Modelling and Simulation of Dynamic Systems

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Lecture - 2
Examples of Models

I welcome you all in this lecture on examples of models which is a sub topic for the course modelling and simulation of dynamic systems. Now before we talk about models, let us see what is the definition of a model. Now as we all know to study the dynamics of any real system the idea of the model of the system is very important.

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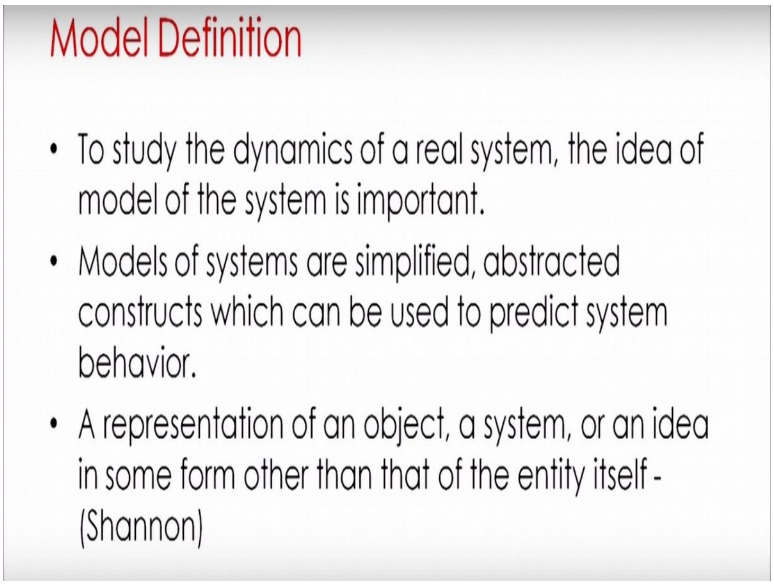

As I have given in my first lecture, say, modelling of a water tank by spring mass damper system or modelling of a car. Again say by spring mass damper system. Now these models are simplified, abstracted constructs which can be used to predict the system behavior. So the model which are created are simplified abstracted construct, and with the help of this model, we can predict the system behavior. Shannon defines the models as a representation of an object a system or an idea in some form other than the entity itself.

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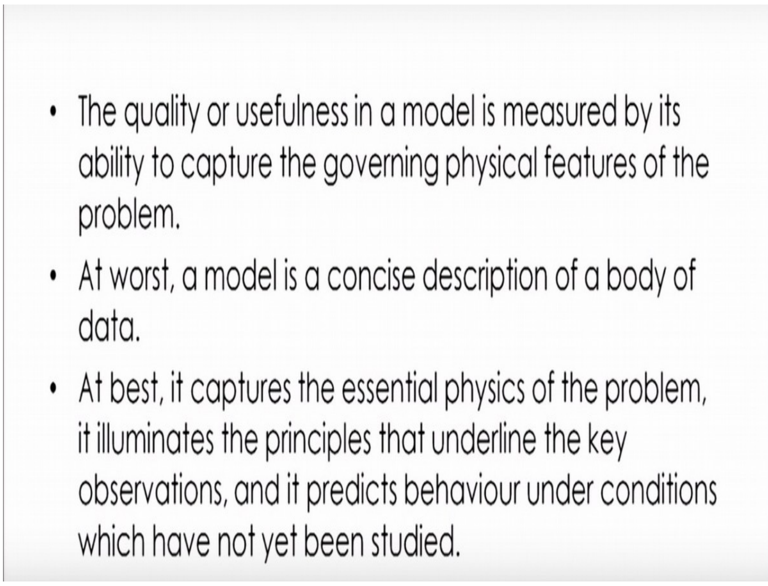

Now the quality or the usefulness of the model is measured by its ability to capture the governing physical features of the problem but, the model which we are creating could be a best model, or a worst model. What do I mean by best or worst model? A worst model could be a concise description of the body of data or at the best model it captures the essential physics of the problem.

It eliminates the principle that underlines the key observations and it predicts the behavior under conditions which, have not been studied. Scaled physical model are well known to engineers, and there are many examples of scaled physical models.
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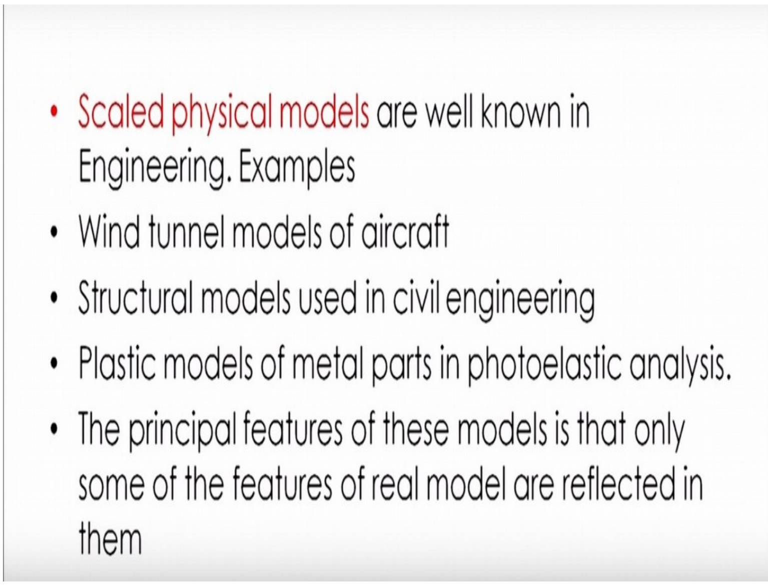

For example, a wind tunnel model of an aircraft and structural models used in civil engineering, plastic models of metal parts which are used in the photo elastic experiments.

In order to determine the isochromatic fringe orders or the principle stress is different, and of course with an addition of certain other supplementary method one can predict the principle stresses as well, so that is done with the help of photo elastic model. the principle features of these models is that only some of the features of the real models are reflected in them. We do not try to incorporate all the features of the physical system into the real model as it may result in complicating the models itself, and will be difficult to analyze at later stages.

