Modelling and Simulation of Dynamic Systems
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Lecture No - 18
System model of electro mechanical systems

I welcome you all in this lecture on system model of electro mechanical systems and this lecture is part of our course on modeling and simulation of dynamic system. So today basically will be looking at the electro mechanical system and as the name indicates that it is going to be a combination of electrical system and mechanical system. So will be seeing that how can be combine that two systems electrical and mechanical system.

Some combination of this we have already seen in case of bond graph modeling where we are combined this systems with the help of power points and here will be seen that how can we derive system equations for electro mechanical systems.

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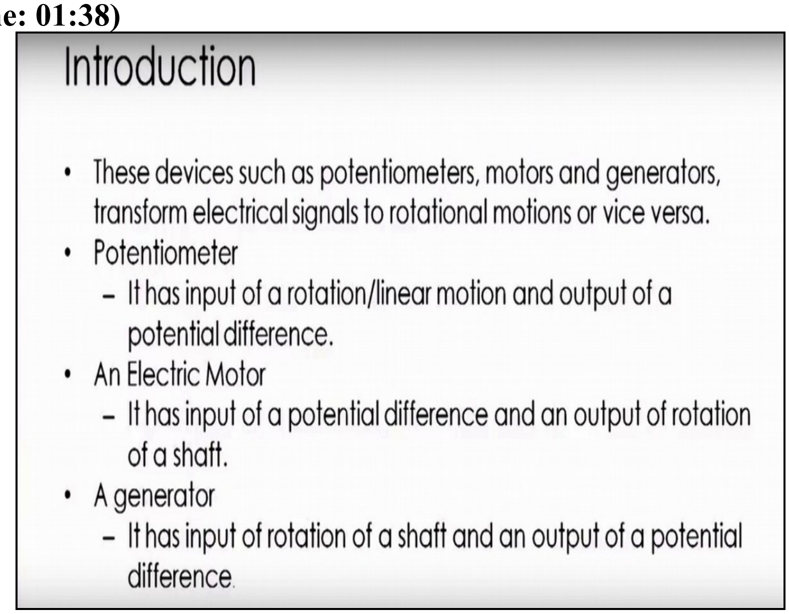

So, these electro mechanical devices are such as potentiometers, motors and generators and they transform the electrical signals into rotational or translational motions and vice versa. So this is what they do. The potential meter for example, it could be rotary potentiometer or it could be linear potentiometer and it has input of rotation or linear motion and output of a potential difference. So here also we have the mechanical and electrical system being combined.

Then if we take example of electric motor and it has of input of potential difference and output of rotation of the shaft. So that is there and the generator which the reverse of electric motor it has got rotation of the shaft has input and the output as the potential difference. So, here in this lecture, I will be discussing about the first two examples. There are many more examples, which you can refer through some of the extended text books.

So first let us take the case of potentiometer. Now we know that potentiometer is used for measurement of rotation or translation. These devices are very much useful in low cost servo motors where we want to major the rotation motor of the shaft. So there these devices are used most in the robotic applications. So here I will be just talking about a linear potentiometer.

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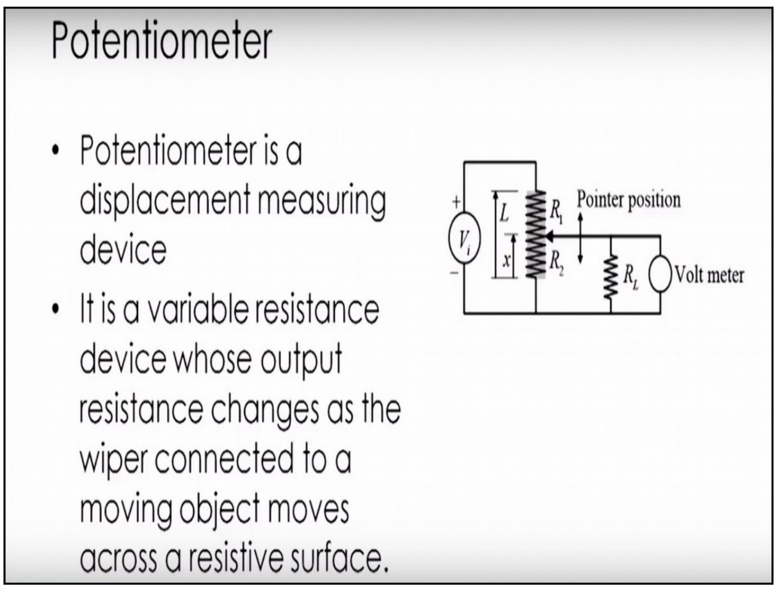

The potentiometer as I said is a displacement measuring device and it is a variable resistance device, whose output resistance changes as wiper connected to a moving object moves across a resistive surface. So here you can see that as this wiper moves across this rustle surface, the resistance changes and based on theses linear motion of this we get the electrical signal or we get the voltage here.

