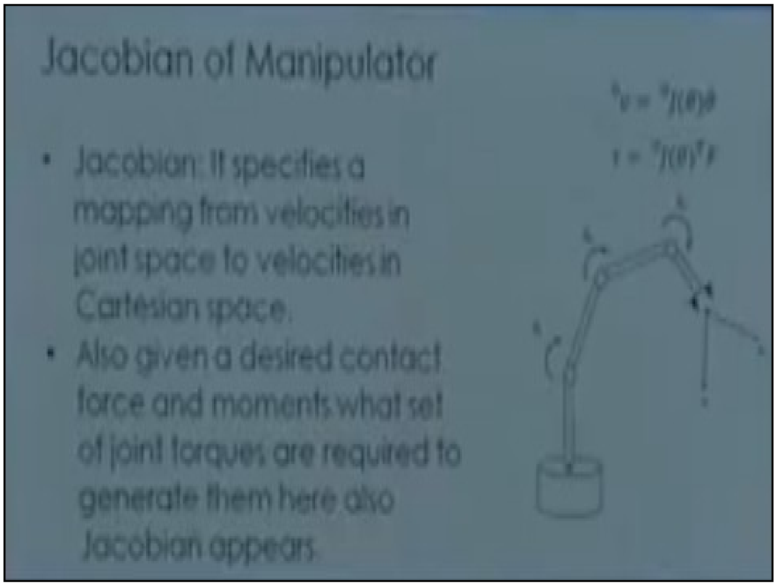

Next is the Jacobian of the manipulator. Jacobian actually is specifies a mapping from velocities in joint space to velocities in Cartesian space. And of the relationship is something like this for this manipulator you can see that the joint space velocities are theta 1 dot theta 2 dot and theta 3 dot and the Cartesian space velocities are save angular velocities are omega linear velocities are v.

So here the mapping between this joint velocity vector theta dot and the Cartesian velocity vector v. This is where the jacobian comes into the picture. Here zero basically means that these are specified with respect to the 0^{th} frames which are nothing but the base frame I have talk to you. So here we will also see that for a given desired contact force and moment what set of joint torques are required is we are interested in knowing that in there also this jacobian comes into the picture. And that relationship is equal to j transpose into f. So this is how this comes into the picture.

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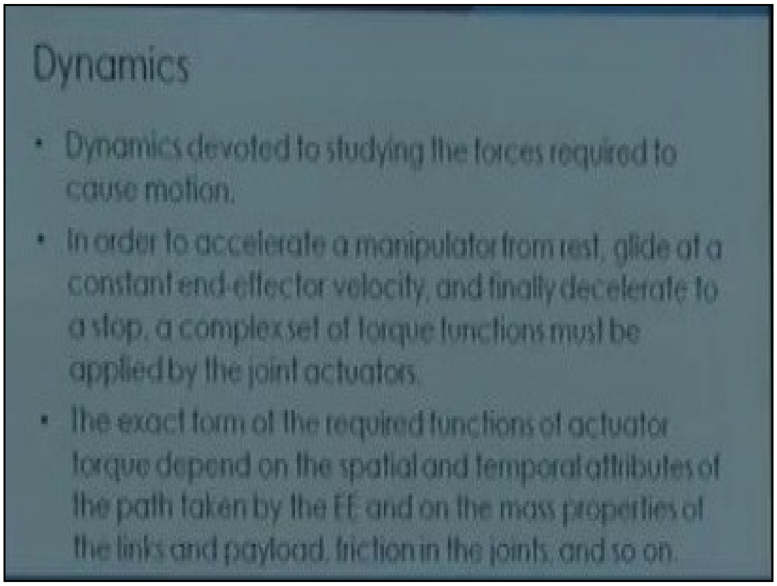

Now comes the actual point that is the dynamics of the manipulator which we are interested in. Now as all of you know that the dynamic inverse is studying the forces required to cause the motion. Now you know in order to accelerate a manipulator from rest glide at a constant end effectors velocity and finally decelerate to a stop. A complex set of torque functions must be applied by the joint actuators.

Then only this type of the motion of the end effectors is possible now the exact form of the required functions of actuator torque depends on the partial and temporal attributes of the path taken by the end effectors as well as the mass properties of the links and payloads. Friction in the joints and so many things affect the dynamics of the manipulator that exact torque requirement of the manipulator.

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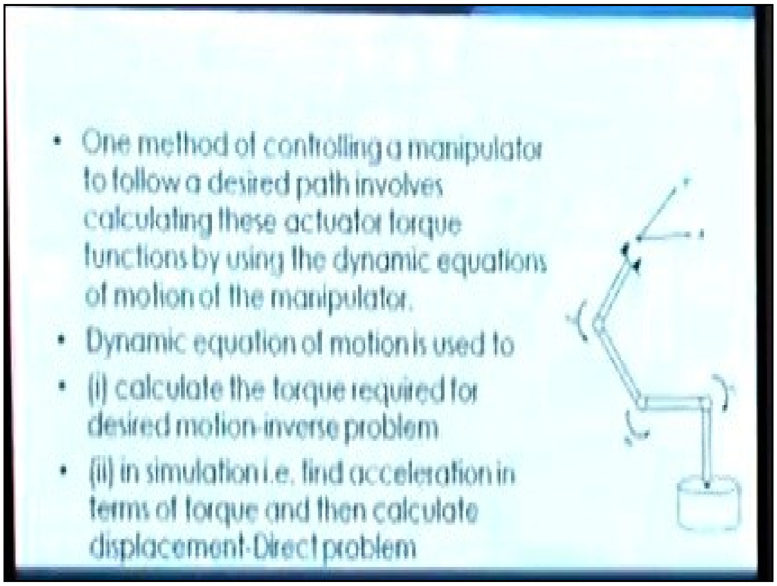

Now these manipulators as I said needs to be control. Because we want to apply an appropriate torque here for a given trajectory to be followed by the manipulator. Now one method of controlling a manipulator to follow a desired path involves calculating these actuator torque functions by using the dynamic equation of the manipulator itself. So if we do that we can control the manipulator appropriately. Now these dynamic equations of the motion of the manipulator can be used for two purposes.

The first purpose what I just talk to you that we calculate the torque required for desired motion and this is what we called as the inverse dynamics problem. And the other purposes it could be the simulation that is if you are given the joint torque. We can find out what are the acceleration velocity and the displacement. So find acceleration in terms of torque and then calculate displacement and this type problem is what are called as the direct problem.