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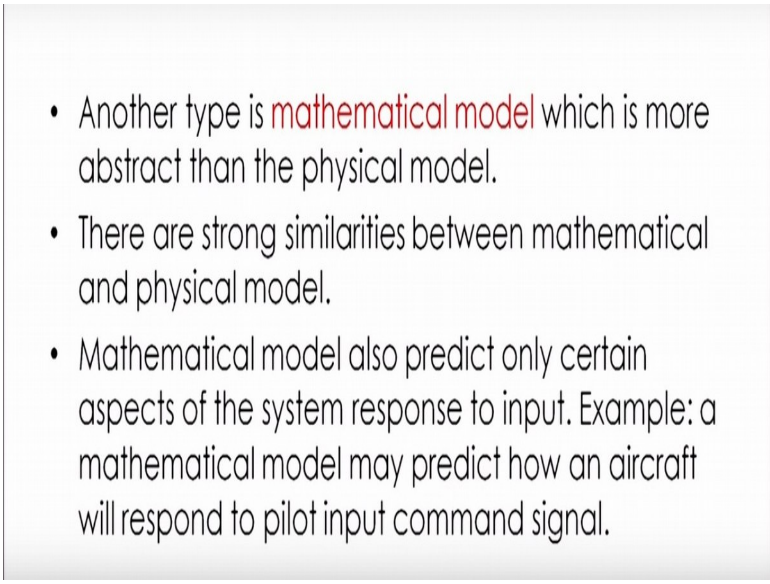

Another type of model is what is called as the mathematical model, which is more abstract than the physical model - of course there is a very strong similarity between the physical model and the mathematical model and this mathematical model also predicts certain aspects of the system response when it is subjected to an input. Now, for example for an aircraft a mathematical model may predict how an aircraft will respond to the pilot input command signals.

So, by seeing the broad examples of the model type as the physical and mathematical models. Let us see a little elaborate classification of the various model types.

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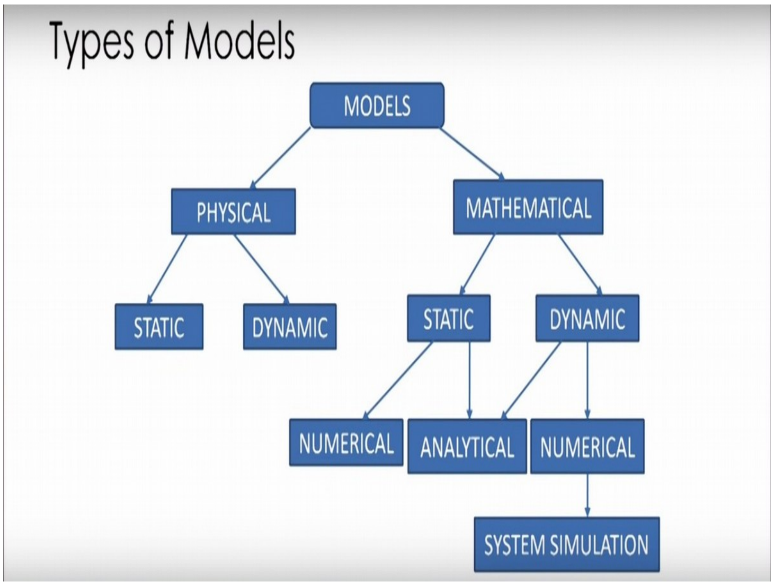

So, as I said models can be classified as the physical model and mathematical model. This physical model again could be classified as static model and dynamic model, similarly the mathematical models can be classified as static model and dynamic model. This static model can further be classified as numerical model and analytical model.

Likewise, the dynamic model can be classified as analytical model and numerical model and of course, from the numerical model one can go for the system simulation. So, this is the classification of types of models. We will be going through each one, example one by one. Now the physical model as the name indicates, physical models means something which is going to be Arial which has got physic, so these are based on some analogy between such systems as mechanical and electrical or electrical and hydraulic system.

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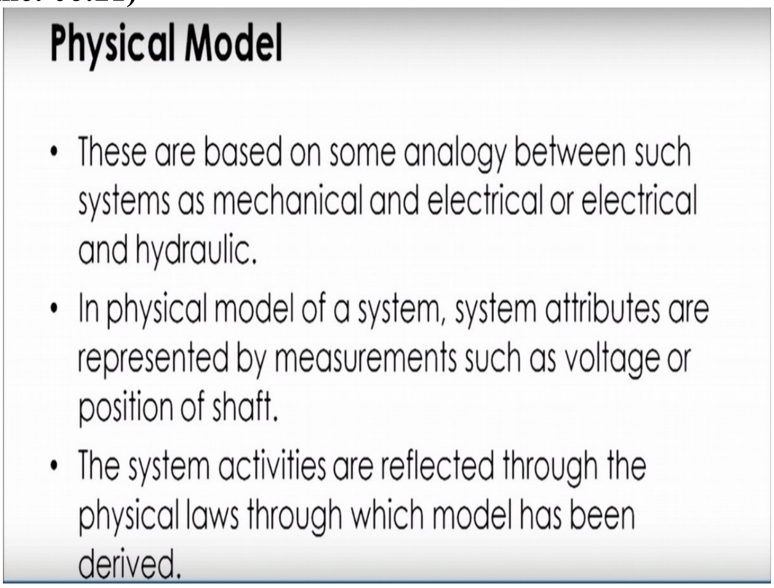

So, these physical models are based on certain analogy, between these types of system. In physical model of a system, system attributes are represented by measurements, such a voltage or position of the shaft in which the system activities are reflected through physical laws through which models have been derived. Now let us see the mathematical model, this type of mathematical model use physical symbolic notations and mathematical equations to represent the systems.

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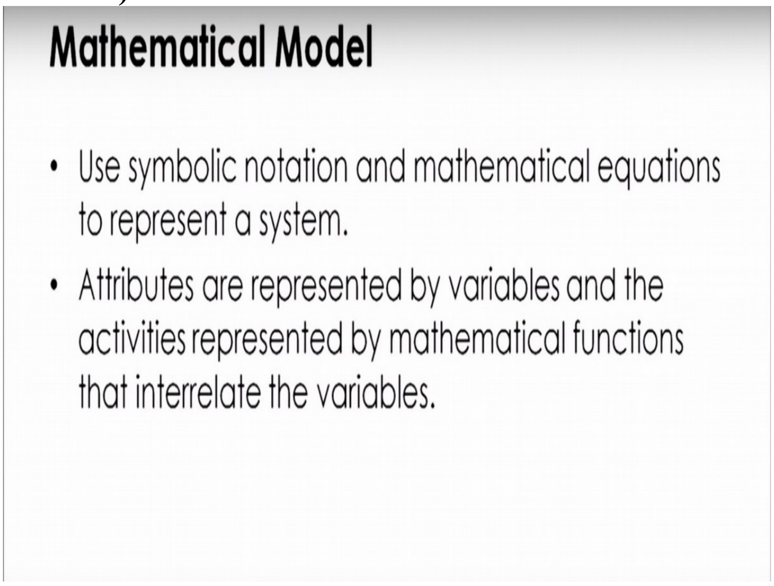

So the expressions could be either symbolic in nature, or it could be the mathematical equations which are used to represent the system; now attributes are represented by variables and the activities are represented by the mathematical functions that interrelates the variables. So, this is what do we mean by the mathematical model.