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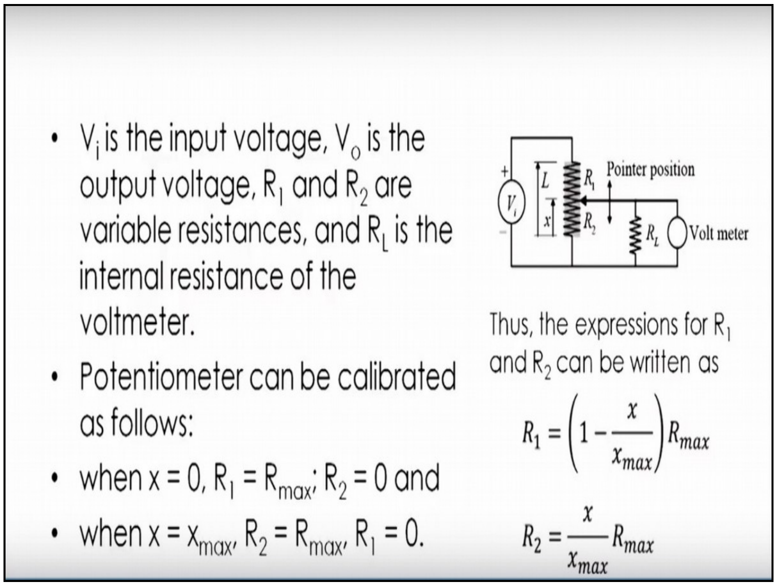

So, in this figure you can see that v i is the input voltage v o which is not return. Here v o is the output voltage and r 1 is and r 2 is variable resistances. And r l is the internal resistance of the voltmeter which we usually take to be of very high value. Now we can calibrate a potentiometer as follows. when x is 0 when this x is 0 r 1 is r maximum because this pointer will be at the position. So our r 1 will be r maximum and at that position r 2 is going to be 0.

Similarly when x is x maximum and that is pointer at these place r 2 going to be r maximum and r 1 is going to be 0. This one is going to be 0 because this x going to be at the maximum position. So these type of thing mathematically we can express this two condition using these relationship. R1 equal to 1 - x by x max into r max and r 2 equal to - x by x max into r max.

So you see here when you are going to put x equal to 0 you are going to get r 2 equal to 0. That this condition as well as when you are going to put x is equal to 0 here you are going to that r 1 equal to r max.

Similarly here when we are going to put x equal to x max here you are going to get r2 equal to r max and when you are going to put r 2 equal to that condition when you are putting x is equal to x max this system will be 0. So will be r1 0. So, basically with the help of these two equations we can calibrate the potentiometer.

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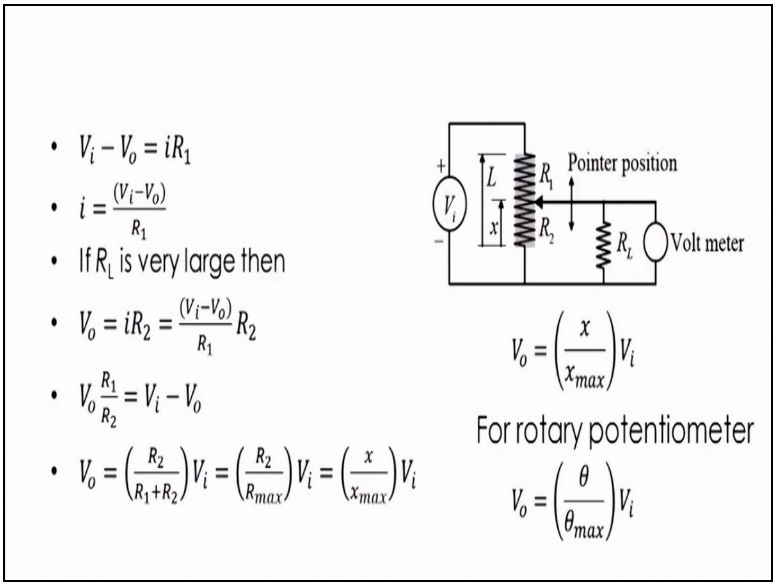

Now here you see that these voltage is v i and of course the voltage here is going to be v o, output voltage so if I apply the Kirchhoff's law here so this v i – v o going to be the i r 1 here. So I can find out the value of i from here that is v i - v o upon r 1. Now if we take r l that is resistance of the voltmeter at very high as I said in the begin itself. I can write v output v o as i into r2 or I can substitute for this i this is going to be equal to v i – v o upon r1 into r2.

What I do here I write vo into r1 by r2 equal to vi - vo. That is I simplify these and these terms here. And form here you can see that I can write expression I can basically take this two left hand side and I can write take expression for vo as r2 by r1 + r2 into vi . And these r2 r1 + r2 is the maximum value of resistance .So r1 + r 2 I just write here r max. So this is r 2 by r max into vi and we have already seen that r2 by r max r2 by r max here is basically x by x max.

So I can substitute this value in my result. So I get vo equal to x by x max into vi. So this is my output voltage and this is equal to x by x max into v i. So now depending on the value of x I will be getting the output voltage. And this is how the potentiometer works. So basically the linear motion x is getting converted into the output voltage v o. And x by x max I know the distance maximum up to which i pointer can move to the extreme values. So this value is also known and for given this value I will be getting and proportional voltage.

So this is how we can explain the potentiometer. Now if your potentiometer is the rotary potentiometer I told you which are used in the low cost servo motors. The rotary potentiometers are used so here instead of this x we are going to have the rotary quadrant that is the theta. So v o equal to theta by theta max into v i. So this is how the rotary potentiometer can be explained or for the rotary potentiometer the angular rotation of the potentiometer is can be measured in terms of the voltage.