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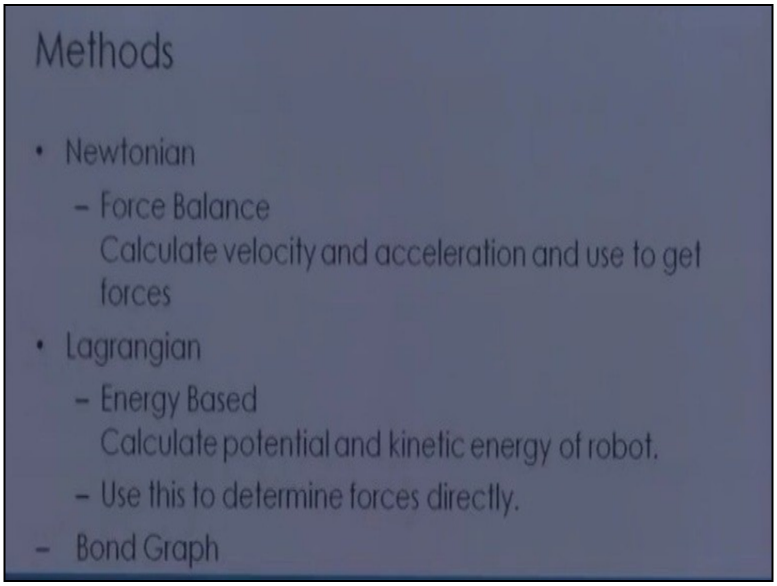

Now to deal with the dynamics of the robot there are three ways with in by which can be done. The first way is the neutron and method. Now this method is essentially based on the concept of force and moment balance. So we have the force balance calculate velocity and acceleration and use to get the forces. And another method is the lagrangian method. This method is the energy based method and it calculates potential and kinetic energy of the robot.

And these kinetic and potential energy terms can be used to calculate the lagrangian which is nothing but kinetic energy minus potential energy and when this Lagrangian is substituted in the Lagrange equation you get the joint torque or the joint forces. And from here we can determine these torques and forces directly.

The method is the bond graph method we have already seen how to derive the system equations using bond graph. So the bond graph method can be use to first draw the bond graph model of the robot and then we can derive the system equations o of the robot algorithmatically.

Using the algorithm that is what the elements into the system and what does the system gives to integrally causer storage elements if we applied that algorithm then we get the system equation automatically from the bond graph.

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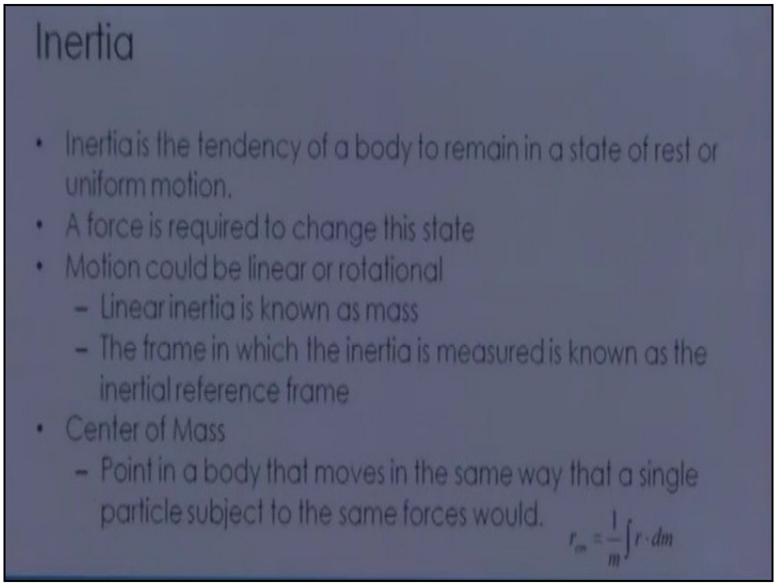

Now before we precede a small discussion about inertia because we are talking about dynamics and without inertia dynamics is meaningless. So a little discussion on the inertia is the tendency of a body to remain in a state of rest or uniform motion.

A force is required to change this is state. Now the motion could be linear or rotational. Linear inertia is known as mass and the frame in which the inertia is measured is known as the inertial reference frame. Center of mass the point in a body that moves in the same way that a single particle subject to the same forces would move and this way we can evaluate the center of mass.

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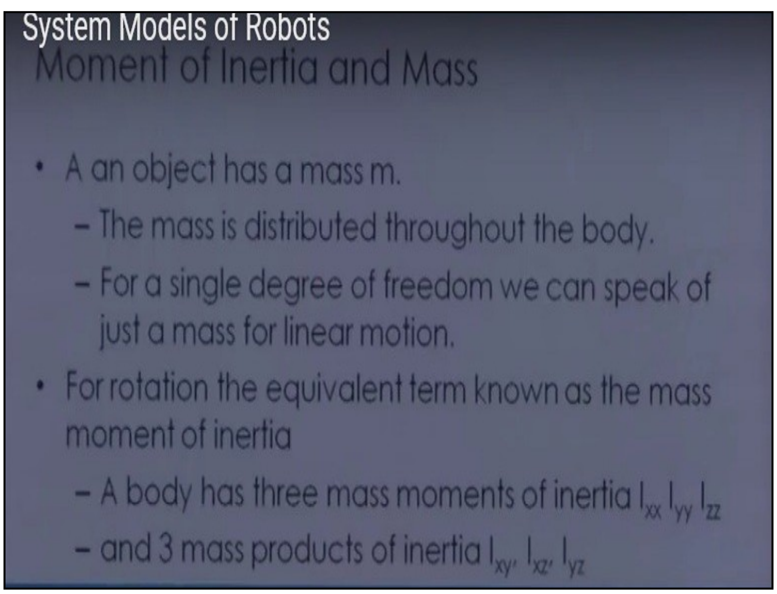

Moment of inertia and mass if a an object has a mass m the mass is distributed throughout the body and for a single degree of freedom degree of freedom I didn't talk to you but degree of freedom as all of you must be knowing is the number of independent variable required to is specify any object.

So for a single degree of freedom we can speak of just a mass for linear motion. For rotation the equivalent term known as mass moment of inertia and a body has got three moment of inertia and three mass products of inertia.