Now these models could further be classified as static model and dynamic model, so the physical model or mathematical model can be classified as static and dynamic model. So, the static versus dynamic model. What are the static models basically.

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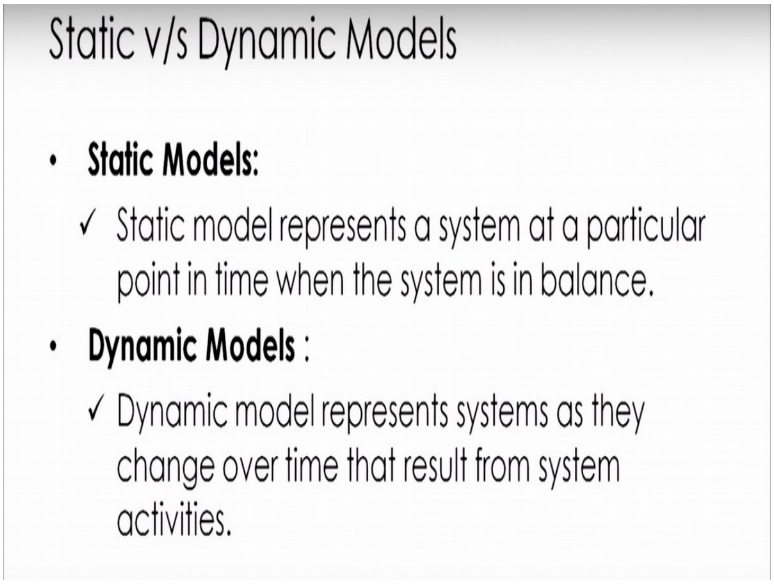

Now the static models represent the system at a particular point in time when the system is in balance or the system is in the state of equilibrium so this is what is called the static model. The dynamic model represents system as they change over time and of course, that change occurs because of certain activity which are there in the system. So these are the dynamic models. Now, models could be further be classified as numerical model versus analytical model.

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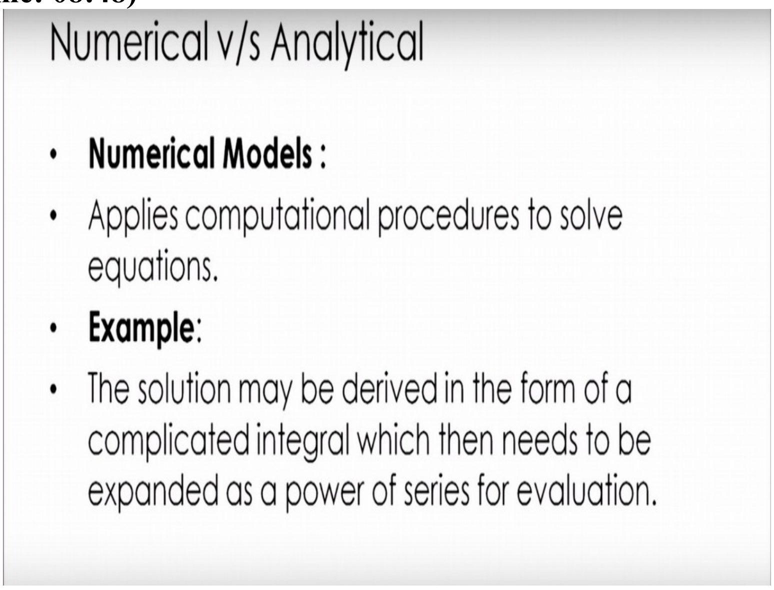

Numerical models they apply computational procedures to solve the equations for example, the solution may be derived in the form of a complicated integral expression which then needs to be expanded say in the form of a power series for evaluation purpose so this is about the numerical model. The analytical model it finds the model, that can be solved and best fit the system being studied.

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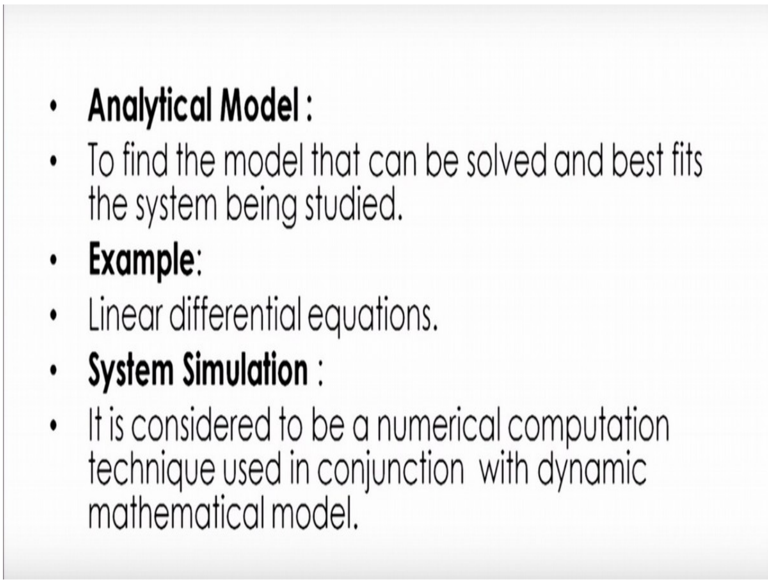

This is how we can work with the analytical model, for example, the linear differential equation and the system simulations basically it is considered to be a numerical computation technique used in conjunction with the dynamic mathematical model. So these are the brief descriptions of the various types of models. Now, let us see a little detail about these types. First we will take up the static physical model. The static physical model.

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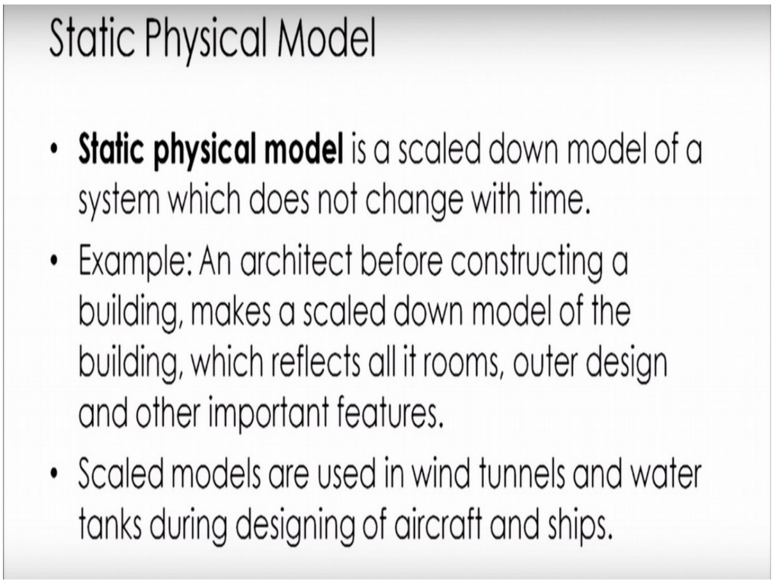

No these static physical models are basically a scaled down model of the real system. So whether we talk about an architect preparing a model of a building or it could be a small model of an aircraft which is to be tested in a wind tunnel or it could be a scaled model of a ship which is to be tested in a tank.