Next example I would like to take that of the dc motor. As here as all of know in mechatronic system electric motors are used as actuators. So you take example of any mechatronic system whether it is your photo copy machine or any Robert or washing machines or any mechatronical system motors or use as the actuators. And there are mostly used in position or speed control system.

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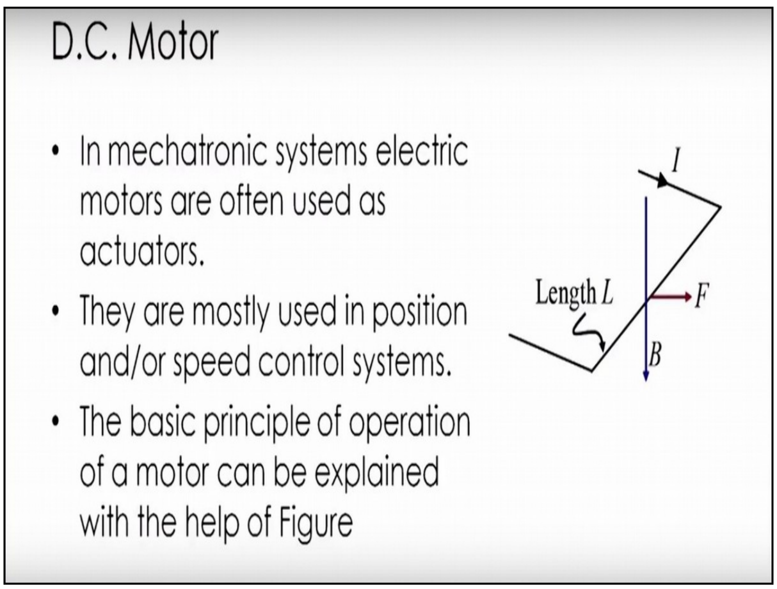

The basic principal of motor can be explain all of you studied in your school as well this basic principal that is current carrying conductor when it is places magnetic field there is the force on the conductor.

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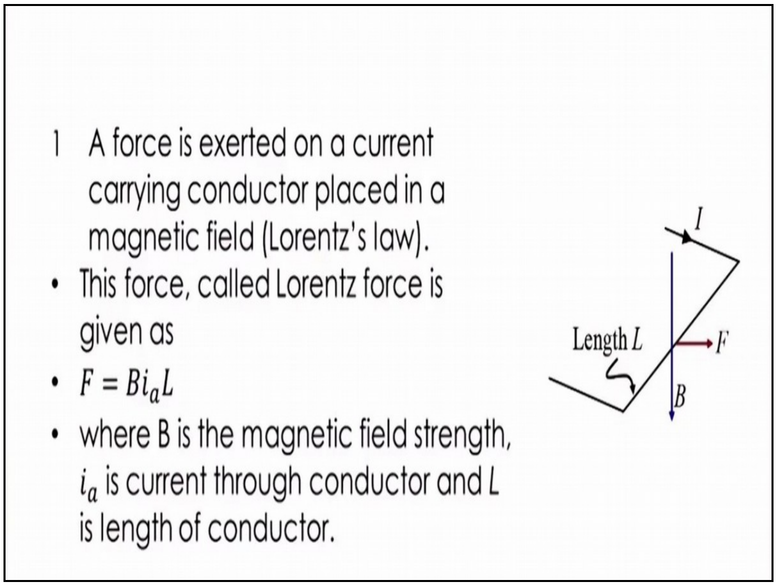

So a force exerted on a current charring conductor placed in a magnetic field this is what we also call it as the Lorentz law. And this force called as Lorentz force is given as f equal to b i a l and this direction can be given by basically you right hand rule. So through right hand rules for a given dedication of the current field you can find out where force is going to act. So this magnitude of forces is bia into l, where b is the magnetic field strength, ia is current through conductor and l is the length of the conductor.

So this is the expression for the Lorentz forces on which is developed in a current carrying conductor in the magnetic field.

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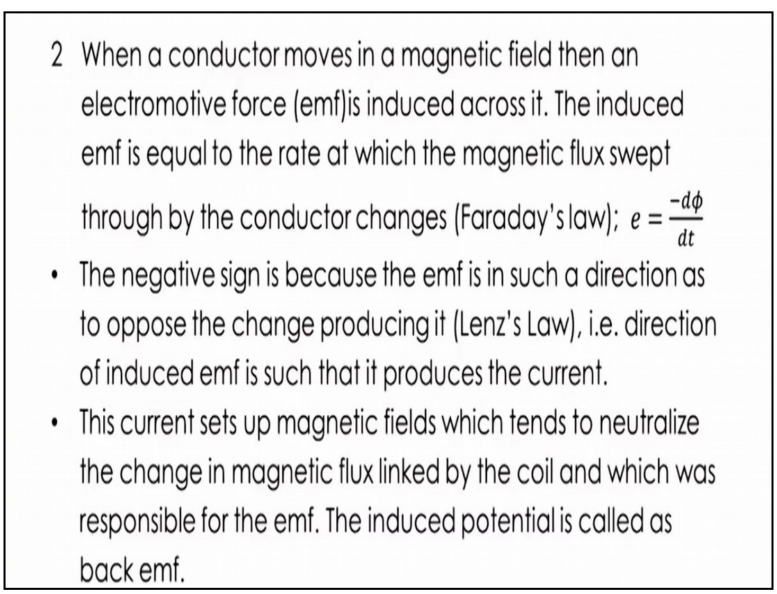

There is another law which is used in the modeling of the motors and that is what is the called as the faradays law. It is stated that conductor moves in a magnetic field then electromotive force (emf) in short is induced across it. An induced emf is equal to the rate at which the magnetic flux switch through the conductor flux shift by the conductor changes or in short the e is equal to d fi by d t.