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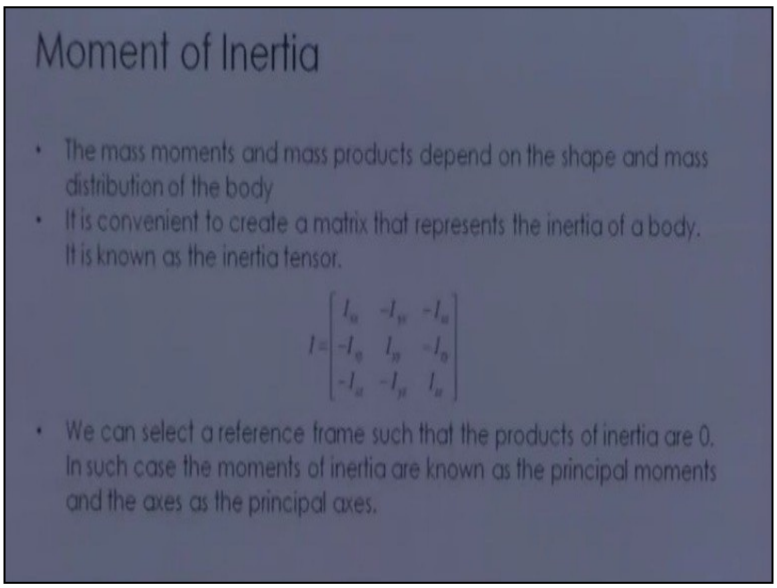

Now the mass moment and mass product depends on the shape and mass distribution of the body it is convenient to create a matrix that represents the inertia of a body and this matrix we call it as the inertia tensor. So here you have ixx iyy izz terms and other are the crossed terms. Now when we can select a reference frame such that the product of inertia. This product of inertias is zero in such case the moment of inertia is known as the principal moment and the axes as the principal axes.

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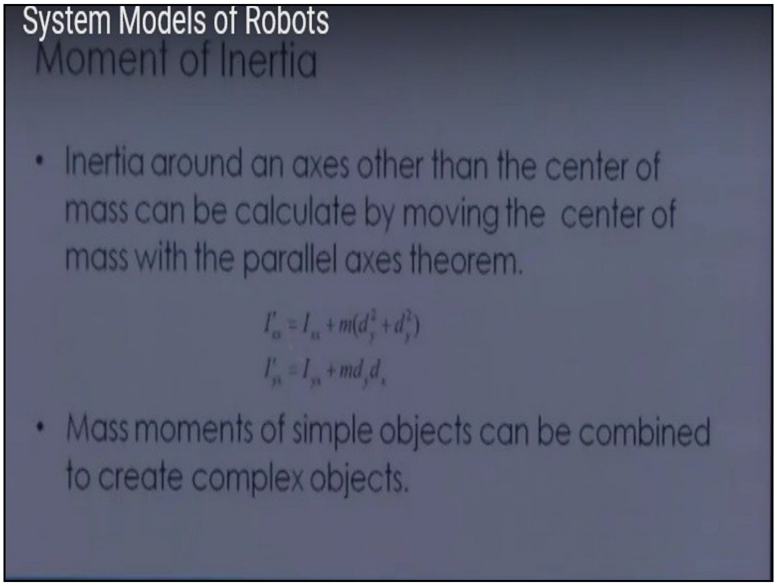

Now we can transfer or you can find out moment of inertia about any other axes using the parallel axes theorem. So these are the basic concepts of the mechanics which I hope all of you must know.

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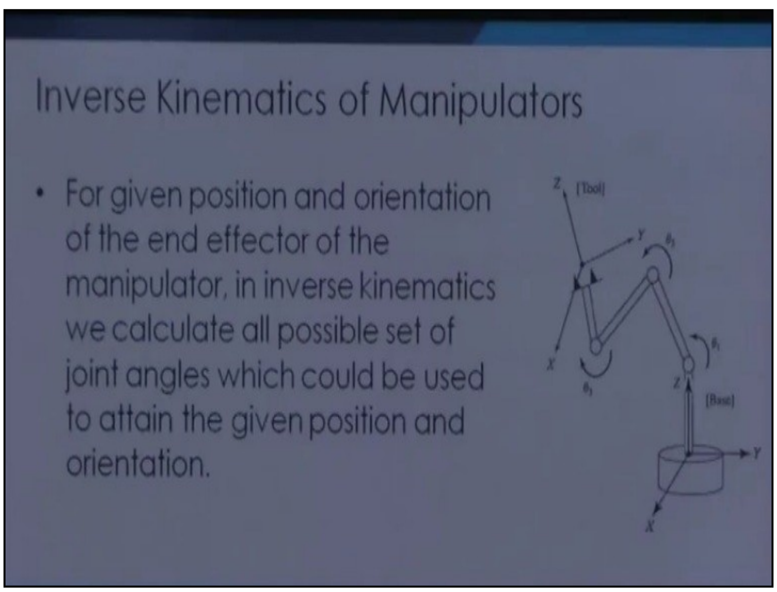

Now let us look at the ways of finding out of the dynamic equation of the manipulator. First as I said Newtonian method so here a first iterate out the serial change of the robot starting at link one and continuing to I minus. 1 here what we do is that calculate the linear and angular position velocity and accelerations of the center mass of each link.

And then once these are there the accelerations and velocities of the center of mass are calculated we can use the Newton Euler equations to calculate the force of the forces on the center of mass of each link. So this is for the linear motion and this expression is for the angular motion.

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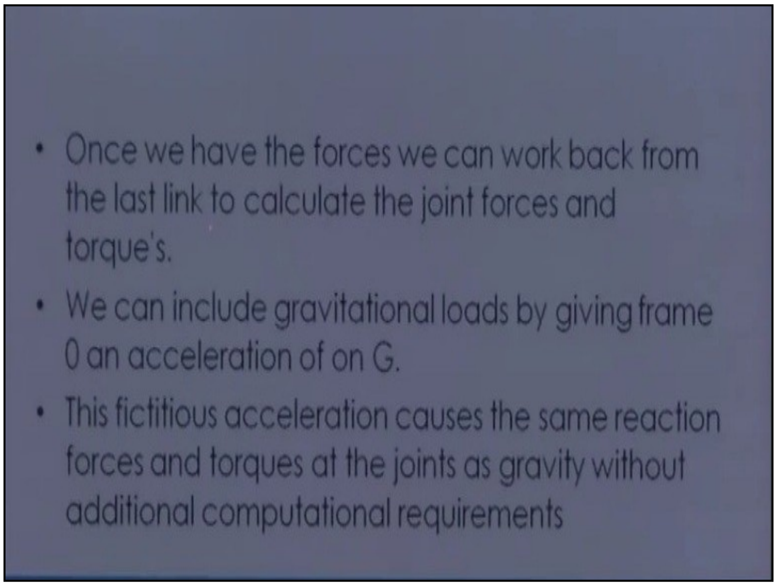

Now once we have the forces we can work back from the last link to calculate the joint forces and torques. We can include gravitational loads by giving frame 0 an acceleration of on G. Now this fictitious acceleration causes the same reaction forces and torques at the joints as gravity without additional computational requirements.