So there could be as I said various example for these an architect, as I said before, constructing a building makes a scaled down model of the building which of course reflects all the features of its room outer design and all other features. Likewise scaled models are used in wind tunnels and water tanks during design of aircraft and ships. So these are basically scaled down model of a system which, does not change with time, so these are static physical model. Next is the dynamic physical model.

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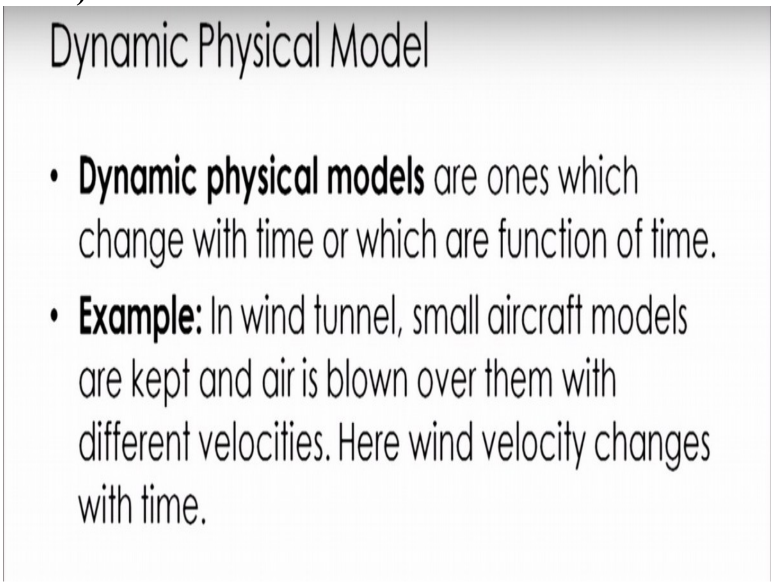

The dynamic physical models are the ones which change with time or which are function of time. So these dynamic models for example, if we talk about a small aircraft model in the wind tunnel if I am going to blow the wind with different velocities now here, we can say that the wind velocity changes with the time. So this type of model is what is called the dynamic physical model.

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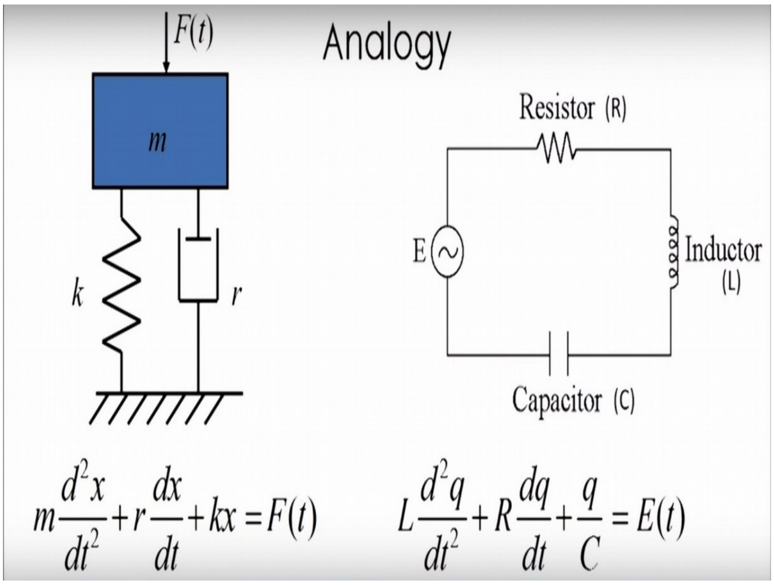

Now these dynamic physical models are very much helpful in system analysis for example, suppose I want to prepare the dynamic model of a car, now I can model the car as a lumped mass and I can model the suspension of the car as a spring damper system. Now in preparing this in dynamic physical models the thing is that, these types of models are always based on certain analogies, these analogies could be between the electrical and mechanical system. For example, the car model which I am talking to you I can model this; I can get the similar behavior with an electrical model which consist of a voltage source a resistor, an inductor and a capacitor.

So, a spring mass damper system is analogs to a LCR circuit which we call it LCR circuit in series. Now, here you see the governing equation or the equation for this spring mass damper system could be given by m d square x by d t square + r d x by d t + k s is equal to ft. Here m is the mass say x is the displacement of the mass r is the damping, k is the stiffness and ft of course is the force, which is being applied on the mass. Now if I take an electrical analog of it the voltage is corresponded to the force resistor corresponded to the damper and inductor correspond to the mass and capacitor corresponds to the stiffness.

If I look at the system equation for the electrical system we get l d square q by d t square + r d q by d t + q by c is equal to e t. Here, you can see that l is analogues to m q is analogue to x r is analogues to r and one by c is analogues to k and f is analogues to e. In this expression q represents the charge. Now the greatest advantage of this physical analogy between the mechanical and electrical system is that the mechanical system which is very huge, say if, I want to model a car and if I want to see the effect of stiffness of suspension systems of the car on the displacement of the mass actually doing this activity with the mechanical system is very tedious and difficult also, but the same activity or the similar corresponding analogues activity I can perform here by seeing the effect of c on the charge.

So this is the advantage of this analogy in practice as I said it is easier to modify the electrical system than the mechanical system, and so electrical analogue of mechanical system can be prepared and studied easily.

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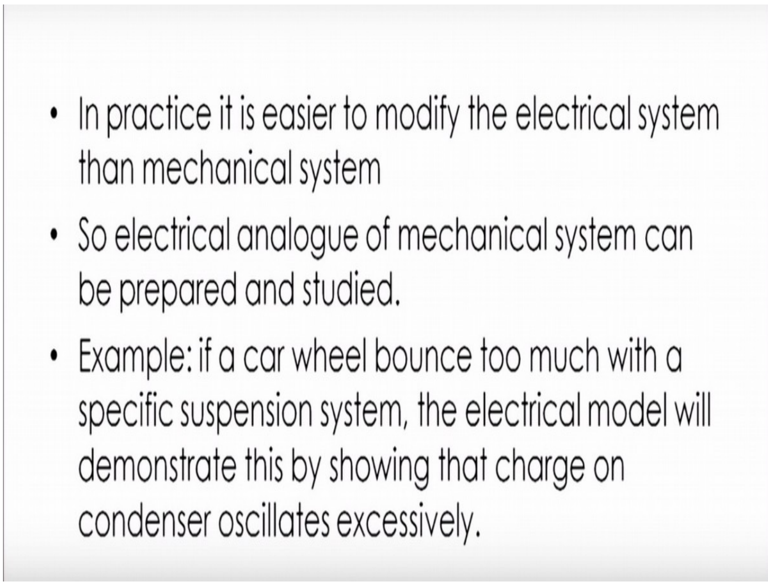

As I said if a car wheel bounce, suppose we take up an example if a car wheel bounce too much with a specific suspension system the electrical model will demonstrate this by showing that charge on the condenser oscillates heavily.

