Modelling and Simulation of Dynamic Systems
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Lecture No - 19
System model of Hydro Mechanical systems

In this lecture on system model of hydro mechanical system which is a sub model are molding and simulation dynamic system course which you're going through. So previously we have seen the various independent systems that are mechanical, electrical, hydraulic, pneumatic, and thermal. And the in this lecture will be wishing the combination the combination of mechanical and hydraulic.

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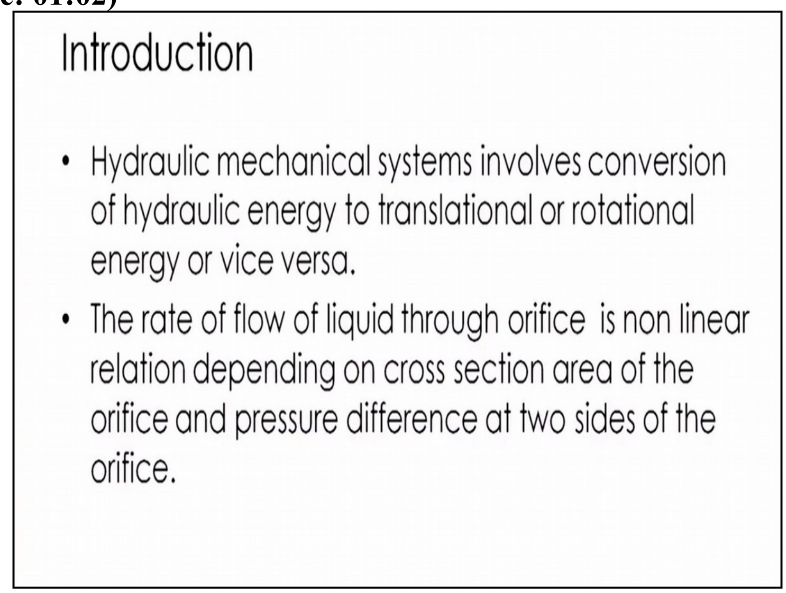

So hydraulic mechanical systems involves the conversion of hydraulic energy to translational or rotational energy or vice versa. That is we can both way hydraulic mechanical or mechanical to hydraulic. And while going to this as we have seen earlier ah when we are discussing about linearizing of the all in a equations that the rate of flow of liquid through orifice is non linear relation depending on cross section area of the orifice and pressure difference at two sides of the orifice.

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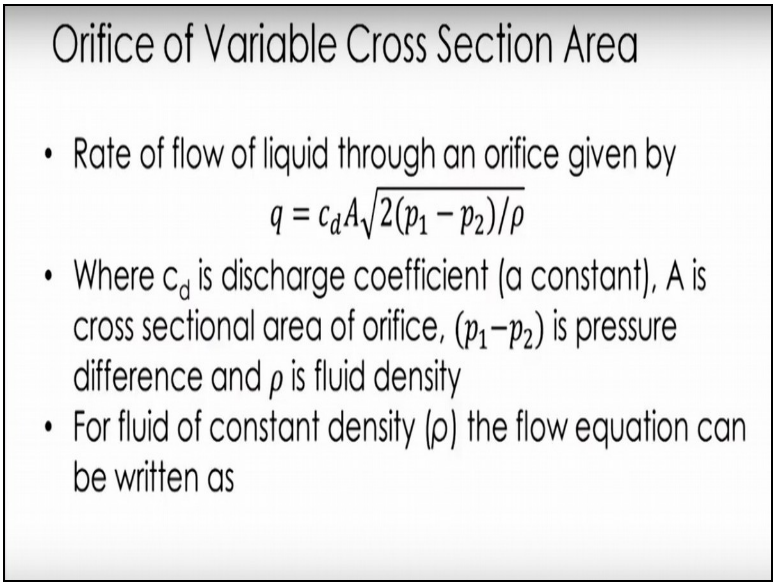

Now when we are talking about hydraulic system here the flow take place a form shape constant consumption area or variable consumption area. So we need to consider that here now, so in that some module on the linearising of the non linear equation. We have seen the case when we assume the orifice area to be constant. So here first we like to see if the orifice area is variable then how what is the relationship between how the discharge takes place.

And then based on this concept will build up a model for hydraulic servo motor so the rate are flow of liquid through on orifice is given by this equation here Q equal to c d that is the coefficient of charge as is the cross section area of the orifice and here p1minus p2 is the presser drop at the stream and the downstream side of the orifice and row is the density the liquid.