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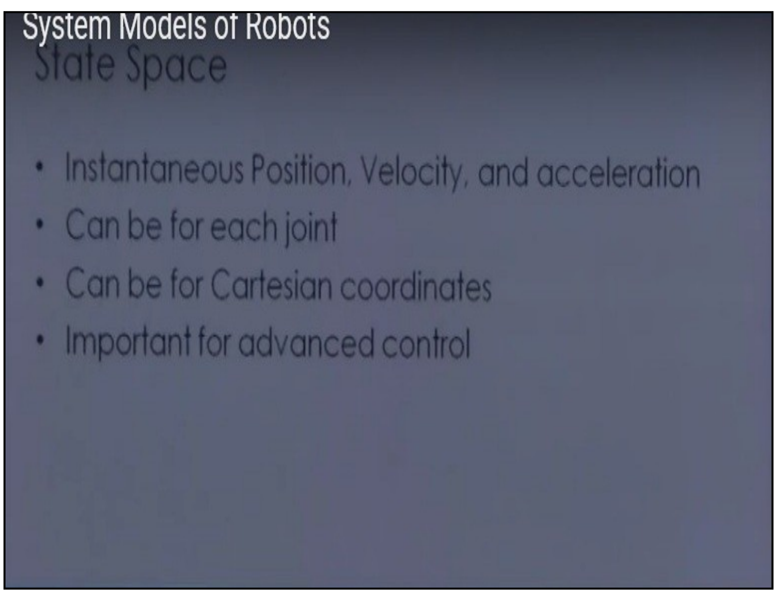

The expressions are usually detailed in the form of state space and for this describe basically that instantaneous position velocity and acceleration and can be for each joint can be for Cartesian coordinates and of course this is the important for the advanced control analysis.

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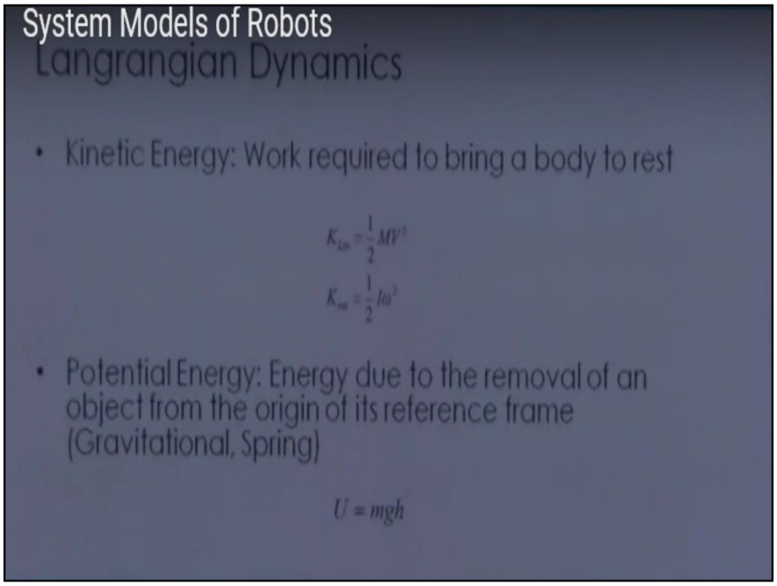

Next method which I like to describe is the lagrangian way of finding out the dynamic equation of the manipulator. So in lagrangian method we know that first we evaluate the kinetic energy to work required to bring a body to rest which we call it us kinetics energy. So for linear motion this is half m v square and for angular motion is half I omega square. And the potential energy is the energy due to the removal of objects from the origin of its reference frame gravitational or a spring. So for gravitational this potential energy is m g h.

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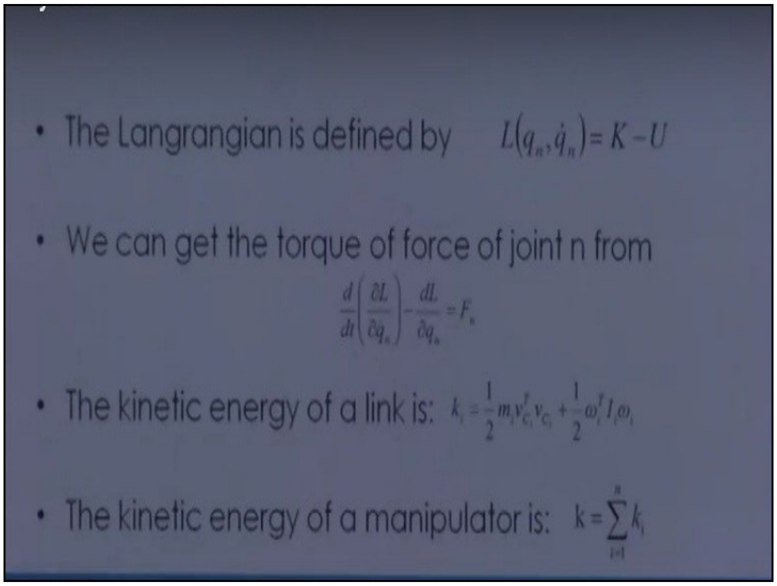

Now this lagrangian is define by k minus u. So l is k minus u. We can get the torque or force of joint from this equation. So this is d by dt dell l by dell q n dot minus d l ad dell l by d q n equal to f n. Where basically this q n is the generalist quadrate. So we can find out basically if you

Talk about the manipulator we can find out the kinematic energy of the link this is transitional kinetics energy. And this rotational kinetics energy for I f link then we can add up all the for kinetics for the entire link we can get an expression.