So this is what we call as the dynamic physical model. Next let us see the static model so first the static mathematical model. A static model gives the relationship between the system attributes when the system is in equilibrium - examples here we can take a case of static mathematical model from industry. Generally there should be a balance between supply and demand of any product in the market, supply increases if the price is higher but on the other hand demand decreases with the increase in price.

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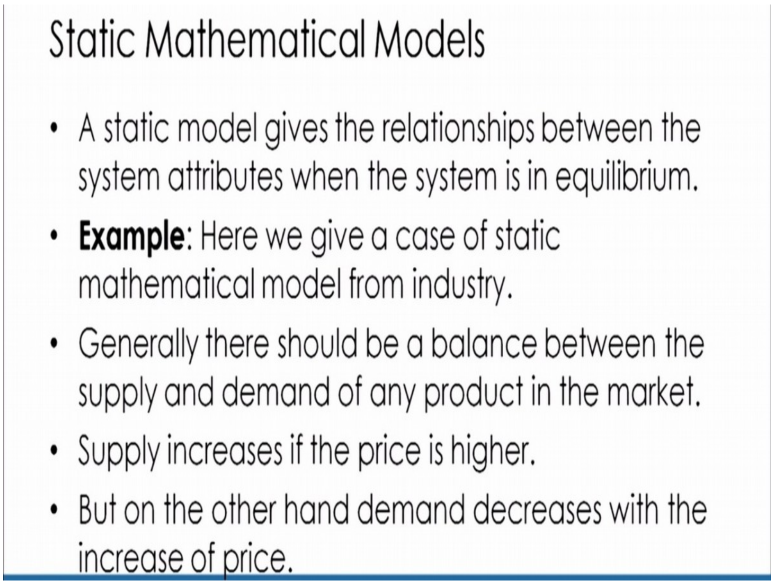

So these are the two factors. Now suppose, if I want to model this thing, I can do that with the aim to find the optimum price with which demands can match the supply.

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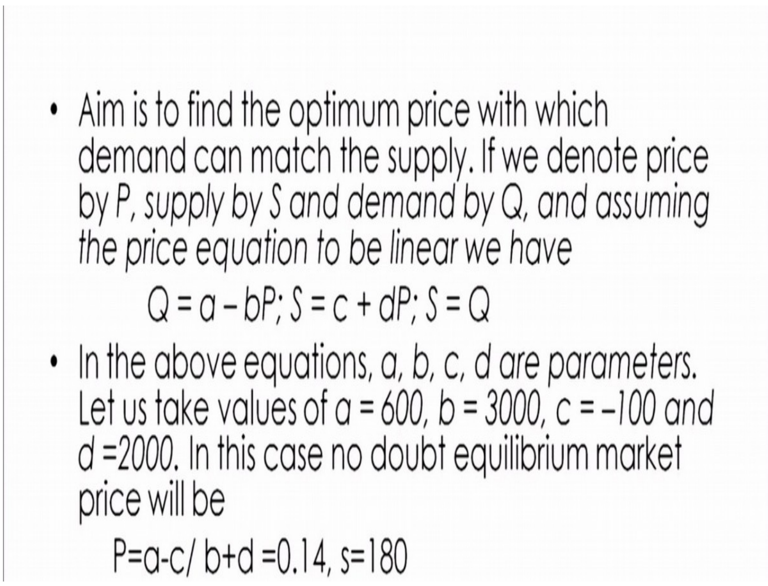

Now, if we denote price by p say supply by s and demand by q and assuming that price equation to be linear we have these equations q is a- bp, s is equal to c + dp and s is equal to q, where here a b c d are the parameters, and for a specific value of this parameters we can find out the equilibrium market price, of course this we can get with the help of these two equations.

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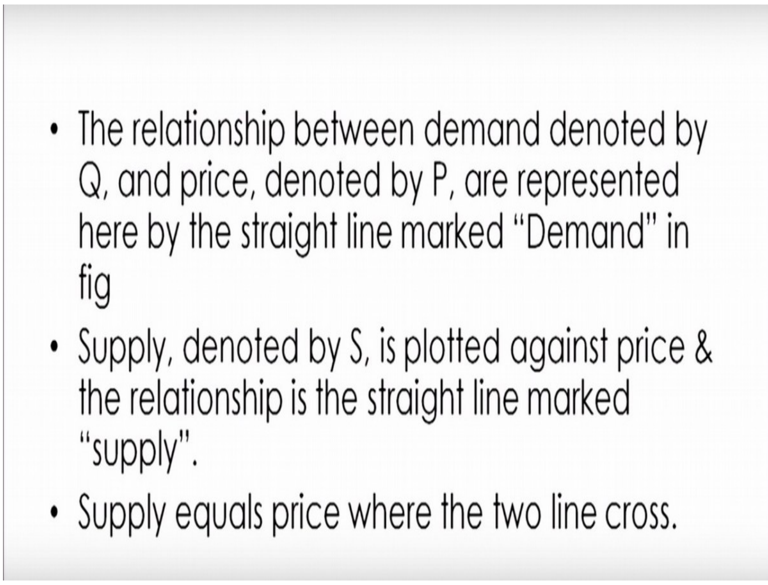

So the relationship between demand denoted by q and price denoted by p are represented here by straight line marked demand in the next figure which is there on the slide similarly the supply denoted by s is plotted against price and the relationship, is a straight line marked as supply, supply equals price where, the two lines cross. Now these things more usually the demand and supply are depicted by the curves, the previous one which we saw here.