Here if we assume that this row is constant then we can be rewrite the equation and q is equal to c a and root p1 minus p2 Now here this c is a constant you can see that here q is the function of area of cross section of the orifice and the presser drop.

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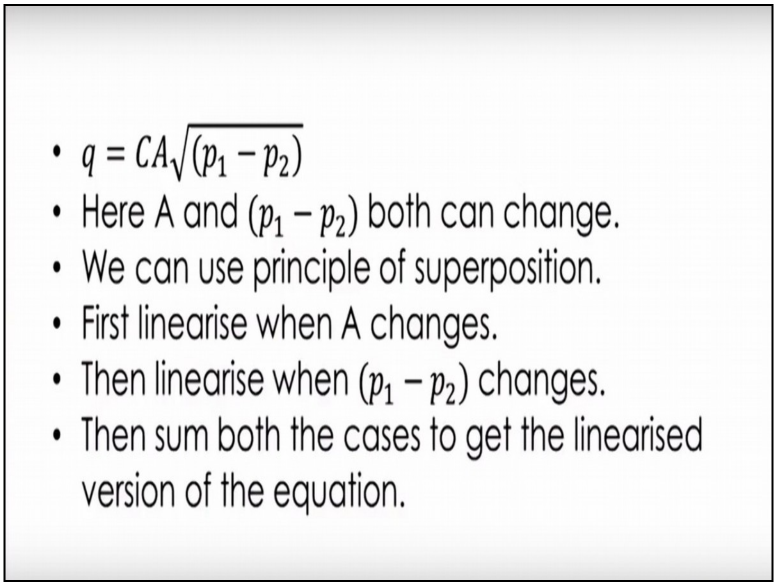

Again here as I said if we are going to consider the general case then both area of correction as well as presser drop both can change. So when both can change while linearising what we actual do is that now ah we using principal of super position using the principal of super position what we do firth we linear when only a changes and the linear when only p1 minus p2 changes.

And then we some both the cases to get the linear of the equation the principal of the super position is applicable only for the linearized version of this equation remember so this principal of super position is applicable only on the in version of this equation. So this way actually we get the effort the complain case that is the case then area of correction and the case in the presser drop is changing.

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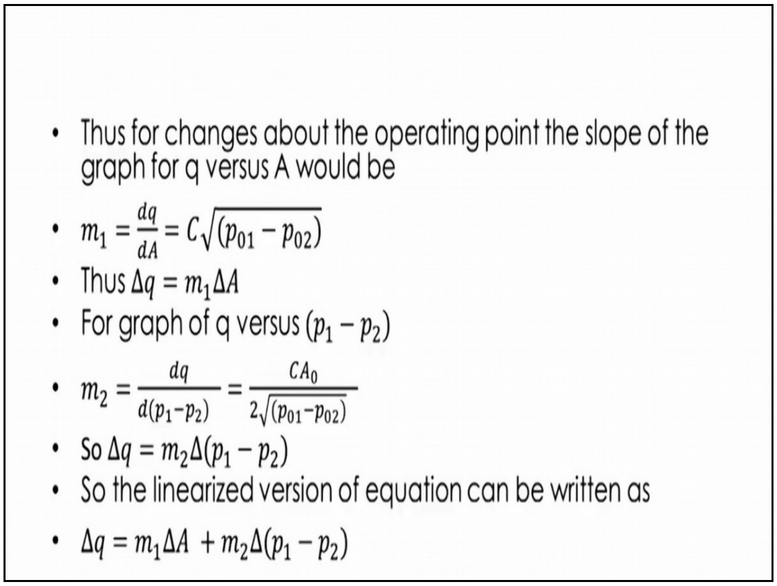

Now to see how it happened thus for changes about the operating point the slope of the graph of q versus a. So first we find out the slope of the graph of discharge area cross section and for this is going to be m1 equal to dq bi da and this will be c is rout of ah po1 minus po2.

They are basically these values are p1 and p2 or the values at the operating point the operating point which we are considering. So form here once I know the value of m1 m1 you can know if I see and find out these values that is the presser drop operating point once i know this i no see then I can find out m1. Now once I had m 1 I just can write del q equal to m1 del a.