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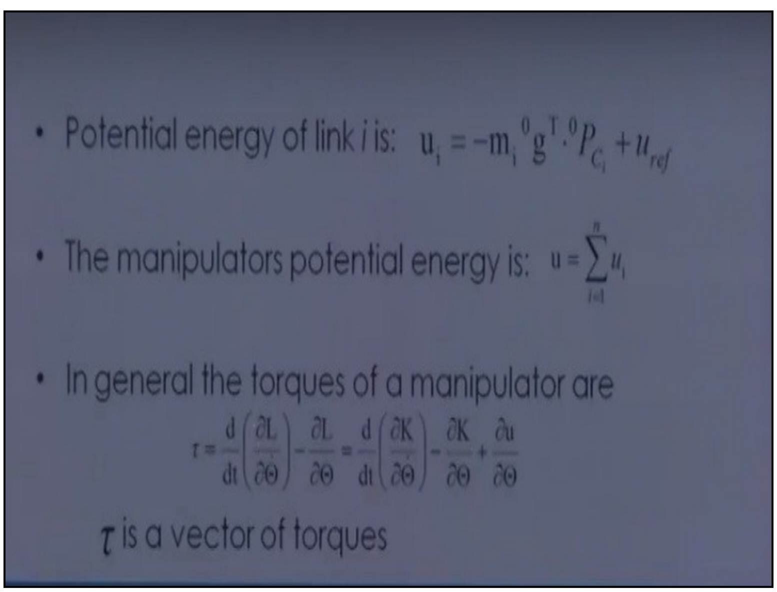

For the total kinetic energy Likewise I can find out the potential energy for the higher link here then the manipulator potential energy can be found out by summing the potential energy of all the links and then we can get the torque expression for the manipulator design. And here I m just substituting for l.

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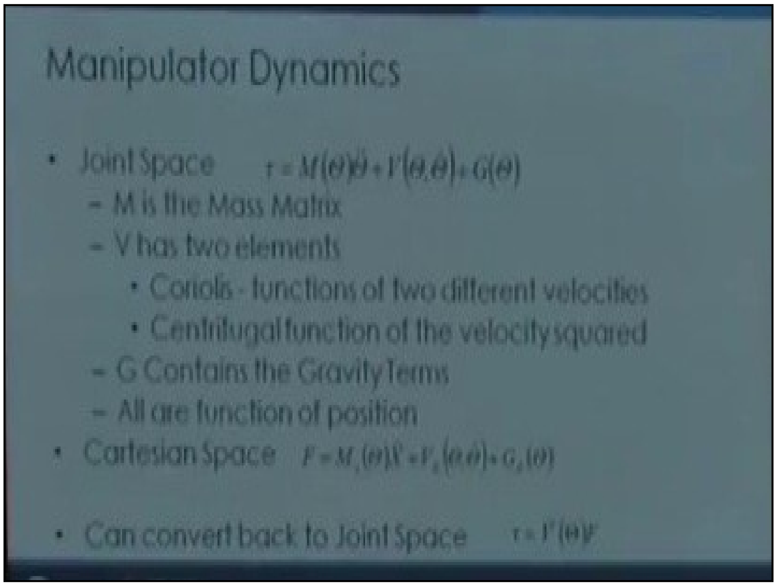

So the manipulator expressions finally it comes out in this form. Where m is the mass matrix and v has two elements the corollas which is the function of different velocities and the centrifugal which is the function of velocity square. We have the g theta that is which contains the gravity terms also. Here we can add up even the friction term also that also can be add it on similarly we can write the expression in Cartesian space this way.

We can convert back to the joint space using this equation which I discuss you some time back that is torque equal to jacobian transpose f.

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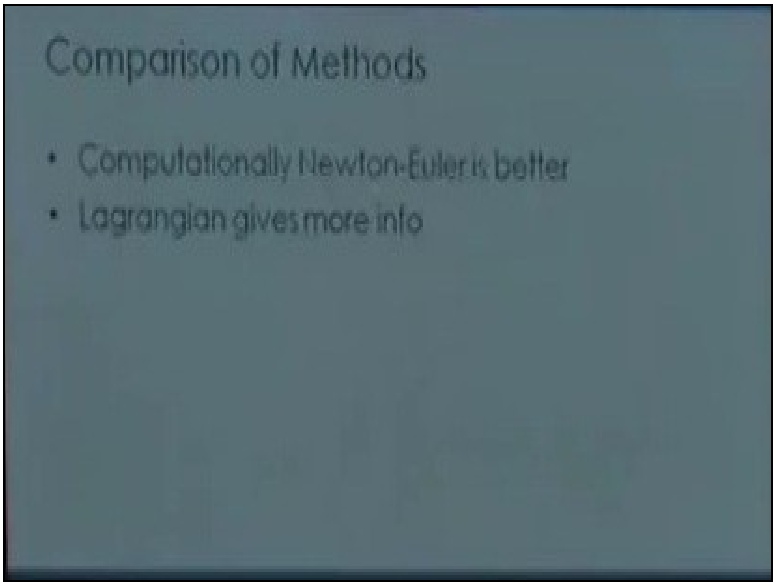

So if you compare this two method the Newton Euler computationally better but lagrangian equation gives more information.

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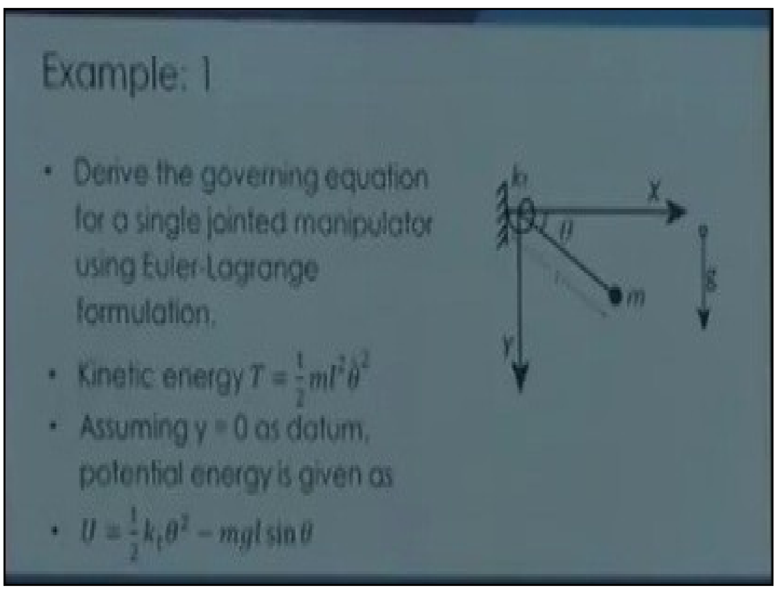

Let us take an example suppose I have the pendulum here basically a mass m at the end of the link l here to actuate this joint I have got a tortional spring. So this tortional springs actuated this joints we can always replace this tortional spring with the help of motor. But here my aim is to explain u how we model it. I am just considering the actuation of this joint with the help of a tortional spring.