And this - and you know which comes from the lens law which basically means that this direction is such that are as directions which are opposes the cause the generating in or the negative sign. Because this emf is direction is such as oppose the change producing it. That is the direction the induced emf such that it produces the current. And the current sets up the magnetic fields which tend to neutralities the change in magnetic flux linked to the coil. And which was responsible for the emf and this induced potential is also called what we called it all as the back emf.

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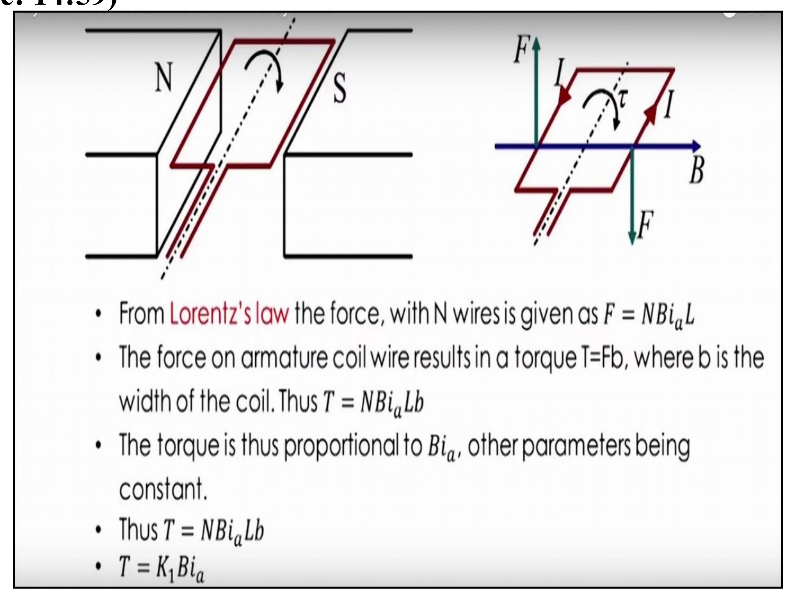

So, I take example of the magnetic field created by a permanent magnetic. Here the North Pole and the south pole of the magnetic and here is the current carrying conductor is placed. Now the Lorentz force law we can find out the forces if there are such wires here that will be just multiplication by n and of the forces which was generated in the single conductor.

So it is a times bia l and of this width of the conductor is be we are going to of torque and the torque is equal to f into b. Because you can see this force and this force the opposing direction.

So let between them and they are going to turn your conductor. So, that torque is f into b where it to be with of the coils. As thus torque will be basically becoming nba l into b. Now if you look at this expression you can see that torque is propositional to bia.

Any numbers of n the conductor of coils is fixed here l is fixed b is fixed because of these are the length parameter. Numbers of coils basically this torque is proportional to b and ia where the b is the magnetic field and ia is the current through conductor. So we can put these are mlb some constants k1 and are the remaining to the contest right at b ia.

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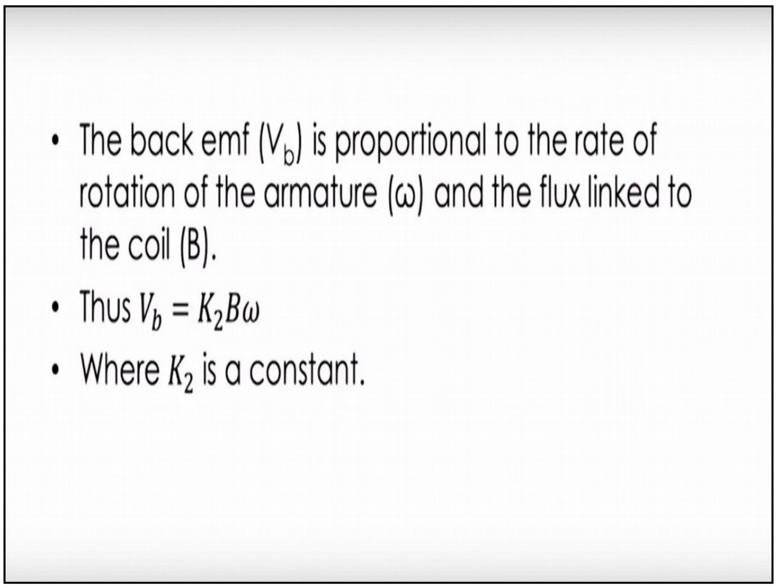

Now using the Faraday’s second law which we have seen here. The first law as the Loners law which is responsible for the Loners forces. And second law from the second law we can see that the back emf is proportional to the rate of to rotation of the armature that is omega flux leaving to the coil that is b. So this v b ho returns as from constant in to b into omega here this k2 is constant.

Now I just so your torque going to a function of b and ia so based on up what you are going to change. You can have the torque and you can get the torque both ways by keeping ia constant varying b and varying up ia and keeping b constant.