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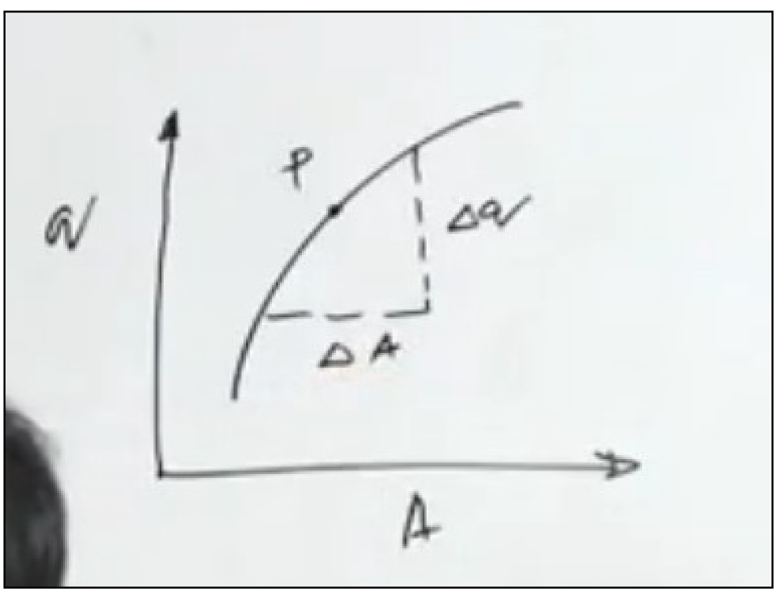

Basically a graph we have seen in case of ah nil in your equation chapter so this is your q this is a and it is nature of graph is something like is and here we have the operating we basically this is what am talking to you that is this is del q this is del a. So here ah our del q equal to m1 del a where m1 basically the slope of the line the operating point is this one. So similarly i can draw a plot for q verse is p1 minus p2 and for that I can find slope m2 is equal to dq append p1 minus p2.

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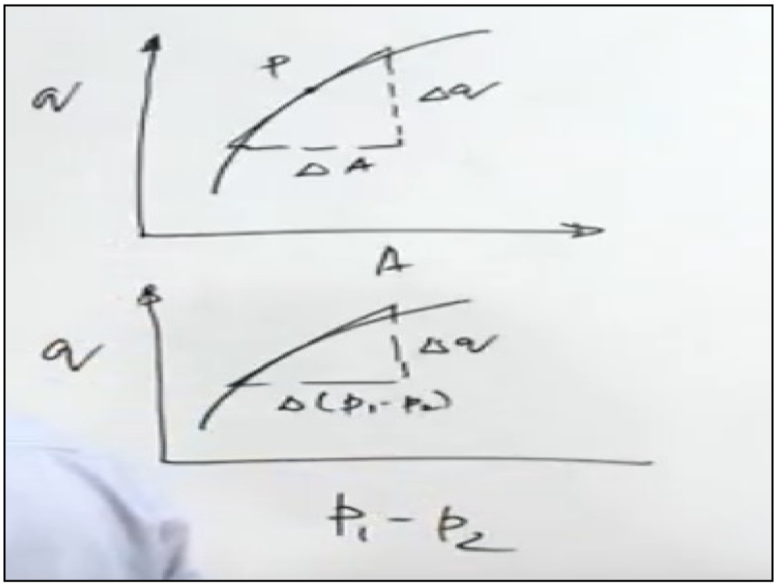

So that is again this case that is I plot q and p1 minus p2 here and this is my plot and so I can have something like this alright so this is my del q and this is my p1 minus p2 so form here if I know zero values and these values at the operating point I can find out the value of m2.

Then I can write the linearilised version of equation can be del q equal to m2 del a plus m2 del p1 minus p2 and then the linearised version for the combine is can be written as q equal to de m1 plus m2 p1minus p2. Here m1 is the slope first case and m2 is the slope furor the second case. Now next will be using this concept to see the derive the system equation the hydraulic servo motor.

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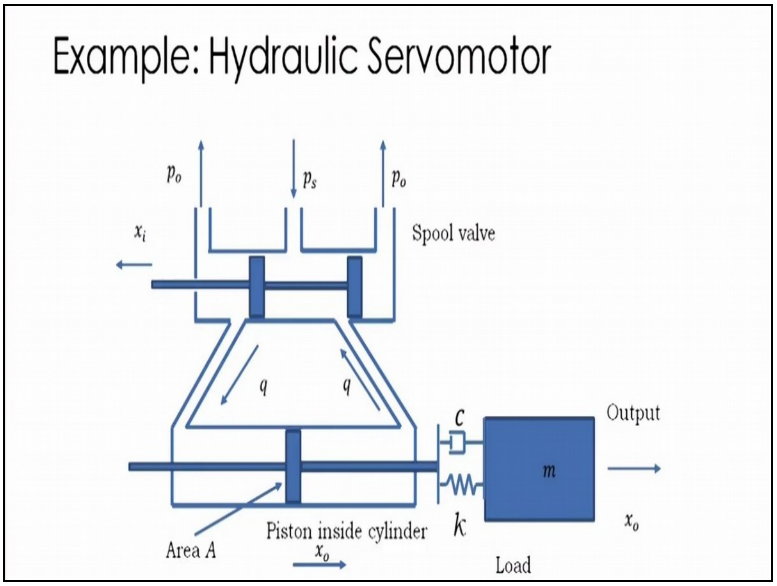