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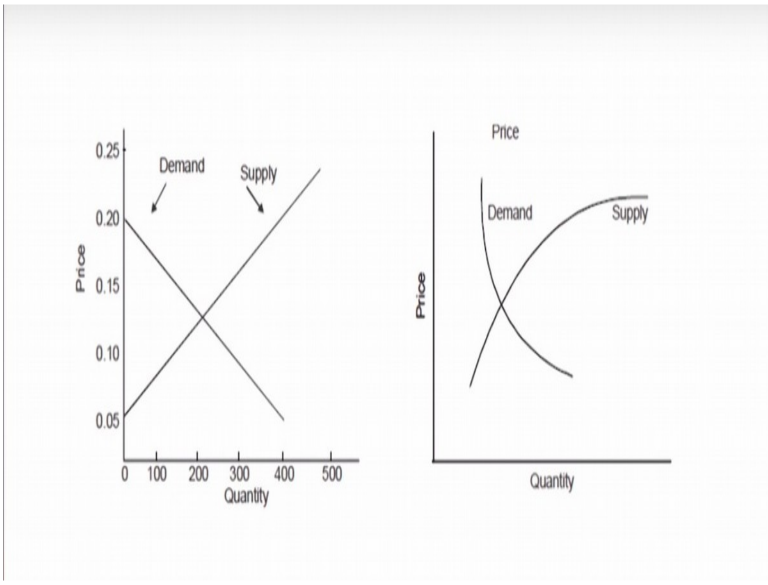

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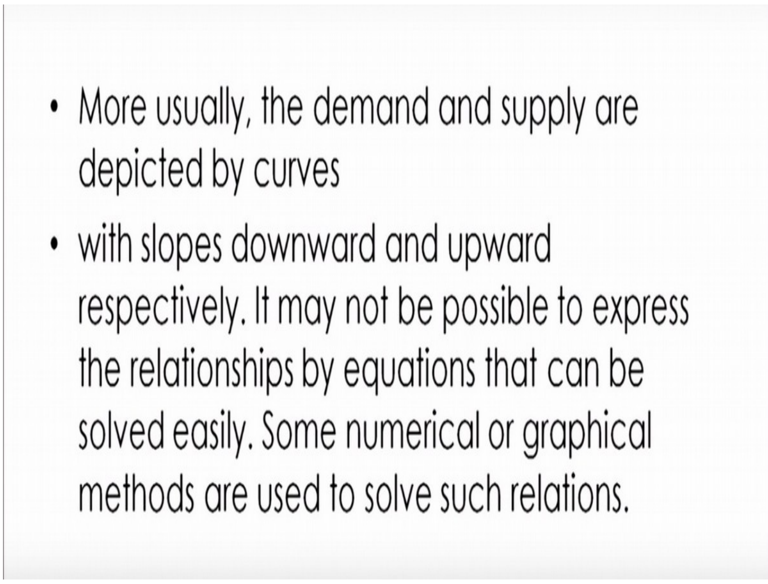

They are represented by the straight lines but, the demand and supply are depicted by the curves which slope downward and upward respectively. It may not be possible to express the relationship by equation that can be solved easily. So, for these case if we are going to represent the supply and demand by curve, we need some numerical or graphical method to get a solution.

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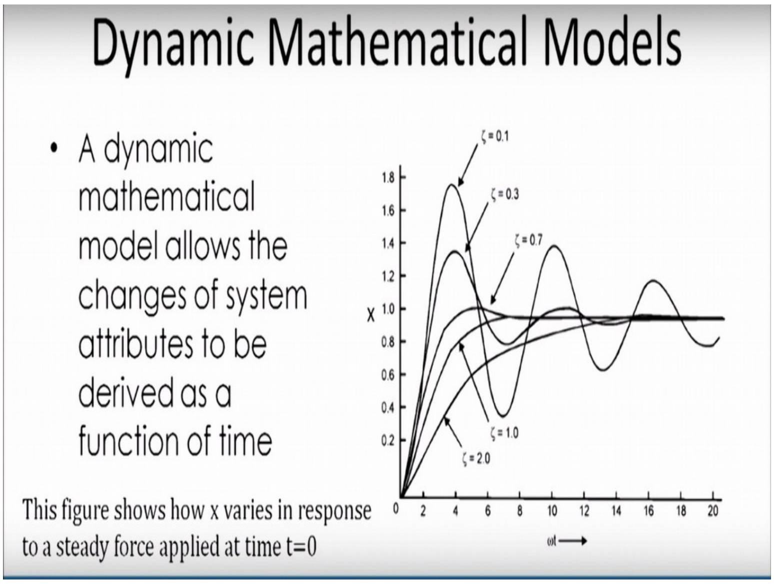

Next models are the dynamic mathematical model. These dynamic mathematical models allow the change of system attributes to be derived as a function of a time. So this plot which I am showing you, it shows how x varies in response to steady force applied at t is equal to zero for various damping ratios. So for various damping ratios how the x that is the displacement of the mass varies with time that has been represented and this plot, shows for the simple spring mass damper system which I have discussed earlier in this lecture.

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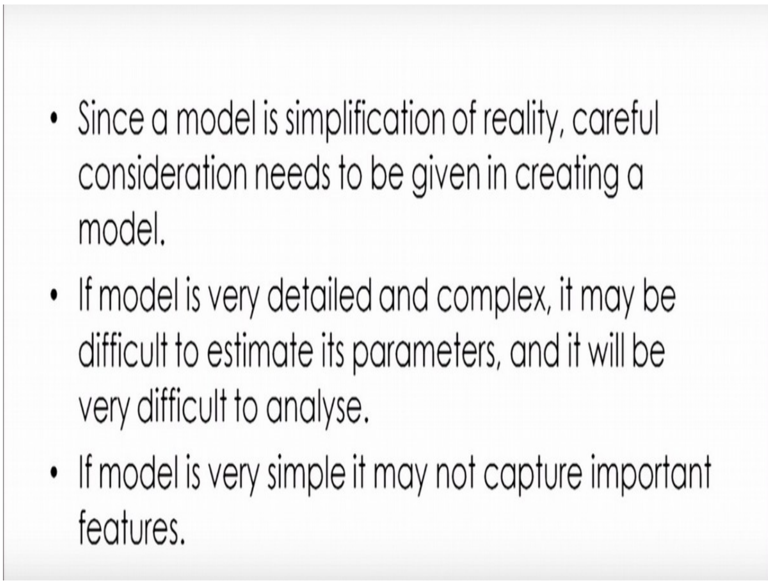

Now, since these models are simplification of reality a very careful consideration needs to be given in creating a model. If, a model is very detailed and complex as I told you that, it becomes very difficult to identify the system parameters and to analyze the model and if your model is very simple then it may happen that, your model may not capture some of the important properties or important features of the system.

So, while preparing a model of the system we need have a clear cut idea of what behavior we want to study, what are the factors which are going to affect the behaviour of the system and, to ensure that we do not leave anything unnecessarily and other way we do not include anything otherwise. Inclusion may not harm if you include some unnecessary thing it may not harm your model but, the thing is that it may take your computation time which is unwarranted.