So let us use the Euler Lagrange formula to find out the dynamic equation for this one. So you can see the kinetics energy is half m l is square theta dot square basically half I omega squared. For I for I this case m l squared so we have half I omega squared term. Assuming y equal to zero as that time the potential energy can be given by half k theta squared that is the energy stored in the spring and minus mg l theta because is going to be l sign theta is so minus m g in to l sign theta.

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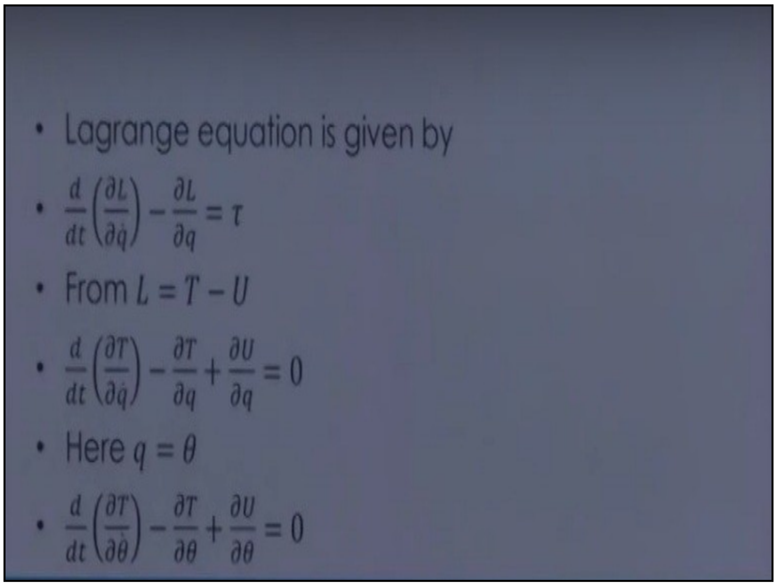

Now I can we have the lagrangian equation. So in order to get a here I find out l which is t minus u substitute this t minus u here in this equation I get Lagrange equation in terms of t and that is kinetics and potential energy. Now here the generalized quadrates in our case are theta so this is going to be a Lagrange equation.

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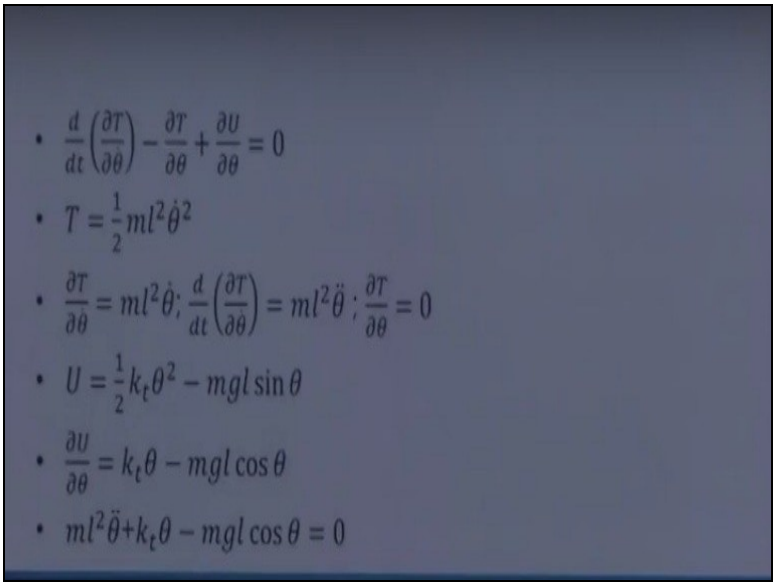

Here now in we know that our t has been half ml theta dot squared. So I can find out the dell t and dell theta dot this one I can find out d by dt of this one which m l h squared theta double dot the I evaluate this term which is zero here then I am have we know the potential here so I can evaluate dell u by dell theta here this one. When I substitute all this terms in this equation this is what we get ml square theta double dot t theta minus m g l co theta equal to zero.

So this way we derive the system equation for this robot. If that was using the lagrangian method where we found out the kinetic energy we found out the potential energy we found out he Lagrange and then we put it substitute in the lagrangian equation and we got the system equation for the robot.

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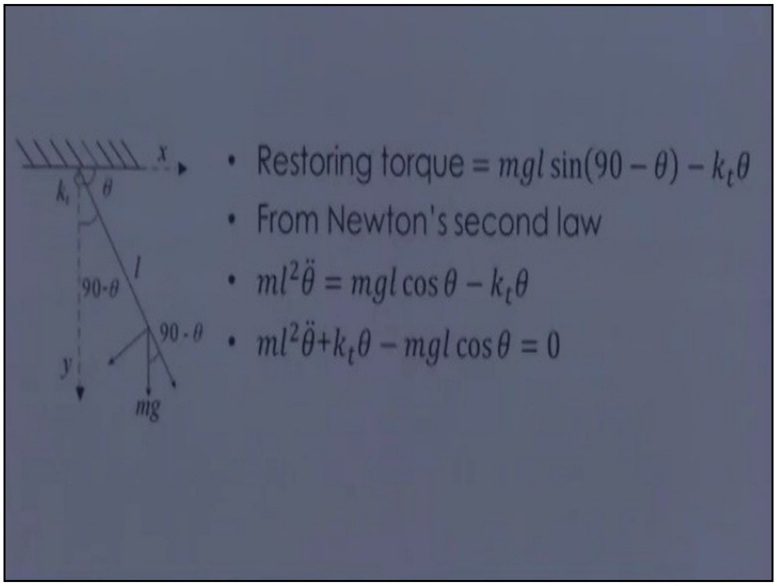

Now if I do it in the Newton's method then ha then here we can see that the restoring torque m j this is mg the force going to be mg sin ninety minus theta which is basically the this term. And the torque term because of the minus k t into theta and this unbalance bb torque is responsible for the acceleration of this point mass here that we equate into alpha.

M l h squared theta double dot and this way we get the same equation. One so here basically what we have done here is mls torque that torque is going to be responsible for this acceleration of this joint and we have the equated with the inertial torque. This is what we done in the Newton's method.