Here I can just describe the system have is spool valve here and this is spool valve is got is One input port say with pressure at ps and two output with exit pressure at po position of this valve can be operator with the help position of piston cylinder arrangement and say xi. Here represents the coordinate for the motion of this piston.

And here you can see that there is the piston inside the cylinder and the cross section area of this champer is a and of course because of flow of liquid this will be crossing the piston to move this direction or another direction and here is my mechanical load. You can see that there is a mass say m and say there is a damping present is represents by c and there is a certain stiffness is presented by K. So this whole thing is spring mass champers system basically represents the mechanical part of it and here we have the hydraulic part.

So this we call it as spool valve and of course I just say this is our load and the output basically this xo so our aim here in driving a system equation is basically to find out the relation between the output that is xo and the input xi. So this we can work it out and let us assume that there is the discharge q from these valves inlet valves and exit valve from here and the pressure this side is a p1 at the side p2 will further talk about.

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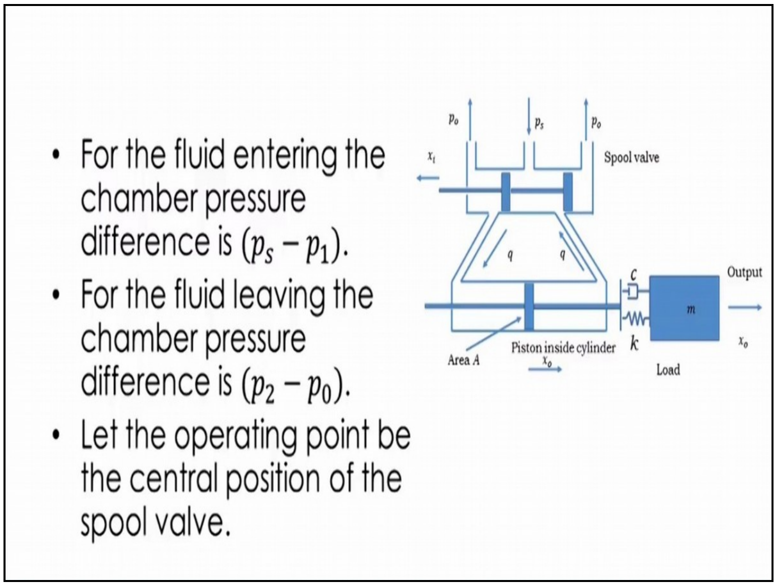

So now here you see that for the fluid entering the chamber pressure what is going to the pressure difference so i just that decide present to be p1 so the pressure fluid entering this champers is ps Minus p1 and the for there fluid leaving the chamber the pressure is p2 minus po so here this side the pressure is p2 and this is the exhaust pressure is po o basically of so here the pressure drop is p2 minus po. And let the operating point the central position of the spool valve.

Now at this position that is the central position and here the ports connecting to cylinder are both closed. The central position these ports are closed so for this condition the q is going to be zero. So whatever be the delta q are the change will be basically q minus 0 or it will be q. So the area of cross section of orifice as I said here because when this piston will be moving the area of cross section will keep on changing here. So the area of cross section of orifice is proportional to basically this xi if xi Messer form the central position.

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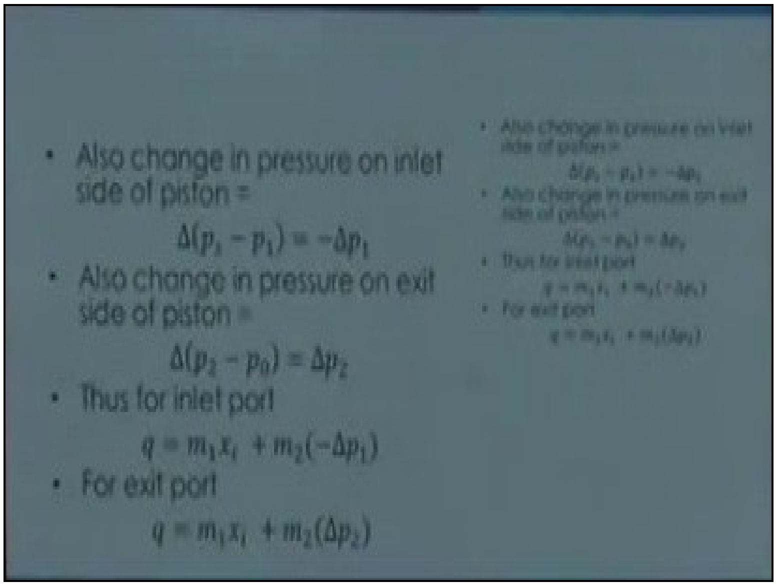