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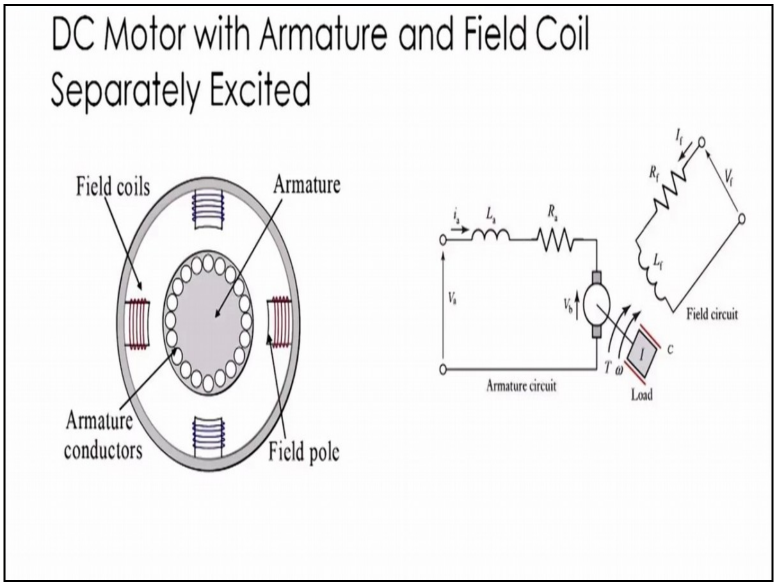

So let us see dc motor with armature and field coil separately excited so this figure shows you the dc motor this is the armature and this an armature conductor through which the armature the current passes. And here we have the field poles and a field coils are there and are this coils and responsible for the generation of the magnetic field b. So here I can just draw is schematic form of this motor here.

So the field coil as I was telling you we have a field circuits here and the voltage supply to the field coils v f current through the field coils i f. Here we have the field coil has got a as certain inductance l f and it is got a certain resistance rf. And this shows the schematic for armature circuit so the armature conductor which is here this conductor will have certain inductance. So la basically the cases of this used here the armature and the f is used for fields.

So this conductor for all are the inductance la and have the resistance of ra and the current I a is passing through the voltage of the v a supply is across the armature the terminal. And you have the back emf be which in be generated that is v b and after these you have the mechanical element so you have shaft here and end of the shaft you can load alright and this load of a the that is an environment is the there is Bering of and that Bering of the generating certain Bering reaction.

Bering torque we can approximate by concentrating with a tamping behavior that equal to tamping constant in to the rotation of the shafts. Now that I was telling you that the torque generator by motor function of two thinks the b and i a.

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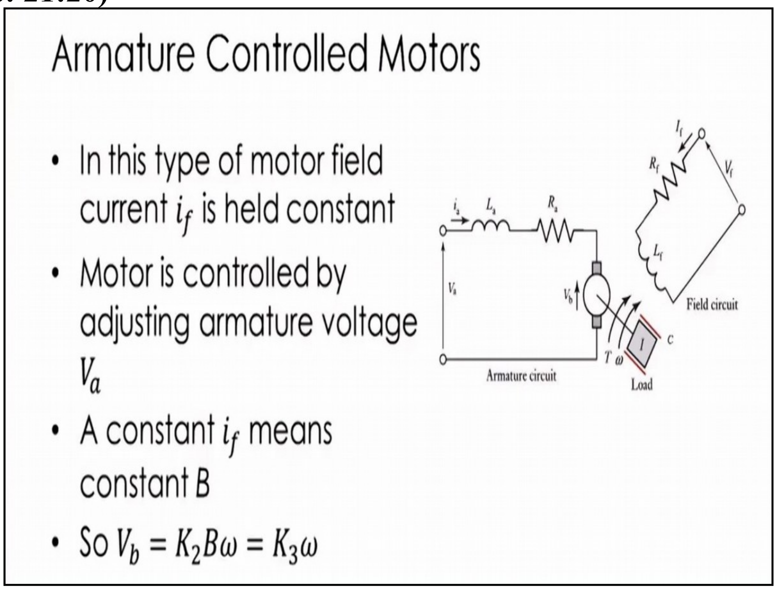

So let us see the first one armature controlled motors now in this type of motor is field current if is held constant so we are keeping this if as fix and the motors is control by adjusting armature voltage v a .so a constant if means that the constant b so as I said armature control motors were keeping the constant .so when you take constant if you want vary the torque ad you need to vary the I a that is the armature current. So here the back emf v b will be equal to k 2 times b omega r times as a said since b is constant I can give it other constant name k3 into omega so this is armature control motor.