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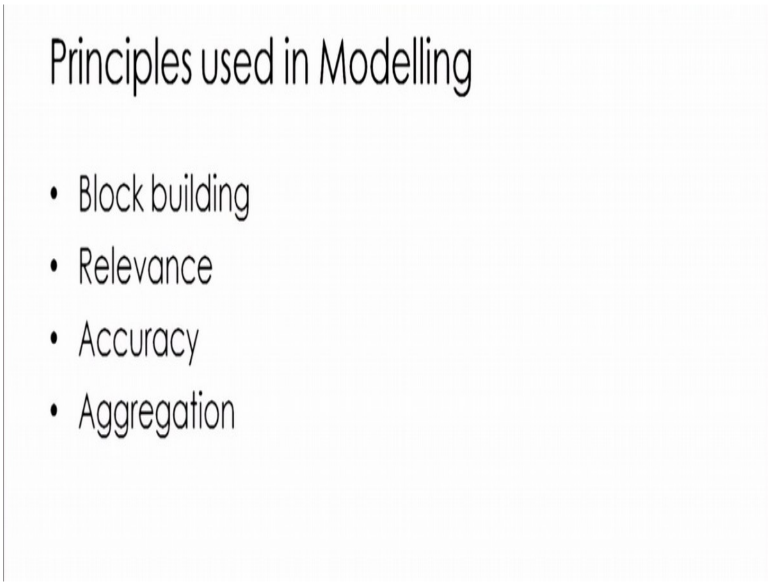

Now, what is the principles which are used in modeling? Let us see so for building up of the models these are some of the principles which are usually considered, say block building relevance, accuracy and aggregation. So, let us see these one by one.

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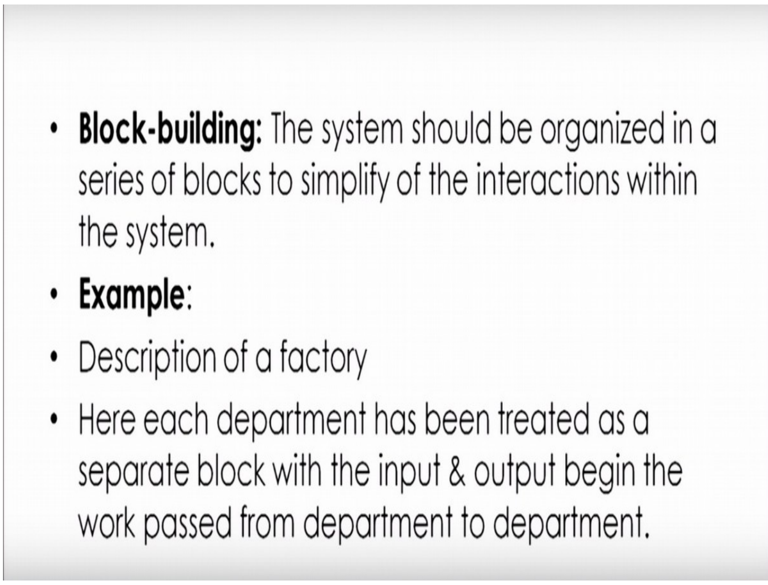

First the block building the system should be organized in a series of block in this block building preparing of the system model through building, we organize the system into a series of blocks to simplify the interactions within the system, for example, if I want to prepare a model of a factory, where I can treat a each department as a separate block with certain input and output, being the work pass from department to department so, this is how we can do the modeling using the block diagram method.

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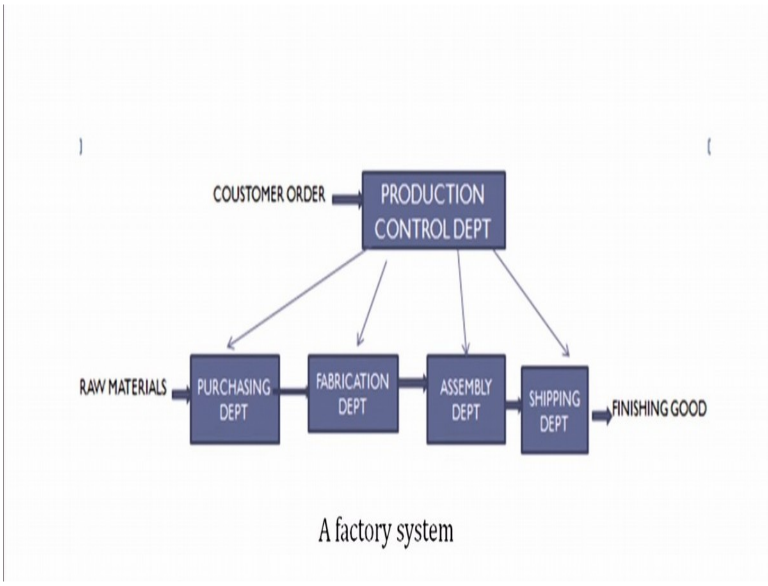

For example, if you have the production control department here and there are certain customer orders coming. then and from here raw material is coming so the customer orders can be communicated to these departments and raw materials comes to the purchase department then, it goes to the fabrication department after fabrication it goes for the assembly, and after assembly it goes for the shipping and of course in turn on what we get is the finished goods. So this is what we call it as the block diagram approach for the system modelling.

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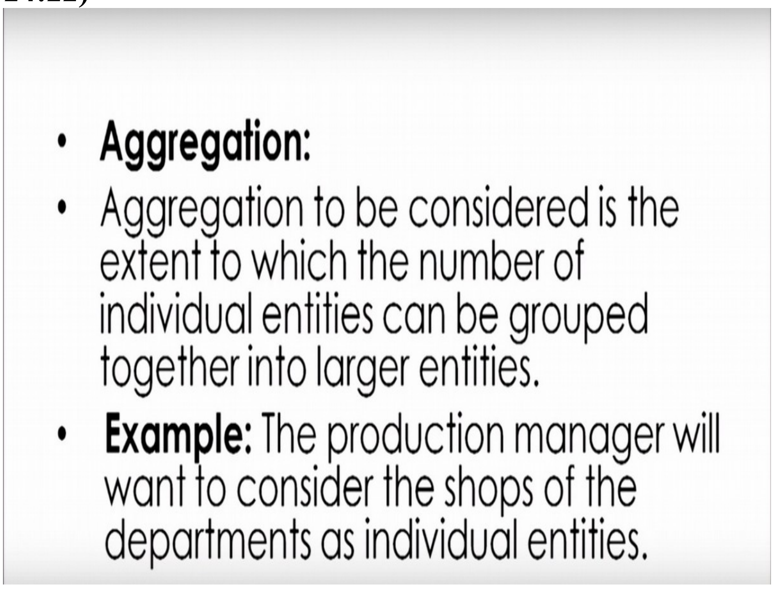

Next is the aggregation, so in aggregation, aggregation to be considered is the extent, to which the number of individual entities can be grouped together in order into a larger entities for example, the production manager will want to consider the shops of each department as individual entities and of course they can be grouped into larger entities.