Now also change in pressure on inlet side of the piston is delta p s minus p1 and this I can call it as minus delta p1 assuming that is constant. And the change in the pressure on the side of the piston is del p 2 minus p0 and i can take it on del p2 assuming that p0 is constant. Thus for the inlet port i can write this expression basically which this expression.

We are getting from this generalized which we have derived basically that is m1 into del a plus m2 del p so here i can write for the inlet port the q equal to m1xi plus m2 into del p0 minus del p1 for the inlet side similarly exit port I can write q as m1xi plus m2 into delta p2.

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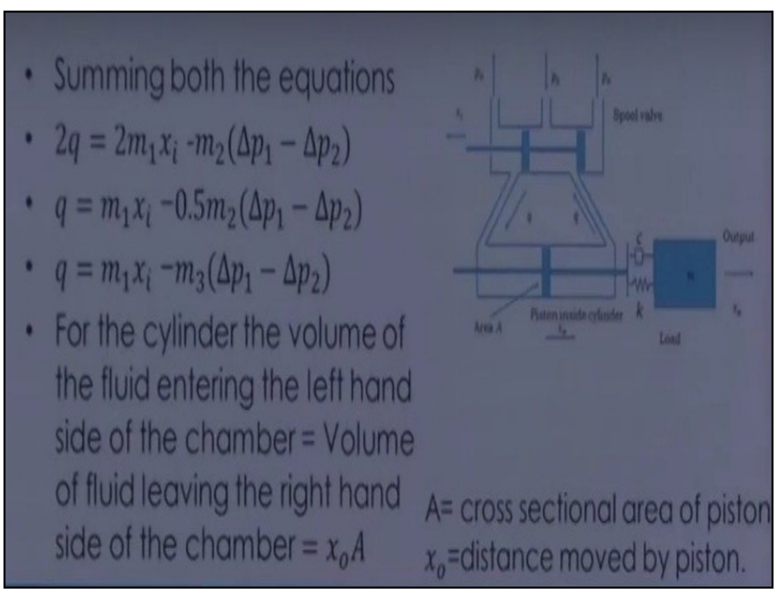

So this way i can write this expression and when sum up both the equations then this is what get to q equal to 2 m1 xi minus m2 del p1 Minus Del p2 are here q will be m1 xi divided by minus point 5 to del p1 minus del p2 are I can write q I just keep this thing as m other constant I write it as say m3 so m1xi minus m3 delta p1 minus delta p2.

This is the expression for q. Now for the cylinder the volume of the fluid entering the left hand side of the chamber equal to volume of fluid leaving the right hand side of the chamber. And this volume will be basically x zero into a where zero is the piston motion and is the cross sectional area of piston and x zero is the distance moved by piston here this piston. so what will be rate of change of volume we can differentiate this ax0 so a it will be a dx by dt and the rate at this the Fluid left side of cylinder that we already take as q. suppose there is some leakage ql to the piston.

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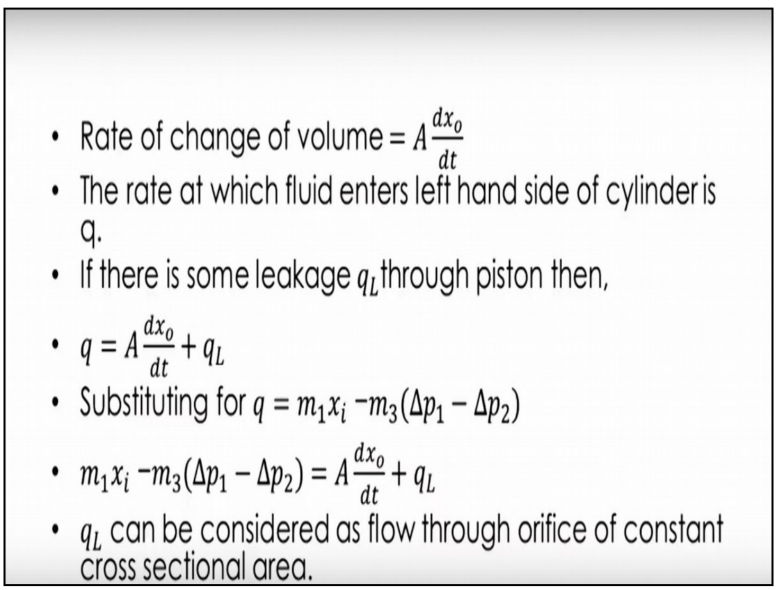