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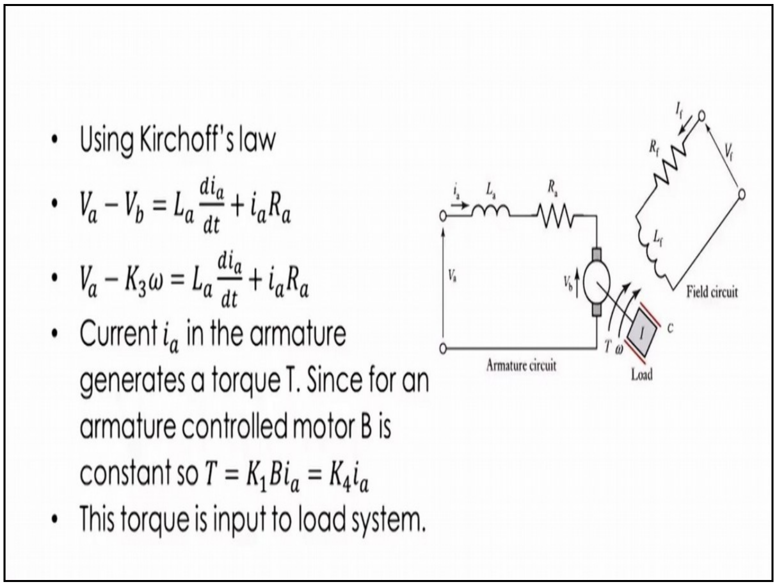

Now using Kirchhoff's law for the armature circuit I can write the v a - v b is equal to la d I a by d t + I a r a so this v b is the back emf and this is the voltage applied alright. So this is this will be the net voltage and this is going to be equal to la d i by d t + I a r a. Now here you see va known to us are this is the one which we are going to apply and vb we have just found out he k 3 into omega assuming b as constant. So here this is k3 omega and this la d i by d t + I a r a.

That current I a in the armature generates a torque since for an armature controlled motor b is constant as i am telling you in armature control motor since we assume that in the if constant so my b is constant so so since b is constant i can write t as k1bia i can take this b also as a constant so this basically k4ia and this torque going to be the input for my load system.

So if we neglect that tortional effect of the shaft of course if you want we can model that also but let us make the problem simpler i neglect the tortional effect of the shaft.

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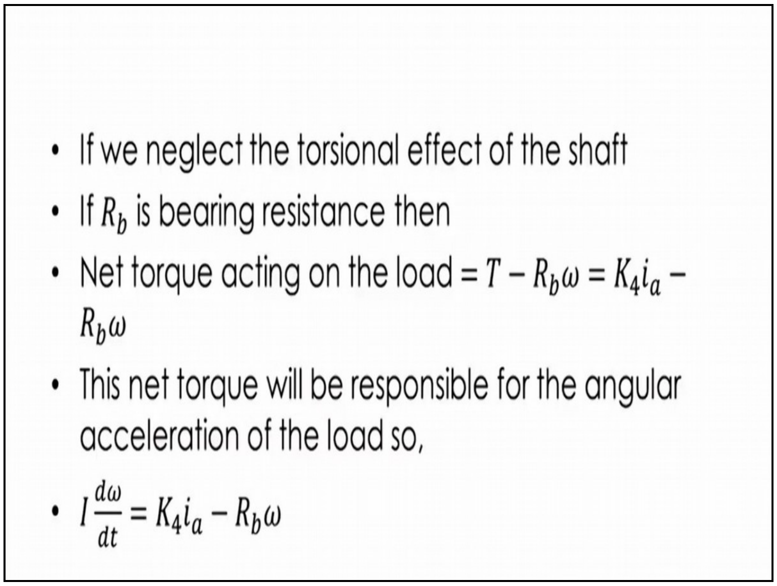

If I represent r b as bearing resistance then the net torque acting on the load is t - r b into omega here basically this are be replaces the see that I am coefficient which up show the previous if schematic figure. So the net torque acting on the load in the t - r b omega and i can substitute for this T that is k4ia - r b into omega and this net torque is responsible for the angular acceleration of the load. So this net talk I can write i d omega by d t because this net are going to be responsible for the angular acceleration of the load.

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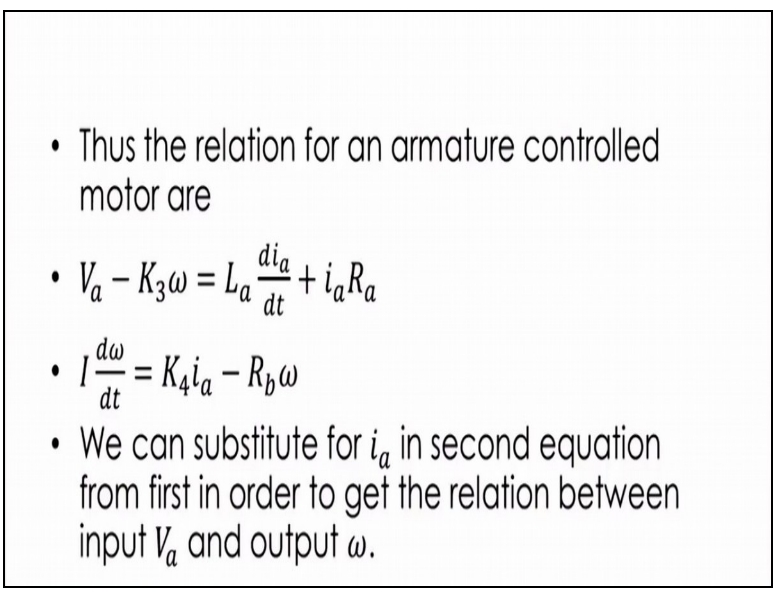

So the relation for an armature controlled motor could be this one so this is basically our what you can that combination of electrical and mechanical terms here also combination of electrical and mechanical terms. So here you can see that this omega is basically the rotation of the shaft.