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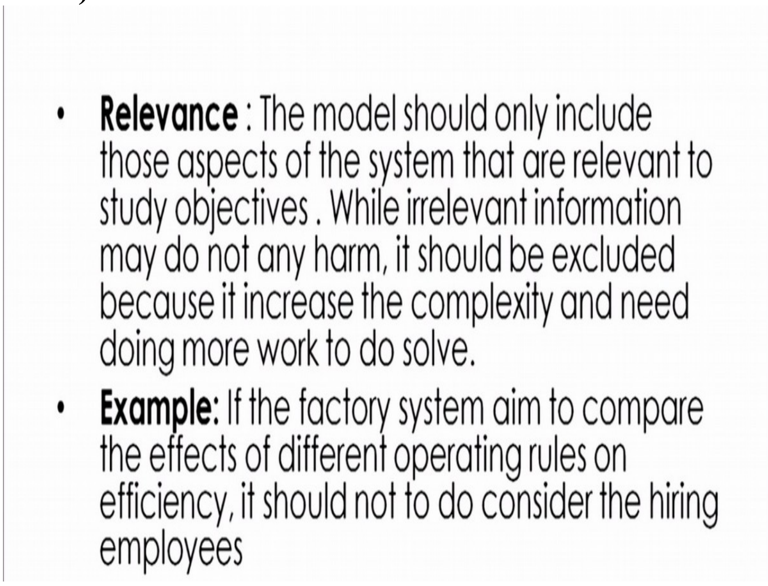

Next is the relevance, the model should only include those aspect of the system that are relevant to the study objective I have been emphasizing on this fact, again and again while irrelevant information may do not do any harm, it should be excluded because it increases the complexity and the need doing more work to solve it. So that it is what is while modelling we have to consider this factor into consideration that is the relevance.

That, is what to consider and what not to consider, for example, if the factory system aims to compare the effect of different operating rules on efficiency it should not consider the hiring of the employees so the hiring of the employees may not be related with it. Next important factor is accuracy, the accuracy of the information for the model should be considered.

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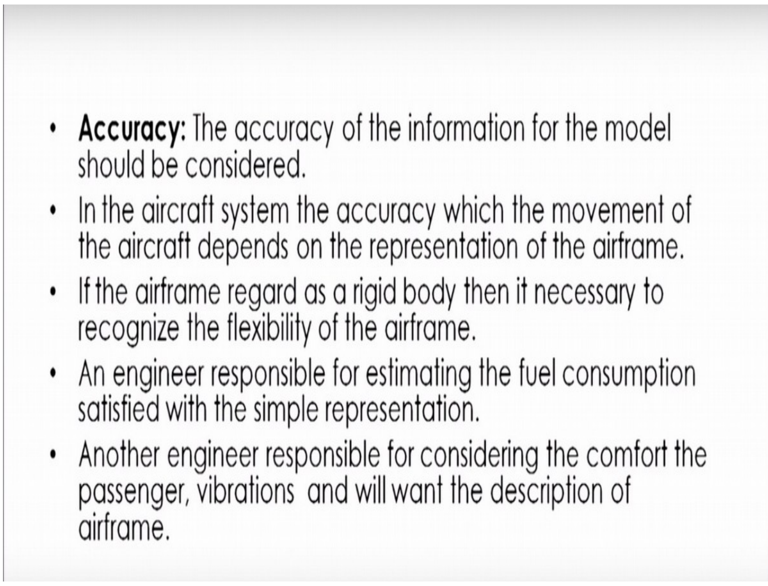

For example, in the aircraft system the accuracy the movement of the aircraft depends on the representation of the airframe. So, if the airframe is regarded as a rigid body then, it is necessary to recognize the flexibility of the airframe.

For an engineer responsible for estimating the fuel consumption these assumptions of rigid body may be simple satisfied or simple representation we can say but, if another engineer is interested in knowing the vibrations or the comfort of the passengers then he cannot assume the airframe to be a rigid body. He has to consider the airframe as a flexible body. So, here for the comfort of the passenger vibration and description of the airframe here it needs to be considered as a flexible body.

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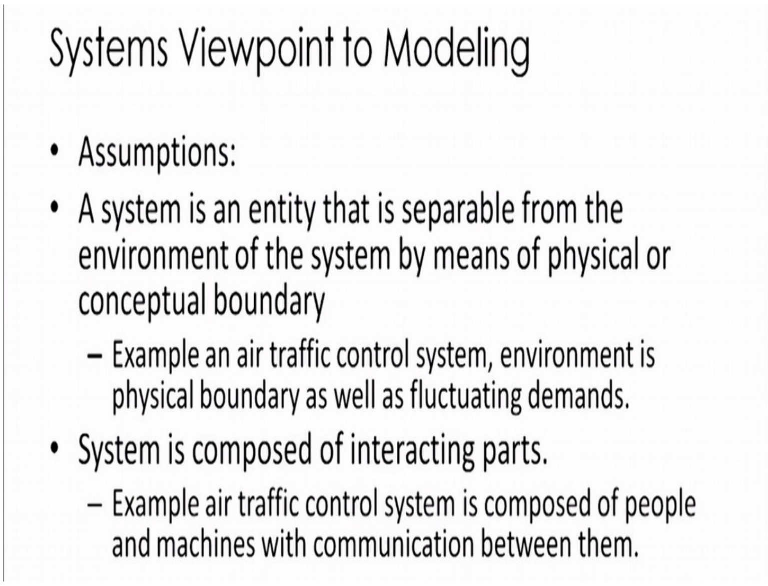

Now at last, here what could be the systems view point to the modeling. So after seeing the various types of models which could be made for a system, what could be the systems view point for the modelling. Now the systems view point for the modelling they are basically two very basic assumptions, which we need to consider always and these assumptions are that a system is an entity that is separable from the surroundings or the environment of the system by means of say physical or conceptual boundaries.

So, we isolate the system from the surrounding or it is possible to isolate the system from the surrounding for example, an air traffic control system environment is physical boundary, as well as the fluctuating demands that could also be the environment for the physical system and the second assumption here could be that the system is composed of interacting parts.

For example, air traffic control system is composed of people and machine with communication between them. So these are the two basic assumptions for the systems viewpoint of the modelling, the first one is that, we are able to separate the system from the surrounding so that, we can focus on the system, we can understand the system and the system is composed of the interacting parts, and these interacting parts can have energy interaction or other interaction between them. So with this I will close this lecture and next lecture we will see the modelling of the dynamic system. Thank You