So in that case what happens basically this q will be having a dx zero by dt plus will be having a term ql. So this ql actually something like this say we have a cylinder and we have a piston here so ql is basically taking from this flow taking place from here.

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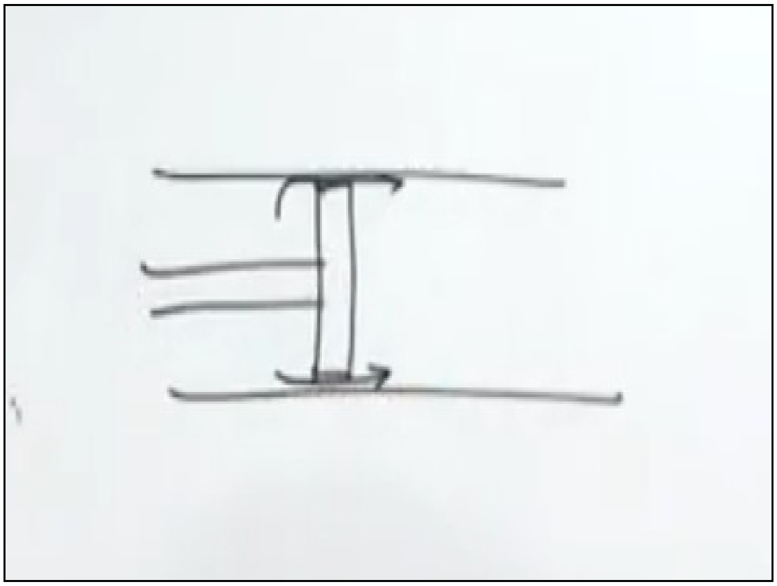

So that q will be a dx zero dt plus ql and substituting for this q this expression. What we get m1 xi minus m3 del p1 minus del p2 is equal to A dx0 is divided by dt plus ql we have already drive this expression for q here what I am doing here I am substituting of this value q into this equation from here we got this equation we get ql. Ql we consider here i telling you of constant cross section because this cross section we can treat to be constant.

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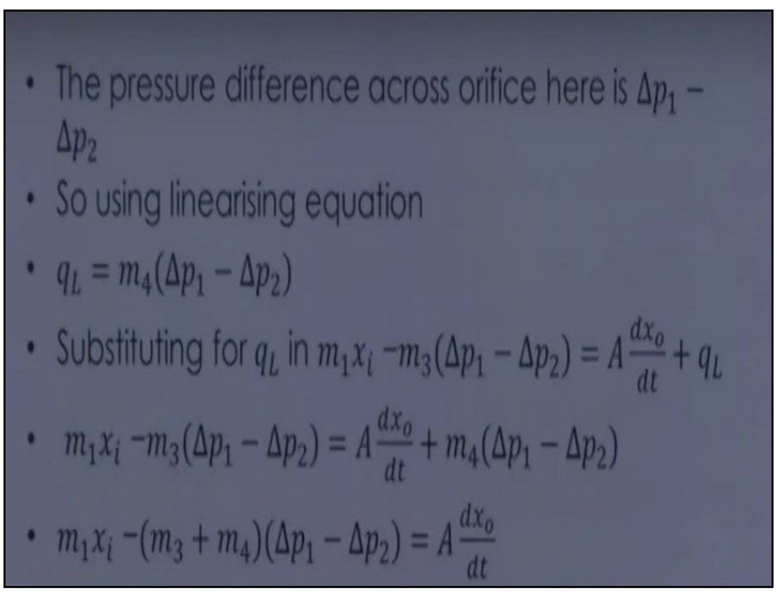

So if we do that the pressure difference will be del p1 minus del p2.because the pressure of del p1 is this side and pressure of that is del p2 i that side so if we linearise the equation for constant cross section area cases this q leakage will be m4 into del p1 minus del p2.