So the form here we get the back emf so v a - this equal to lid by d t + I a r a and this is for the equation.

For the load that is the inertial force in equal to the torque of the motor - you whatever torque because of the bearings are the tamping top whatever of the want and from here I was telling you h we are more interested knowing here the relationship between the input v a output the omega. So from here if I can eliminate substitute i a from here then I can get equation in terms of equation v a on the omega.

But you can see that there equation are in terms of derivation. So a best way of doing that would be that we take the Laplace transform convert this differential equation into the algebraic equations and get the relation between the omega and v a. That we can always do and that I am not dealing and I am leaving that as an exercise to you.

Now let us see the before we proceed for there I just wanted to show to the power of bond graph and if I represent that is motor equation both this equation can be represented are this equation are model automatically using this bond graph model.

Here you can see this is armature portion here the inductance here of the armature the resistance of the armature and this is voltage supply and this end your getting the back mf so this is vb the back emf here and that of course equal to k4 into omega.

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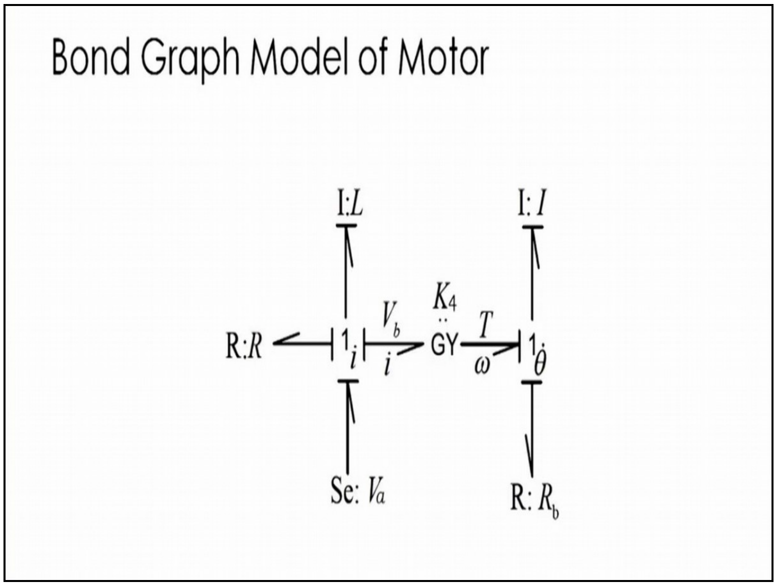

As per the gyrator relationship and here of course you have the load and here you have the bearing the resistance. So if you able to draw this bond graph you automatically get those two system equation one at which option in the previous line. next let us move to the field control motor so the previous case the previous case there we assume that if that is field current to be constant now in this type of motor armature current is held constant.

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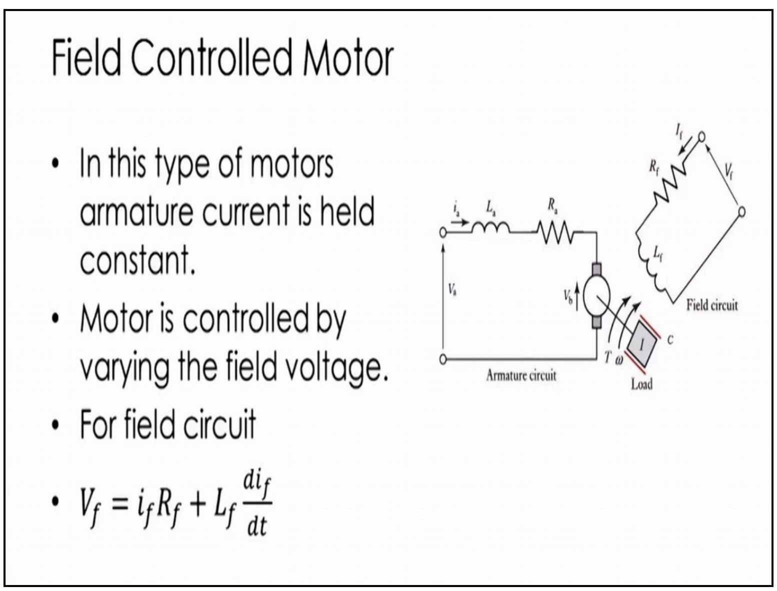

So basically armature current is held constant in this type of that is field control motor and this motor is control by varying the field. Voltage so here you can see for the field circuit again using Kirchhoff's law I can write v f equator that is total supply voltage v f equal to I f r f + l f dif by d t . And what field the current leads to production of a magnetic field the toque act set on the coil.