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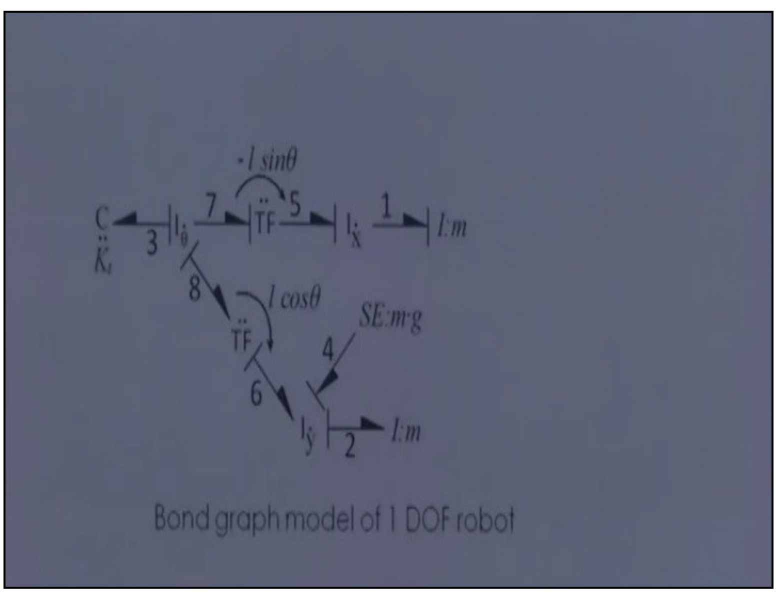

We can also use the bond graph model. We can draw the bond graph model for this Robert for that we can need to just find out the position the velocity of the tip x and y direction. We can draw this bond graph model on I m not going to explaining you I m leaving it as an exercise to you but this is the bond graph model which you are going to get for this case.

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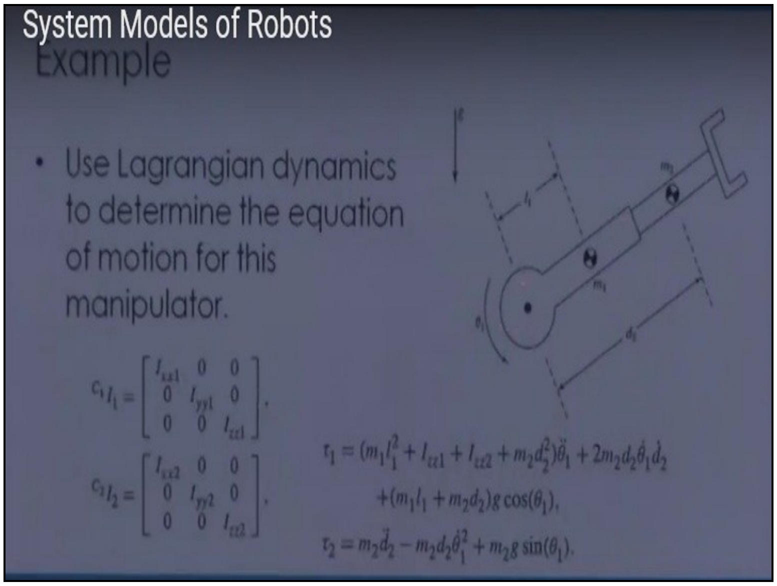

Next two three examples I will just explain to you how we do that. Next let us consider this problem this is the 2 degree of theta and first degree of theta is here through the prismatic joint. So for this we have the inertial property given I one is given here about the center of the mass.

And I two mass center of the mass of link two this is given and if you use the same Lagrange dynamic equation finds out the kinetic energy find out the potential energy. Find out the Lagrange and then write the Lagrange equation for the in terms of two generalized quadrate. Here we are going to have the generalized one is joint rotation and another is this joint linear motion. So if you do that this the result which we are going to get.

I m not explaining you because of lake of time here but this dynamic equation or systemic equation you are going to get. (Refer Slide Time: 31:51) I have already talked about Newton Euler formulation and I told you that algorithm of two parts.

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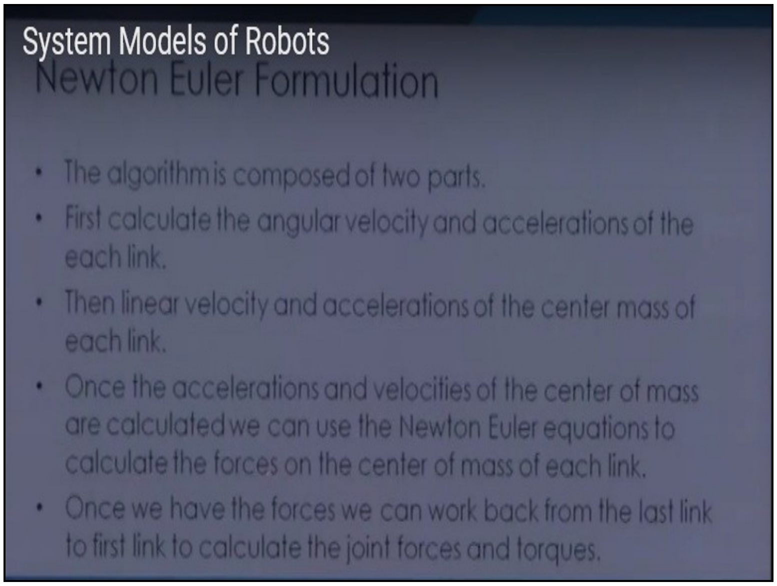

First we calculate the angular velocity and acceleration of each link. Then we calculate the linear velocity and acceleration and velocity of center of mass is calculated the we use the Newton oilers equation. To calculate the center of mass of each link and once we have this forces we can work backward to last link to first link to conclude to calculate the joint forces and torque .this have already explained how we can take case of the g of the manipulator.