Where m4 is again the slope of the graph for ql versus del p1 minus del p2 will be substituting here the equation q l is this equation we can putting here so what we have is basically m1 xi minus m3 into del p1 minus del p2 is equal to A dx0 is divided by dt plus m4 del p1 minus del p2 plus this ql im substituting here m1xi minus m3 plus m4 is bring it together del p1 minus del p2 is equal to A dx0 is divided by dt .

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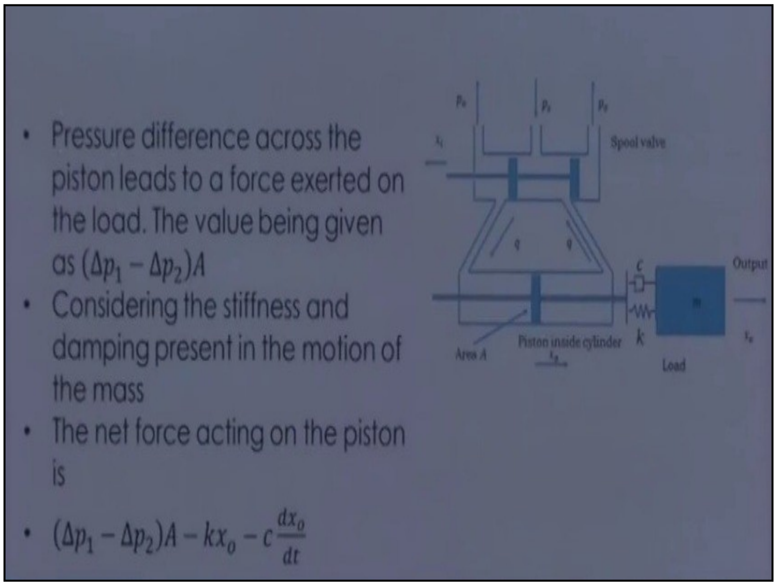

So the pressure difference across the piston on the load the value of this will be del p1 minus del p2 into the area of cross section here this side the pressure is del p1 and this side the pressure is del p2 this pressure difference is multiplied by basically giving you the force. And considering the stiffness and damping present in the motion of the mass the net force acting on the piston we can find out the force this which will be accelerating the mass.

This is the hydraulic force basically minus because of the stiffness presents and minus force because of the damping force in this system.

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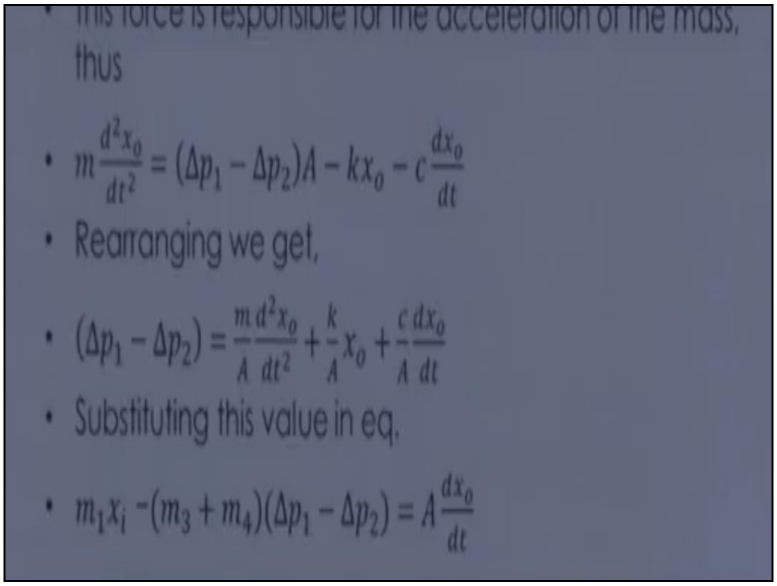

So here I just said this force is responsible for the acceleration of the mass. So this net force I can write as mass into acceleration and here this is you can see the second derivative of the xo. Now we can rearrange this we have del p1 minus del p2 is equal to m is divided by A d squared xo is divided by dt squared plus K is divided by A x0 plus C is divided by A dx0 is divided by dt.

And now we can substitute this value we derived the substitution here let us see we can do the substitution here I am going to substitute for del p1 minus del p2.