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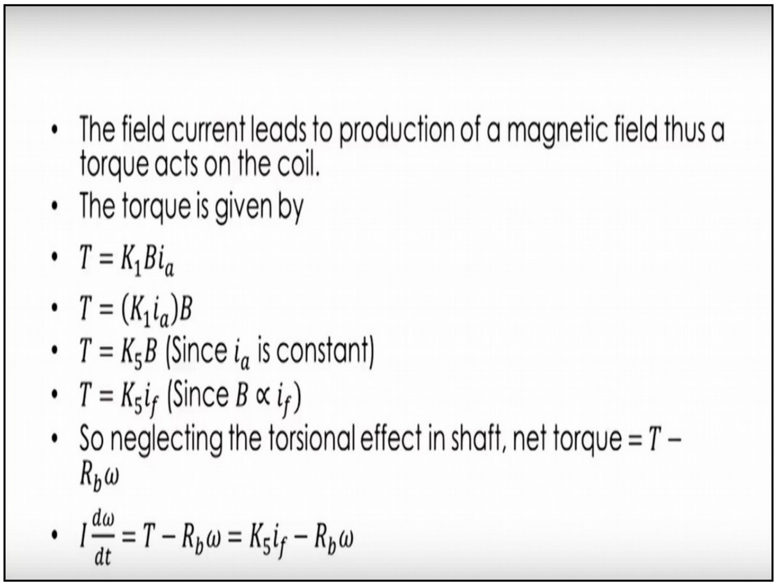

This torque given by k1 bia .now you see that i here since we are going to change the v because why we are going to change the v because here i am changing the current i am going to sorry i am going to controlled by the varying field voltage armature current is held constant. So here my b is changing and armature current I a held constant. So what I can do it that this k1ia I can make it constant into b.

So I give some name to it k1is is some k5 into b since I am assume the armature current to be constant here. Now this again this b is function of I f so more the if more the b going to the i reply this b with I f so this torque k 5 into I f. Now once we are go the torque expression neglecting the tortional effect in shaft the net are I can the write s t - r b w v is again the bearing resistance. So that r b w if I aha h subtract from the t I will have net torque.

And this net torque is going to be responsible for the acceleration of the shaft. so this it i omega by d t equal to t - r b omega are i can substitute for this k5ah if - r b omega thus the behavior of field current motor can be described as this two equation. That is v f equal to if r f + lf dif by d t this equation basically we wrote for the field circuit field coil circuit.

This equation is basically that is inertial force sorry inertial torque here that equal to that are generate - this is the what you can that the bearing force the tamping force are the reaction torque here because of the bearing alright.

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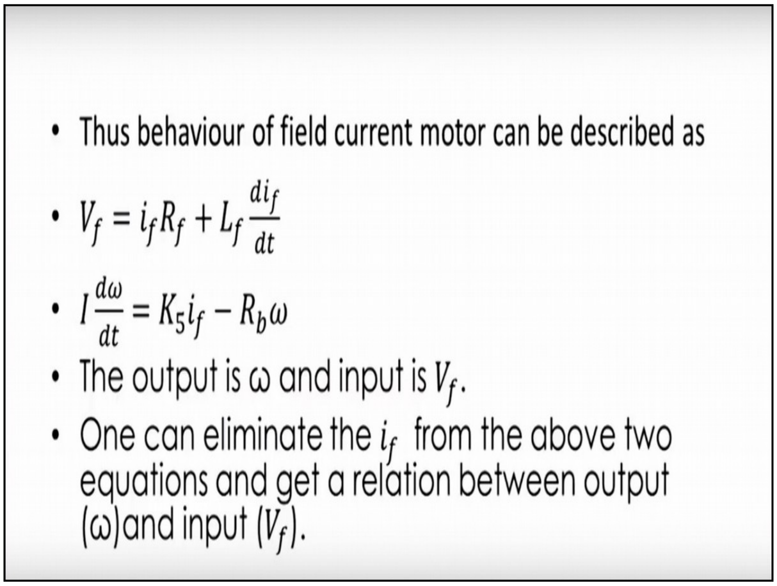

So the output here you can see that it is omega and input is v f .so again if I want to have the relationship between omega and the v f what we can do is that again we can take the Laplace transform and convert this differential equation into the algebraic equation and get it relationship.

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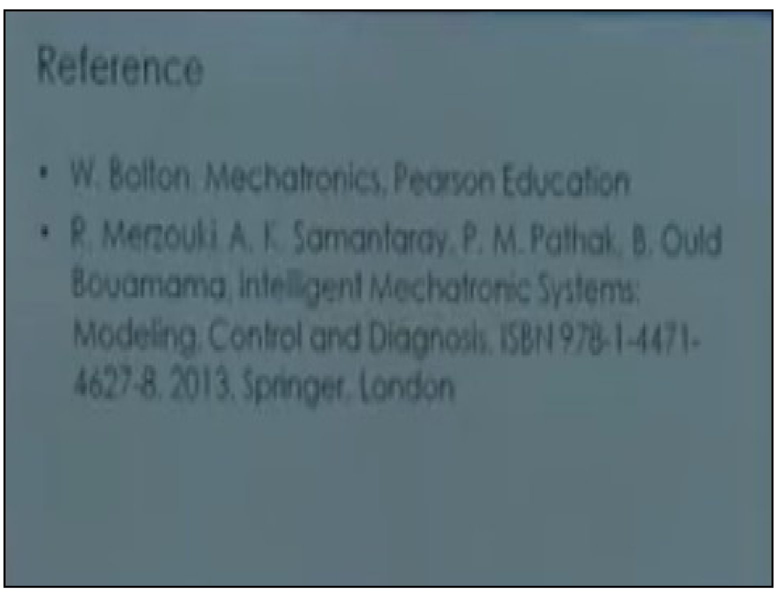

So these are the references if you want for the read it is our book that is intelligent mechatronic system and you can refer the W Bolten Mechatronic system from Pearson education. Thank you