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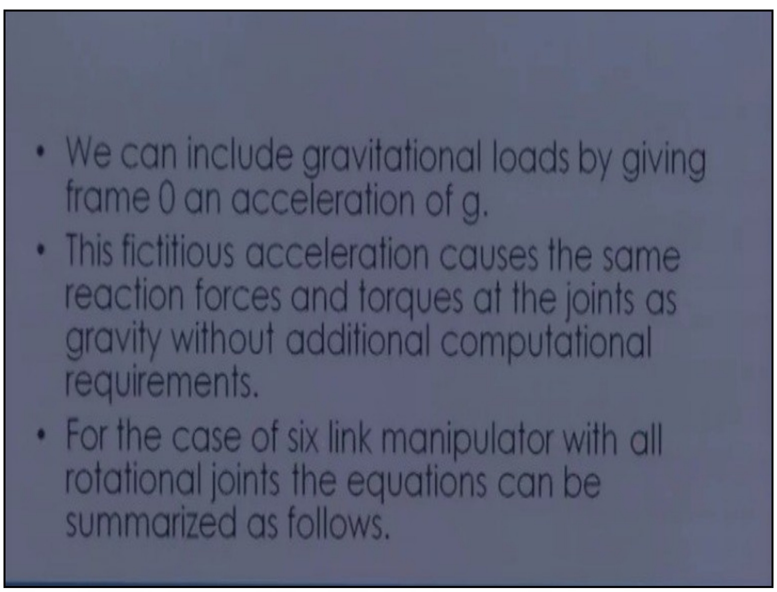

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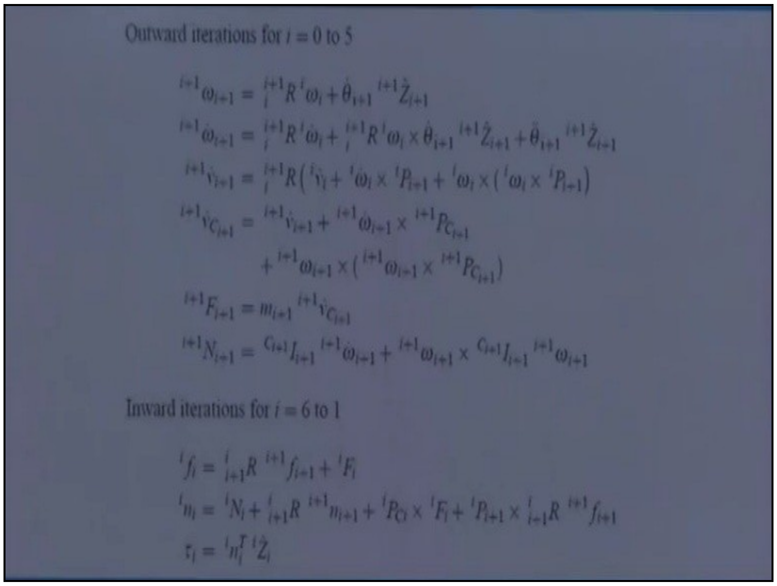

Now for the case of six link manipulator with all rotational joints the equation can be summarize as follows. So these are going to be a set of equation which we call it us the outward iteration for I equal to five .basically here we find out the angular velocity you can see for the I the link I plus one the frame then we are finding out the acceleration.

Here we are finding at the linear acceleration of the origin of frame of I plus one with respect to I plus one then here we are finding out the acceleration of the center of mass then we are calculating the inertial forces and we are calculating the inertial torque. so this is what we done in the outward iteration and here then we go for the inward iteration that is I equal to six to one and here we are calculated the forces in the link top in the links and moment in the links and then we find out the jet z component of that which gives us essentially the joint torque.

And this is the last example which I will just explain to you again I am not doing this problem you can use the same algorithm which I just explain to you using that is the Newton oilers equation.

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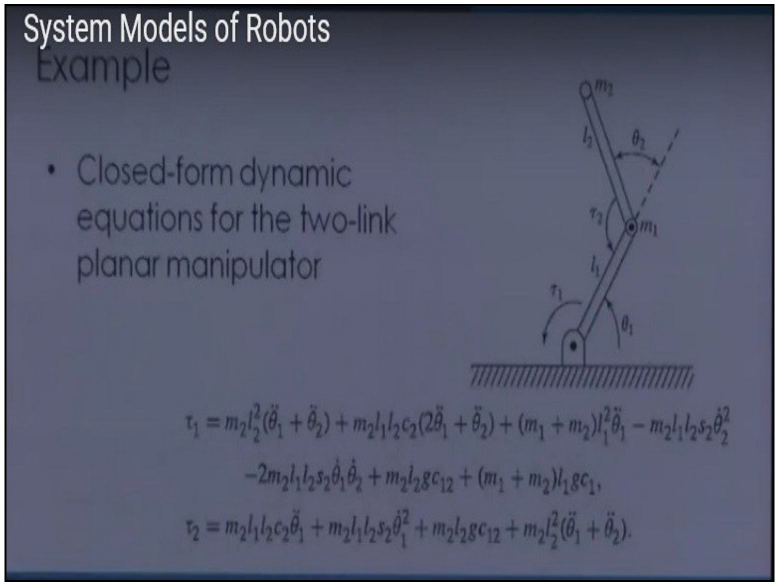

So this is a two link manipulator the modeling can be simplified by assuming the point masses here m one and m two the link lens are l one and l two these joints are theta one and theta two. This problem is finding out the joint torques tauv one and tauv two. So if you applied the same algorithm. There are the values which were going to get the first joint torque and the second joint torque.

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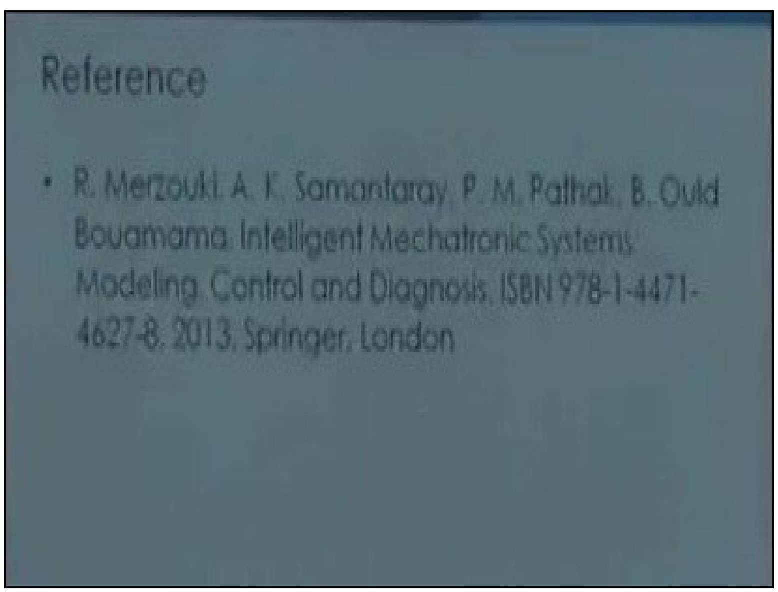

So that is all and you can refer our book for further study. There is a very popular book by G J Kaerk. And which is published by the Pearson that also you can refer and that will help you in understanding the modeling of the robots. Thank you.