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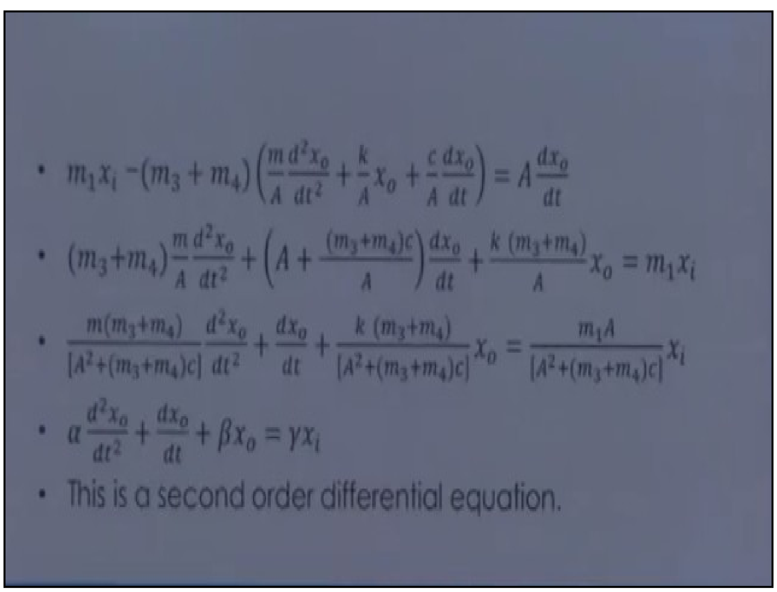

If I do this is what I get so m1xi minus m3 plus m4 this is the substitution for del p1 minus del p2 from here into this equation so K is divided by A x0 plus c is divided by A dx0 is divided by dt is equal to Adx0 is divided by dt. Now what we do we simplify the expression for the m3 plus m4 this is the substitution for del p1 minus del p2 from here into this equation for K is divided by A x0 plus c is divided by A dx0 is divided by dt is equal to Adx0 is divided by dt.

Now what we do simplify the expression for the m3 plus m4 m is divided by A d2x0 is divided by dt squared basically now I am writing the expression in terms of the coefficient of the derivative. So here the coefficient of derivatives x0 is divided by dt 2 plus is this one here I am just segregating the term coefficient of dx0 is divided by dt here.

So this plus a this term and this K m3 plus m4 is divided by A into x0 is going to m1 xi and further I can simplify ah this and to do that simplification is what I do here simplify this term and then multiply and divide this term so divide whole expression by this term A squared plus m3 plus m4 into c and of course I have m3 plus m4 into m here a get cancelled basically from here one A from here and one a from here a get cancel so this is the coefficient of the second derivative term plus dx0 is divided by dt plus.

We have the coefficient into x0 here this coefficient into xi. And further if I write these term I give some name for Alfa d squaredx0 is divided by dt squared plus you can see here this is dx0 is divided by dt plus write this beta x0 and say plus this is gamma xi.

So you can see here what I can get here get A differential equation of second order this equation is basically differential equation I get here the relationship between the input displacement x1 and the output xo and of course there are various coefficient alpha, beta, gamma and this coefficient can be evaluated basically here m term we have m3 and m4 terms be there and this way we can work it out this one and here basically the mass and m1 m2 m3 and m4 are represented somehow the slopes.

Which we have drive the linearising the expressions. So with this I would like to close this ah sub chapter and

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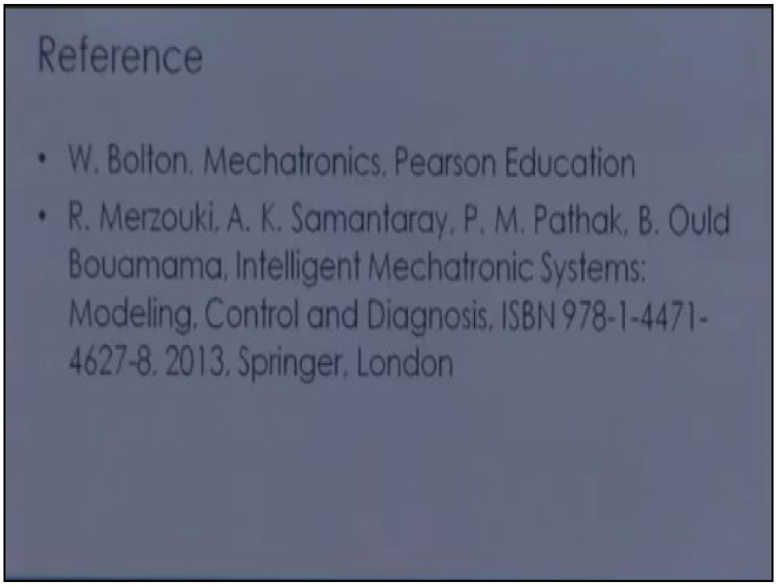

You can refer Bolton Mechatronics and our book Intelligent Mechatronic system: modeling, control, and diagnosis for the reading. Thank you.